Research article

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A conformal transformation approach to wide-angle illusion device and absorber

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Abstract: We theoretically investigate the illusion device designed by a conformal transformation that can render an elliptic defect behave like a flat mirror. Different from illusion devices consisting of the complementary medium and anti-object with negative permittivity and permeability, our proposed illusion device requires only isotropic positive permittivity medium. It offers a possible route to eliminate the lateral shift in the conventional quasi-conformal carpet cloak. Interestingly, with the same conformal transformation, we can achieve an impedance-matched flat absorber by simply varying the shape and the refractive index of the defects. Both the illumination device and the absorber in our design have broad bandwidth, wide-illumination range and polarization-insensitive performance.

Keywords: illusion optics; metamaterial absorber; transformation optics.

1 Introduction

Supported by the development of metamaterials, transformation optics (TO) offers capability to steer electromagnetic waves along arbitrary trajectories [1–9]. This

extraordinary feature attracts much attention, promoting the applications of TO in the design of versatile devices including cloaking devices [10–17], superlenses [18–20], superscatters [21], antennas [22], etc. Under the guideline of the TO theory, Lai et al. proposed the concept of “anti-object” and complementary medium [23] that can cancel the scattered field produced by a pre-specified object. Based on this method, the concept of illusion optics was proposed in 2009 [24], triggering global research interests on theoretical designs [25–28] and experimental realizations [29, 30] of different illusion devices. Recently, Chen et al. has adopted a multi-folded transformation to design a remote illusion optical device, which can remotely produce a hidden region without causing additional scattering in the far field [31, 32]. However, all these illusion devices based on anti-object concept and the complementary medium usually requires the negative refractive index medium. Therefore, resonate units are necessary in the practical implementation, leading to significant loss and narrow bandwidth that dramatically affect the illusion performance.

Besides illusion optics, TO can also be applied to design metamaterial absorbers. Narimanov and Kildishev theoretically proposed optical black holes that can absorb light with broad frequency band [33]. The specifically designed dense optical media was proposed to realize the photonic black hole and the continuous-index photon traps that can be utilized to design optical high-Q-factor cavities [34]. Later, the experimental demonstration was performed by Cui et al., who reported the first omnidirectional electromagnetic absorber in the microwave regime [35]. Chen et al. applied the transformation optics to realize Schwarzschild black hole and investigated its absorption property [36]. Sheng et al. mimicked the gravitational lensing through a micro-structured optical waveguide, which can also be applied to an omnidirectional absorber [37]. Above all, both illusion optics and absorber design guided by transformation optics have been demonstrated separately, but never been realized by using one single coordinate transformation.

In our work, we adopt a conformal transformation to design an illusion device that can transform an elliptic PEC defect to a flat mirror. This illusion device only requires

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materials with positive refractive index, which can be achieved by simply drilling holes on dielectric slabs in practical implementations. The transformed medium has isotropic refractive index, making the whole device insensitive to the polarization of the illumination. Moreover, a broadband wide-illumination angle and polarization-robust metamaterial absorber can also be built under this conformal transformation, by properly choosing the shape and the refractive index of inner defects. Therefore, we can apply one transformation to achieve two different devices which share the broad bandwidth, wide-illumination range and polarization-insensitive features.

2 Materials and methods

Starting from a slab (region between two red straight line in Figure 1A) that can be regarded as the virtual system, we use a conformal mapping

$$w = \gamma \ln \left(\frac{1}{e^z - i w_0} + i y_0 \right) \quad (1)$$

where $z = x + iy$, $w = u + iv$, and $y_0 = [w_0 / (e^{2(x_0+d)} - w_0^2)]$ (d is the thickness of the slab in the virtual system) to map the slab to a grating (region between the red straight line and red wavy line in Figure 1B), the physical system. The coordinates systems in virtual and physical space are represented by $z(x,y)$ and $w(u,v)$ respectively. A similar transformation has been applied to design plasmonic gratings [38] and flat metasurfaces with compact dimensions [39]. Under such a conformal mapping, a regular Cartesian mesh in Figure 1A is transformed into the grids as shown in Figure 1B. The left boundary of the slab is fixed at the position $x = 0$ so that only one side of the physical system will be transformed into the curve, while the other side remains as a straight line. With the increasing thickness of the slab, the right boundary of physical space is changed from a continuous wavy curve into several isolated ellipses as illustrated in Figure 1C, D. The boundary of the ellipse can be expressed in the form,

