Research article

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Convolution operations on time-domain digital coding metasurface for beam manipulations of harmonics

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Abstract: Time-domain digital coding metasurfaces have been proposed recently to achieve efficient frequency conversion and harmonic control simultaneously; they show considerable potential for a broad range of electromagnetic applications such as wireless communications. However, achieving flexible and continuous harmonic wavefront control remains an urgent problem. To address this problem, we present Fourier operations on a time-domain digital coding metasurface and propose a principle of nonlinear scattering-pattern shift using a convolution theorem that facilitates the steering of scattering patterns of harmonics to arbitrarily predesigned directions. Introducing a time-delay gradient into a time-domain digital coding metasurface allows us to successfully deviate anomalous single-beam scattering in any direction, and thus, the corresponding formula for the calculation of the scattering angle can be derived. We expect this work to pave the way for controlling energy radiations of harmonics by combining a nonlinear convolution theorem with a time-domain digital coding metasurface, thereby achieving more efficient control of electromagnetic waves.

Keywords: time-domain digital coding metasurface; nonlinear convolution theorem; time-delay gradient; nonlinear scattering-pattern shift.

1 Introduction

Considerable efforts have been invested towards controlling electromagnetic (EM) fields in desired fashions. Since the generalized Snell's law was proposed [1], space-domain metasurface [2] – a two-dimensional (2D) equivalence of a metamaterial [3–5] – was proposed to manipulate the propagation behaviors of EM waves by introducing controlled abrupt phase shifts associated with reflected and transmitted waves on the metasurface. Owing to the flexible and elaborate designs of meta-atoms and their arrangements, several physical phenomena such as holographic imaging [6] and orbital angular momentum [7] have been achieved. Recently, to simplify the design approach and realize digitalization, space-domain digital coding metasurface [8, 9] was presented, in which meta-atoms with binary phase states were introduced. For a 1-bit space-domain digital coding metasurface, only two types of basic elements with full reflection amplitudes but opposite phases are employed, denoted as “0” and “1”. For a 2-bit space-domain digital coding metasurface, digital particles turn into “00”, “01”, “10”, and “11”, whose reflection responses are characterized by four phase states of 0°, 90°, 180°, and 270°, respectively. Various functions such as random diffusions [10, 11] and anomalous beam reflections/transmissions [12, 13] have been realized by appropriately designing the coding profiles.

However, there is an inherent limitation of passive space-domain digital coding metasurfaces [14]; once fabricated, their scattering patterns are fixed. To overcome this limitation, a programmable space-domain digital coding metasurface [15, 16] that adopts active elements

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(e.g. varactors and PIN diodes) to construct dynamic coding elements was proposed to facilitate a variety of attractive applications, such as polarization converters [17], reprogrammable holographic imaging [18], and reconfigurable antennas [19, 20] by programming different coding sequences using a field-programmable gate array. Even though a programmable space-domain digital coding metasurface can provide rich and varied functions, it is still limited to a linear and time-invariant system depending on static phase distribution for regulating co-frequency waves, which cannot be used to generate and control nonlinear effects. However, optical nonlinear effects, which can be motivated by 2D materials with various exotic optical properties [21–28], have been widely used in super-resolution imaging [29], frequency conversion with greatly relaxed phase-matching conditions [30], and in all-optical switching and memories at the nanoscale [31]. Meanwhile, the nonlinear effects exhibit considerable application potential for 5G wireless communication and radar detection because these effects allow obtaining low/ultra-low sidelobe patterns with uniform amplitude unit cells [32, 33], realize beam scanning without phase shifters [34, 35], and achieve simple direction identification for the received signals [36, 37]. Hence, research on nonlinear effects in the microwave and millimeter-wave domains, and even the terahertz domain, would be significant.

To achieve nonlinear modulation, a time-domain digital coding metasurface [38, 39] that is not limited by Lorenz reciprocity was proposed to facilitate the nonlinear performances of EM waves, such as complete optical isolation [40] and frequency shifting [41], by introducing the temporal gradient phase; this phase provides new

approaches for realizing the scattering-pattern control of harmonics. Despite being one of the most important applications of the time-domain digital coding metasurface, there is a lack of systematic research on harmonic beam manipulations that focus on increasing the capacity of wireless communication systems by significantly decreasing the signal-to-noise ratio and realizing multi-channel transmissions. It is worth noting that Liu et al. proposed a linear scattering-pattern shift for the first time by performing convolution operations on a space-domain digital coding metasurface [42], thereby providing an efficient method to achieve the predesigned co-frequency scattering beams. However, their method cannot be adopted to control harmonic propagation behaviors, and currently, there is no literature available on nonlinear convolution operations for harmonic control.

In this paper, to the best of our knowledge, we propose a nonlinear convolution theorem based on a Fourier transform relation between coding and far-field patterns for the first time to achieve flexible and continuous control of the harmonic wavefront. By introducing a time-delay gradient into the time-domain digital coding metasurface, as shown in Figure 1, we can achieve a dual-beam harmonic scattering and steer it to an arbitrarily pre-determined direction, with negligible distortion to the shape of the far-field scattering patterns. To validate this theory, we first propose a broadband time-domain digital coding metasurface composed of active meta-atoms possessing tunable reflection states to realize nonlinear phenomena with high efficiency using phase modulation (PM). Then, two samples are provided to examine the performance of scattering-pattern shift for negative first-order harmonic

Figure 1: Illustration of nonlinear convolution operations on harmonics to realize a dual-beam scattering to arbitrary directions in the upper space.

considering nonlinear convolution operations on the proposed time-domain digital coding metasurface. All measured results agree well with analytical predictions. By combining the nonlinear convolution theorem with the time-domain digital coding metasurface, a flexible control of harmonic behaviors can be easily implemented in practice, including the manipulation of harmonic intensities, phases, and beam directions, thus paving the way for simplified and compact communication and radar systems in the future.

2 Theory and design

For the space-domain digital coding metasurface – based on the classical antenna theory [43] – the aperture field distribution $E(x_\lambda)$ and the far-field scattering pattern $E(\sin \theta)$ are related as per the following Fourier transform:

$$E(x_\lambda) \cdot e^{j2\pi x_\lambda \sin \theta_c} \leftrightarrow E(\sin \theta) * \delta(\sin \theta - \sin \theta_c) = E(\sin \theta - \sin \theta_c), \quad (1)$$

where $e^{j2\pi x_\lambda \sin \theta_c}$ describes the electric-field distribution with uniform amplitude and gradient phase along a certain direction, the electrical length x_λ is equal to x/λ (λ is the operation wavelength), and θ is the deflection angle with respect to the normal direction. Equation (1) implies that once an arbitrary $e^{j2\pi x_\lambda \sin \theta_c}$ at the aperture field is achieved, the scattering pattern $E(\sin \theta)$ can be flexibly and continuously deviated by $\sin \theta_c$ in an angular coordinate through convolution operations [42]. Furthermore, an anomalous scattering angle is defined as $\theta_c = \arcsin(\lambda/\Gamma)$, where Γ denotes the physical length of the period, and it can be calculated while neglecting the bit number of the space-domain digital coding metasurface, if Γ is set. However, the current convolution theorem is limited to linear time-invariant system [42], wherein only the static phase distribution on the aperture is considered while conducting beam steering; this makes it incapable of regulating harmonics generated by a dynamic system.