$$(e^{2x_0} - w_0^2)e^{\frac{2u}{\gamma}} + \left[2w_0 - 2y_0(e^{2x_0} - w_0^2) \right] e^{\frac{v}{\gamma}} \sin \frac{v}{\gamma} + (e^{2x_0} - w_0^2)y_0^2 - 2w_0y_0 - 1 = 0 \quad (2)$$

It is worth noting that increasing the slab thickness d will lead to the decrease of the size of those isolated ellipses. Several periods are presented in Figure 1B, D with the periodicity $2\pi\gamma$. In this paper, we only consider one period to reduce the simulation time consumption.

All the contour lines in Figure 1B cross with each other at 90 degrees revealing the conformal property of this transformation. Hence, the refractive index of the transformed space can be calculated using $n = n_0 |dz/dw|$, where n_0 is the refractive index in the virtual space [40]. The refractive index distribution of the transformed space can be expressed as

$$n(u, v) = n_0 \left| \frac{dz}{dw} \right| = n_0 \left| \frac{e^{\frac{w}{\gamma}}}{i w_0 \gamma (e^{\frac{w}{\gamma}} - i y_0)^2 + \gamma (e^{\frac{w}{\gamma}} - i y_0)} \right| \quad (3)$$

which is isotropic but spatially variant that can be achieved by drilling holes with different diameters or periodicities in a dielectric slab. Note that the refractive index will always be positive provided that the

Figure 1: Schematic diagram of the conformal transformation. (A) The slab (area between red straight lines) in the virtual space can be transformed into (B) the periodic grating (area between the red straight line and the red curve) in the physical space. The parameters were $y_0 = 0.005$, $w_0 = 0.005$, $d = 4.5$, $\gamma = 1$. (C) The slab (area between blue straight lines) in the virtual space can be transformed into (D) the periodic grating (area between the blue straight line and the several blue ellipses) in the physical space. The parameters were $y_0 = 0.005$, $w_0 = 0.005$, $d = 5.5$, $\gamma = 1$.

refractive index of the virtual space is positive. Apparently, increasing the thickness of the slab in the original space will enlarge the refractive index range in the transformed space and therefore increase the difficulty of practical implementation. This indicates that a proper choice of parameters is critical for our design.

3 Results and discussion

We analytically retrieve the light trajectories in the transformed illusion device and black hole. In our calculations, an assemble of light rays are launched from vacuum to the transformed medium. We first plotted several predefined light trajectories in the original space with different incident position. For illusion device in Figure 2A, the predefined light trajectories in the original space should be several straight parallel light rays illuminated to a flat PEC reflection plane then reflected back. And for black hole in Figure 2B, the predefined light trajectories in the original space should be several straight parallel light rays illuminated to infinity. We choose a series of equidistant points on each light ray and calculate their coordinates. After that, based on the conformal coordinate transformation equation, the points

Figure 2: Contour plot of refractive index distribution of transformed medium and the corresponding retrieved trajectory of light illuminating (A) a PEC elliptical defect embedded in an illusion device and (B) the black hole with the position in the same refractive index indicated by the dashed black ellipse.