Hence, to achieve harmonic control using the convolution operation, dynamic modulations, e.g. amplitude modulation (AM) and PM in terms of time, have been introduced into the programmable space-domain digital coding metasurface. To achieve this design, a time-domain digital coding metasurface was proposed to adjust the wave amplitude, phase, and spectrum characteristics dynamically with the aid of digital signal processors [38, 39, 44, 45], thereby providing a novel method to generate and modulate harmonics. In contrast to programmable space-domain digital coding metasurface that employs

quasi-static modulation to regulate co-frequency waves, the time-varying reflection or transmission coefficient is adopted to provide freedom to control the harmonics with respect to time. Because an arbitrary-period time-varying signal can be expressed as a linear combination of functions $e^{jk\omega_0 t}$ ($k=0, \pm 1, \pm 2 \dots$), which are a set of orthogonal bases, the reflection coefficient $\Gamma(t)$ that changes over time can be given by

$$\Gamma(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}. \quad (2)$$

For the reflection-type time-varying system, when the monochromatic incident wave $E_i(t)$ is assigned as $e^{j\omega_c t}$ (ω_c is the carrier angular frequency), the relationship between the spectral domain and time domain of the typical dynamic reflection model can be derived as

$$E_r(\omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k E_i(\omega_c - k\omega_0) \leftrightarrow \Gamma(t) \cdot E_i(t) = E_r(t), \quad (3)$$

where $E_r(t)$ and $E_r(\omega)$ represent the time-domain and frequency-domain reflected waves; $\omega_0 = 2\pi/T$ and a_k respectively denote the modulation angular frequency and the coefficient of the k th harmonic component. From Eq. (3), it is obvious that the incident energy at the carrier frequency can be shifted to high-order harmonic frequencies ($\omega_c - k\omega_0$, $k = \pm 1, \pm 2 \dots$) due to the introduction of the time-varying signal $\Gamma(t)$.

To demonstrate this function, periodic time-varying signals shown in Figure 2A and B are proposed to generate harmonics. As shown in these figures, square-wave signals with oscillating reflection amplitudes (phases) and uniform phases (amplitudes) are used for AM (PM) of the incident waves. Therefore, by substituting the time-varying signals described in Figure 2A and B into Eqs. (2) and (3), the reflection amplitudes and phases of the k th-order harmonic aroused by AM and PM in the spectral domain can be respectively expressed as

$$E_{r-AM}(\omega_c + k\omega_0) = \begin{cases} (S\Delta A + A_2)e^{j\varphi}; & k=0 \\ \frac{\Delta A}{k\pi} \sin(k\pi S) e^{j(\frac{\varphi}{T} - k\pi S)}; & k = \pm 1, \pm 2 \dots \end{cases}, \quad (4)$$

$$E_{r-PM}(\omega_c + k\omega_0) = \begin{cases} Ae^{j\varphi_2} (Se^{j\Delta\varphi} - S + 1); & k=0 \\ \frac{2A}{k\pi} \sin\left(-\frac{\Delta\varphi}{2}\right) \sin(k\pi S) e^{j\left(\frac{\Delta\varphi - \pi}{2} - k\pi S + \varphi_2\right)}; & k = \pm 1, \pm 2 \dots \end{cases}, \quad (5)$$

where the duty cycle S is equal to τ/T , the amplitude difference ΔA is expressed as $A_1 - A_2$, and the phase

Figure 2: Calculated harmonic intensities/phases distribution excited by different periodic time-variant reflected signals. Square-wave-type time-varying reflectivity used for AM (A) and PM (B). Calculated harmonic amplitudes and phases under different modulation conditions: (C) $A_1=1, A_2=0, \varphi=0$, and varying S for AM; (D) $\varphi_1=180^\circ, \varphi_2=0^\circ, A=1$, and varying S for PM; (E) $A_1=1, A_2=0, \varphi=0^\circ, S=0.5$, and varying t_0 for AM; (F) $\varphi_1=180^\circ, \varphi_2=0^\circ, A=1, S=0.5$, and varying t_0 for PM.

difference $\Delta\varphi$ is described as $\varphi_1 - \varphi_2$. Moreover, τ and T denote the pulse width and period of the square-wave signal, respectively. Note that the amplitudes and phases of the harmonics motivated by AM or PM are affected by S (both for AM and PM) and ΔA (for AM) or $\Delta\varphi$ (for PM). Several studies on harmonics generation and manipulation that adjust the amplitude and phase difference have been reported in Refs. [39, 44, 45]; however, this modulation strategy requires a well-designed complex control circuit, which is absolutely essential to realize dynamic switching of the amplitude or phase differences; this has a significant impact on the modulation frequency (ω_0). Hence, to simplify the process of temporal modulation and reduce implementation difficulty, only the effect of

the duty cycle on the harmonics is examined, as shown in Figure 2C and D. The reflection amplitudes and phases of harmonics can be controlled flexibly and arbitrarily by both AM (Figure 2C) and PM (Figure 2D) as the duty cycle increases from 0.3 to 0.9. The corresponding square-wave parameters inserted in Figure 2C for AM are $A_1=1, A_2=0$, and $\varphi=0^\circ$. As shown in Figure 2D, the parameters of PM are $\varphi_1=180^\circ, \varphi_2=0^\circ$, and $A=1$. Consequently, by manipulating the reflection coefficients of the dynamic metasurface from the time dimension, tunable harmonics can be obtained easily.

Thus far, we successfully shifted the carrier signal to the harmonic components using the time-domain metasurface; however, owing to the strong correlation

(Figure 2C and D) between the harmonic amplitude and phase, it is difficult to predesign the phase gradients at the harmonics flexibly without changing their amplitudes. Therefore, to perform the convolution operations on harmonics, the time shifting property of the Fourier transform with

$$\Gamma(t - t_0) \leftrightarrow e^{-j\omega_0 t_0} \Gamma(j\omega) \quad (6)$$

is proposed to lead an additional phase shift $e^{-jk\omega_0 t_0}$ to the k th-order harmonic while maintaining a constant amplitude. To illustrate this principle, the dependence of delay time t_0 is proposed in Figure 2E and F. For PM, three different delay time values ($t_0 = 0, T/4$, and $T/2$) are considered with $\varphi_1 = 180^\circ$, $\varphi_2 = 0^\circ$, $A = 1$, and $S = 0.5$. Furthermore, from Figure 2F, the additional phases are clearly observed at the high-order harmonics, which are consistent with the mathematical predictions mentioned above. Further, the same phenomena are observed in Figure 2E under AM. The extra phase shift provides new degrees of freedom to control the propagation features of harmonics by breaking the tight link between the amplitudes and phases, which makes the time-domain digital coding metasurface suitable for conducting the convolution operations on all harmonics.

Then, the relationship between the anomalous scattering angle θ_c^k and time shift $t_0(x)$ is revealed for accurate deflections of the harmonic beams. Using Eq. (1) and introducing the additional phase shift $e^{-jk\omega_0 t_0(x)}$, the Fourier transform between the aperture fields and the far-field scattering patterns of the k th-order harmonic can be derived as

$$\begin{aligned} E(x_\lambda^k) \cdot e^{j\Phi_{\text{cons}} - jk\omega_0 t_0(x)} &= E(x_\lambda^k) \cdot e^{j2\pi x_\lambda^k \sin \theta_c^k}, \\ \leftrightarrow E(\sin \theta^k) * \delta(\sin \theta^k - \sin \theta_c^k) &= E(\sin \theta^k - \sin \theta_c^k), \end{aligned} \quad (7)$$

in which the electrical length x_λ^k is equal to x/λ^k (λ^k is the operation wavelength of the k th-order harmonic), and θ^k is the deflection angle. To maintain $t_0(x)$ in a rational range (no more than $T/|k|$, $k = \pm 1, \pm 2, \dots$), Φ_{cons} , a constant phase item, is introduced into Eq. (7) without impacting the phase gradient. Therefore, from the left side of Eq. (7), we have

$$\Phi_{\text{cons}} - k\omega_0 t_0(x) = 2\pi x_\lambda^k \sin \theta_c^k. \quad (8)$$