coordinate in the transformed medium that corresponds to those equidistant points on each light ray can be deduced. Finally, by connecting these transformed points in sequence, the light ray in the transformed medium can be obtained. As shown in Figure 2A, supposing an elliptical defect filled with perfect electric conductor (PEC) is embedded in the transformed space, light rays hitting the PEC defects are reflected at an angle same as the incident angle without any deflection. An observer in the far field will see light rays propagating in exactly the same way as reflected by a flat mirror. This kind of “illusion effect” is similar to the carpet cloak, but in our case, the “hidden region” (i. e. the elliptic defect) is isolated and flying above the metal plane. More importantly, light rays reflected from our illusion device do not experience any unwanted lateral shifts. On the other hand, when the thickness of the slab in Figure 1C is infinite, the ellipse in Figure 1D will be shrunk into points, and the region at the right side of the straight line will be filled with transformed medium. The refractive index at the branch point in the transformed space is infinite, therefore, the light will be guided to infinitely approach the branch point as depicted in Figure 2B. Thus, this system can be regarded as a black hole, and the light

illuminated from the free space will be totally confined by the transformed medium when passing the “event horizon”, the boundary indicated by the left blue straight line in Figure 1D. From Figure 1D, the length of the “event horizon” can be found to be infinite. As for the black hole in Figure 2B, since we only consider around one periodicity to reduce the calculation time consumption, the length of the “event horizon” in this condition represented by the top boundary of the transformed medium is 10. However, the huge refractive index range will make the practical implementation extremely complicated. Therefore, in the following, we propose and demonstrate a new scheme for absorber design by utilizing this transformed medium as an impedance match layer to guide light to a homogenous lossy dielectric inner defect for absorption. The boundary of this inner defect is chosen as the position where the refractive index equals to n_1 , as indicated by the black dashed ellipse in Figure 2B. Then, by simply filling the internal ellipse core with the homogenous lossy material of which the real part of refractive index equals n_1 , light rays will be bent by the surrounding transformed medium, and then confined and absorbed by the lossy dielectric ellipse defect core. Hence, the lossy dielectric ellipse together with the surrounding medium behave as an impedance matched absorber. The parameters are chosen as $y_0 = 0.005$, $w_0 = 0.005$, $d = 6$ and $y_0 = 0.005$, $w_0 = 0.005$, $d = 20$ for the illusion device and the black hole, respectively. It is worth noting that the refractive index for both devices are spatially variant and positive everywhere, thus requiring no resonant elements in the design. This feature significantly simplifying the realization process and avoid introducing high loss in the practical implementation.

Using the commercial finite-element method software (COMSOL Multiphysics), we perform full-wave simulations to verify explicitly the illusion effect that transforms an elliptical PEC defect to a flat PEC mirror. The parameters normalized to the wavelength for the coordinate transformation are chosen as $y_0 = 0.005$, $w_0 = 0.005$, $d = 6$. Figure 3B shows a Gaussian beam projecting upon an elliptical PEC defect in free space. As expected, the beam is scattered to various directions by the PEC obstacle. However, with the presence of the illusion device surrounding this PEC ellipse defect, the light propagation patterns will be altered in the way as if there is a bare PEC plane, as indicated in Figure 3C. A light ray incident upon the surface of the PEC ellipse defect will be reflected at an angle equal to the incident angle. In this case, the field above the upper boundary of the simulation region will be similar to the plane reflection case shown in Figure 3A. Therefore, the observer outside the upper boundary will not figure out the existence of ellipse PEC defect. As a comparison, the

Figure 3: Demonstration of the illusion effect. Electric field distribution when a TE Gaussian beam is illuminated to the (A) flat PEC plane (B) elliptical PEC defect in free space and (C) elliptical PEC defect surrounded by graded medium with incident angle 30° from left top boundary of the simulation region. Magnetic field distribution when a TM Gaussian beam is illuminated to (E) flat PEC plane (F) elliptical PEC defect in free space and (G) elliptical PEC defect surrounded by graded medium with incident angle 30° from left top boundary of the simulation region. (D)

Electric field distribution along the top boundary of the simulation region in (A)–(C). (H) Magnetic field distribution along the top boundary of the simulation region in (E)–(G). The parameters were $y_0 = 0.005$, $w_0 = 0.005$, $d = 6$.