By calculating the derivative to position x in Eq. (8), the scattering angle at the harmonics can be easily obtained as

$$\theta_c^k = \arcsin \left(\frac{\lambda^k}{2\pi} \frac{d(-k\omega_0 t_0(x))}{dx} \right). \quad (9)$$

It is obvious that Eq. (9) is consistent in form with the generalized Snell's law described as $\theta_c = \arcsin \left(\frac{\lambda_0}{2\pi} \frac{d\Phi}{dx} \right)$ [1] where λ_0 is the working wavelength in free space, and $\frac{d\Phi}{dx}$ represents the spatial phase gradient. It is clear that $\frac{d(-k\omega_0 t_0(x))}{dx}$ in Eq. (9) plays the same role as $\frac{d\Phi}{dx}$ to provide the necessary phase gradient for bending harmonic waves into arbitrary directions; this indicates that the phase gradient associated with time delay provides an efficient approach for wavefront manipulation. Therefore, to regulate the harmonic beams based on the nonlinear convolution theorem, we need to arrange time delay of different meta-atoms in advance according to Eq. (9) to help deviate the scattering pattern $E(\sin \theta^k)$ of the k th-order harmonic by an arbitrary $\sin \theta_c^k$. In addition, we find that when the duty cycle is equal to 0.5, the reflection power at all even harmonics can be offset (excluding the 0th-order under AM, as shown in Figure 2E), which fits well with the theoretical prediction based on Eqs. (4) and (5). Hence, once the delay time $t_0 = 0$ and $T/2$ are introduced, stable anti-phase states (phase differences equal to $k\pi$, $k = \pm 1, \pm 3, \dots$) can be precisely obtained for all odd-order harmonics with the same amplitude, and this can be used to restrain the radiations of harmonics according to the diffusion theory [10, 11, 14].

To validate the principle of the scattering-pattern shift at harmonics with realistic materials and structures, we design a broadband time-domain digital coding metasurface that can realize the PM of the incident EM waves, as shown in Figure 3A. In addition, the zoomed-in view of the meta-atom is depicted in Figure 3B. As shown in Figure 3B, two rectangular metallic patches are linked by a PIN diode (smp1320-040LF, Skyworks Solutions, Inc.) deposited on the top of the substrate (F4BM265) with the dielectric of $2.65 + i0.001$. Furthermore, a metallic plate is placed on the bottom of the entire structure to block all transmitted EM waves and provide more reflection phase shift. Both the metallic patches and metallic plate are made of copper with a conductivity of 5.8×10^7 S/m, and have thicknesses of 0.018 mm. The geometric dimensions of each coding particle are optimized using commercial software (CST Microwave Studio 2016), with varied forward currents to obtain the required reflection phase and amplitude responses, as shown in Figure 3C and D under the illumination of the x-polarized plane wave (Figure 3A). (see Supplementary Note S1 for simulation method of the typical meta-atom) Figure 3C and D respectively illustrate the simulated reflection amplitude and phase spectra as a function of the frequency, showing almost uniform

Figure 3: Simulated reflection responses of the typical meta-atom.

Schematics of (A) the proposed time-domain digital coding metasurface and (B) the typical meta-atom. (C, D) Simulated reflection amplitude and phase responses of the digital particle applied to PM under different forward states.

high reflection amplitudes (larger than 0.9, Figure 3C) but nearly opposite phases (160.6° to 196.9° , Figure 3D) in the spectrum of the X-band, which is especially appropriate for the PM experiments (see Supplementary Note S2 for experimental reflection responses of the typical meta-atom). The optimized dimension parameters of the meta-atom are given as: $P=10.0$ mm, $a=2.3$ mm, $c=3.0$ mm, $d=0.55$ mm, $g=0.3$ mm, and $h=4.0$ mm.

In addition, to explain the reflection responses of the basic coding particle, the electric-field distributions when the PIN diode embedded in the typical meta-atom is in the “OFF” and “ON” states at different frequencies (8.0 GHz, 10.0 GHz, and 12.0 GHz) are revealed in Figure 4A–F. The electric-field is mainly concentrated at the gap of the upper metallic structure (Figure 4A–C) when the PIN diode is off, owing to the strong electric resonance. While the resonance dies away as the PIN diode switches on (Figure 4D–F), it induces an anti-phase phenomenon between the two operation states (Figure 3D). Moreover, to feed the PIN diodes inside the meta-atoms, equal-length direct current feeder lines (Supplementary Figure S1B) are designed at the edge of each meta-atom columns, as the electric field is the weakest at points A (Figure 4A), and it slightly affects the original performance of the typical unit cell.

As shown in Figure 3A, each PIN diode in the same column possesses identical forward voltage, and thus the

reflection states of each column can be adjusted independently to meet the phase demands for both theoretical calculations and experiments. To demonstrate the performance of the proposed theory, we provide two examples with the same original time-domain coding patterns but different gradient coding patterns constructed separately by 1-bit (Figure 5B) and 2-bit (Figure 6B) time-domain digital coding sequences, as shown in Figures 5 and 6, to realize a scattering-pattern shift at the negative first-order harmonic. It is easy to generate “0” and “1” elements with opposing phases but uniform amplitudes as 1-bit time-domain digital coding particles by inserting a reasonable time delay kt_0 for the k th-order harmonic. The higher-bit cases can also be conducted in the same manner.

Owing to its high conversion efficiency (40.53%), PM is selected to generate the negative first-order harmonic with the following modulation conditions: $\varphi_1=180^\circ$, $\varphi_2=0^\circ$, $A=1$, $S=0.5$, and $T=100$ μ s using the proposed dynamic metasurface shown in Figure 3A. Note that AM is not adopted because of unavoidable energy attenuation, which results in the reduction of the conversion rate (from carrier frequency to harmonic frequency) as well as the limitation of the meta-atom designs such that they cannot possess an amplitude response. In the first example (Figure 5), we choose the delay time as $t_0=0$ and $T/2$ to construct the time-domain digital coding particles “0” (white in Figure 5A–C) and “1” (dark gray in Figure 5A–C)

Figure 4: Simulated electric-field results of the typical meta-atom.

(A–F) Simulated electric-field distributions of the proposed coding particle when the embedded PIN diode is under the “OFF” (A–C) and “ON” (D–F) operation conditions at 8.0 GHz, 10.0 GHz, and 12.0 GHz, respectively.

Figure 5: 1-bit case for nonlinear convolution operation.

1-bit time-domain coding patterns and their calculated 3D (A–C) and 2D (x - z plane, D–F) scattering patterns for negative first-order harmonic motivated by the PM to illustrate the nonlinear convolution theorem, when illuminated by the x -polarized plane wave at 10.0 GHz.

at all odd-order harmonics. In Figure 5A, the original two beams symmetrically distributed in the x - z plane with a small deflected angle θ_1 can be realized at the negative first-order harmonic with the help of the time-domain coding sequence S1 “0 0 ... 1 1 ...” along the x direction (Figure 5A), when illuminated by the x -polarized plane waves at 10.0 GHz. Note that, in the full text, each code in the time-domain coding sequence only represents one

meta-atom. Furthermore, Figure 5B shows a gradient time-domain coding sequence S2 “0 0 1 1 0 0 1 1 ...” along the x direction, which can be used to produce a similar scattering pattern as shown in Figure 5A; however, it has a large deviation angle θ_2 . Thus, when S1 adds S2, the mixed time-domain coding sequence can be obtained in Figure 5C, and by observing the three-dimensional (3D) scattering patterns from Figure 5A–C, it is clear that the original

Figure 6: 2-bit case for nonlinear convolution operation.

2-bit time-domain coding patterns and their calculated 3D (A–C) and 2D (x - z plane, D–F) scattering patterns at the negative first-order harmonic motivated by PM to illustrate the nonlinear convolution theorem, when illuminated by the x -polarized plane wave at 10.0 GHz.

two-beam scattering pattern can deviate from the normal axis to θ_2 without distortion of the shape of the pattern, which is consistent with the theoretical prediction based on the principle of nonlinear scattering-pattern shift. In addition, to further demonstrate this principle, a quantitative analysis is conducted by extracting the 2D scattering patterns in the x - z plane. As shown in Figure 5E and F, the deviation angle θ_4 (Figure 5F) from the z -axis is almost identical to that (θ_2) in Figure 5E. However, on comparing Figure 5F and D, the reduction in the gain and increase in the beam width due to the decrease of the effective aperture area as the reflection beam is tilted along oblique directions [43] can be obviously observed. Beyond that, the deviation angle θ_1 (Figure 5D) and θ_3 (Figure 5F) are almost equal; thus, such a principle is well validated by the calculation results in Figure 5A–F (see Supplementary Note S3 for the detailed calculations of the deflected angles θ_1 , θ_2 , θ_3 , and θ_4).

To generate asymmetric harmonic beams and provide higher angular resolution during beam scanning, the second example shown in Figure 6 is used to evaluate the performance of the nonlinear scattering-pattern shift by utilizing a 2-bit time-domain digital coding metasurface. In this example, the modulation conditions for the original scattering pattern are the same as those in the first example, except for different delay time. Here, the delay time parameters $t_0 = 0, T/4, T/2$, and $3T/4$ are selected to perform time-domain digital coding particles for “00” (white in Figure 6A–C), “01” (light gray in Figure 6A–C), “10” (dark gray in Figure 6A–C), and “11” (dark in Figure 6A–C) states, at the negative first-order harmonic. From

Figure 6C, it is clear that the original 3D scattering pattern in Figure 6A can be shifted away from the normal axis as desired, by adding the initial coding sequence S3 “00 00 ... 10 10 ...” (Figure 6A) to the gradient coding sequence S4 “00 00 01 01 10 10 11 11 00 00 01 01 10 10 11 11 ...” (Figure 6B). The corresponding 2D scattering patterns in Figure 6E and F show that the deviation angle θ_7 (Figure 6F) with respect to the z -axis is equal to the anomalous scattering angle θ_5 (Figure 6E) generated by the gradient time-domain coding sequence S4. Meanwhile, on account of the original pattern shape shown in Figure 6A being unperturbed (Figure 6C), equal deflected angles θ_1 (Figure 6D) and θ_6 (Figure 6F) can also be observed by comparing the 2D scattering patterns presented in Figure 6D–F (see Supplementary Note S3 for the detailed calculation of the deflected angles θ_5 , θ_6 , and θ_7). In addition, with an increase in the coding bits, the performance of harmonic-beam control through the nonlinear convolution theory can be enhanced further.

3 Experiment

To validate our findings, a broadband dynamic time-domain digital coding metasurface sample (Supplementary Figure S1B) with dimensions of $340 \times 500 \text{ mm}^2$ was fabricated using printed circuit board technology, according to the geometric parameters of the proposed meta-atom. For the measurements, a microwave signal generator (Keysight E8257D) and a spectrum analyzer

(Keysight E4447A) were simultaneously used to transmit normal plane waves and receive scattering signals, respectively, with the help of high-gain antennas connected with low-loss cables in a microwave anechoic chamber (Figure 7A). The operation frequency of the feed antenna was 10.0 GHz for conducting PM. An interface circuit board was designed to provide dynamic forward currents of each column under the programmable control of an embedded system (NI Compact RIO platform).

Figure 7B shows the calculated and measured reflection amplitudes of the harmonics generated by adopting PM (Figure 2B) with $S=0.5$ and $T=100\ \mu\text{s}$. The measured reflection power at the harmonics coincides well with the theoretically calculated one (Figure 7B), which is obtained by substituting the simulated complex reflectivity ($A_1=0.94$, $A_2=0.94$, $\varphi_1=-79^\circ$, and $\varphi_2=-275^\circ$) at 10.0 GHz (Figure 3C and D) into Eq. (5). Because of the fabrication flaws and test errors during the experiments, a slight

discrepancy between the calculated and measured reflection powers at the harmonics is observed in Figure 7B; however, this discrepancy has no significant effect on the correctness of the aforementioned theory about the generation of harmonics.

In addition, two cases (Figures 5 and 6) that illustrate the performance of the nonlinear convolution operations on harmonics are both considered during the experiment, as shown in Figure 7C–G. To measure the far-field scattering features at the harmonics, the sample and feeding antenna, mounted 1.8 m apart on a supporting board (Figure 7A), are located on a stage that can be rotated in steps of 0.1° . The receiving antenna (not shown in Figure 7A) connected with the spectrum analyzer is located far from the sample, to record the far-field scattering pattern of the negative first-order harmonic in different directions. Figure 7C–G shows the measured horizontal-plane scattering patterns for the negative

Figure 7: Measured harmonic intensities distribution and 2D/3D scattering patterns of the time-domain metasurface.

(A) Photograph of the experiment environment. Calculated and measured reflection amplitudes of harmonics generated by PM (B) with $S=0.5$. Measured 2D scattering patterns at the negative first-order harmonic of the proposed time-domain digital coding metasurface encoded by time-domain coding sequences S2 (C), S1/S3 (D), S4 (E), the modulus of S1 and S2 (F), and the modulus of S3 and S4 (G), under normal incidence at 10.0 GHz.

first-order harmonic of the sample, which are generated under the same PM conditions as the ones adopted in the first and second examples. A pair of scattering beams appeared at angles of $\pm 5.0^\circ$ (almost equal to the calculated results of 5.06° , as shown in Supplementary Table S1) with respect to the surface normal at 9.99999 GHz (the negative first-order harmonic) in Figure 7D; this agrees well with the calculated results shown in Figures 5D and 6D. Moreover, the corresponding 2D far-field scattering patterns of the metasurface encoded with the gradient time-domain coding sequences S2 and S4 are revealed in Figure 7C and E, respectively; the measured deflection angles are also exactly equal to the calculated ones (Figures 5E and 6E). By encoding the metasurface with the coding sequence, which is the modulus of the original sequences S1 and S2, the scattering-pattern shift can be validated by observing the experimental 2D scattering patterns (Figure 7F), which agrees well with the calculated one shown in Figure 5F. In addition, in the second example, the measured 2D scattering pattern (Figure 7G) formed through the nonlinear convolution operation is obtained, as expected, in Figure 6F. The excellent agreement between the calculated and experimental results confirms the validity of the theoretical analyses presented in the previous section.

4 Conclusion

In summary, this study proposed a general method for realizing the flexible and continuous regulations of the propagation behaviors of harmonics. Based on Fourier transform, a principle known as nonlinear scattering-pattern shift was proposed for the first time – to the best of our knowledge – to steer the harmonic scattering patterns to any predetermined directions through nonlinear convolution operations; the corresponding calculation formula for the scattering angle was also obtained for predicting the deflection angle at the harmonics. To validate the performance of the nonlinear convolution theorem, two time-domain digital coding metasurfaces excited by PM with different delay time were used to realize the desired two-beam scattering manipulations at the negative first-order harmonic. The experimental scattering patterns were highly consistent with the calculated ones, which validated the principle of nonlinear scattering-pattern shift. Our work is the first one to introduce the convolution theory from the linear to the nonlinear domain, thereby creating a new avenue for generating various beams at arbitrary harmonics.

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