electric field magnitudes along the top boundary of the simulated region in the cases of plane reflection, the bare PEC ellipse, and PEC ellipse surrounded by the illusion device are compared and denoted by black, blue and red curves, respectively, in Figure 3D. In two dimension scenario, the electromagnetic field can be decoupled into TE (Electric field along out of plane direction) and TM (Magnetic field along out of plane direction) waves. Thus, in order to demonstrate that this illusion device is polarization independent, we consider both TE (sub Figure A–D) and TM (E–H) polarized incident waves. To explore the influence of the defect size, simulation results of the illusion device with different defect size at TE polarized incidences are plotted in Figures S1 and S2. The parameters normalized to the wavelength for large and small PEC defects are chosen as $y_0 = 0.08$, $w_0 = 0.08$, $d = 3.25$ and $y_0 = 0.08$, $w_0 = 0.08$, $d = 4$, respectively. As comparison, the illusion effect of the same devices in Figures S1 and S2 under TM polarized incidences are also considered and plotted in Figures S3 and S4. All the simulation results demonstrate that the performance of this illusion device is polarization and defect size independent.

The design scheme of our illusion device also provides a potential solution to eliminate the lateral-shift problem in carpet cloak design [41]. Traditional polarization-independent carpet cloak design has to rely on quasi-conformal mapping. However, quasi-conformal carpet cloak still has fatal drawback which makes the cloak be detectable. With our conformal transformation method, such drawback can be solved. In Figure S5, we present two quasi-conformal mapped carpet cloaks. One is designed to hide curved PEC bumps (in Figure S5A), and the other one is targeted to conceal three PEC elliptical objects (in Figure S5B). The corresponding refractive index distribution and retrieved ray trajectories of these two cloaks are plotted in Figure S5, from which we can find that both cloaks have a distinct

lateral shift of the reflected beam compared to the reflected beam from the virtual reflecting plane. As a comparison, we consider three PEC elliptical objects surrounded by media designed with our conformal transformation method in Figure 4. With proper designs, our method can create a virtual reflecting plane below the elliptical objects. Moreover, owing to our conformal mapping method, no lateral shift will be induced by the device as validated by the retrieved ray trajectory in Figure 4. The reason is that under our conformal transformation, the transformed physical space has isotropic property, which is different from the anisotropic medium given by the quasi-conformal transformation. Thus, the vertical compression in the quasi-conformal mapping is no longer required. The light can be focused to the top point of the PEC ellipse defect and the lateral shift can be avoided.

Figure 4: Contour plot of refractive index distribution of transformed medium and retrieved light trajectory of light illuminated to three isolated PEC ellipses surrounded by our proposed transformed media with parameter $y_0 = 0.005$, $w_0 = 0.005$, $d = 6.5$. The equivalent reflection plane is denoted by the black dash line. The red and black line indicate the trajectory of the plane reflection and device, respectively. The contour plot shows the refractive index distribution for the system.

Figure 5: (A) Schematic of practical implementation scheme of the illusion device. (B) Simulated propagation pattern with practical implementation design by drilling holes on dielectric slab with spatially variant radius at 4 GHz. A Gaussian beam is illuminated to the illusion device.

To demonstrate the feasibility of our illusion device, the practical implementation in microwave regime is proposed. The schematic of the illusion device with practical structure is plotted in Figure 5A. The refractive index range of the designed medium covers from 0 to 3 as presented in Figure 2A. It indicates that part of the structure requires the material with refractive index smaller than 0, which will undoubtedly cause big loss and increase the difficulty in the practical design and implementation. Hence, a background material with refractive index $n = 8$ is applied to increase the refractive index of the device, and the residual region with $n < 1$ is neglected. Considering the large refractive index range $n = 1$ to 24, the structure is divided into seven parts (see Figure 5), of which the permittivity is listed in Table 1. To satisfy the requirement of spatially variant refractive index distribution, we adopt the scheme of drilling holes with variant radius based on the effective medium theory, and the radii of holes are decided by the following equation,

$$r = p \sqrt{\frac{n^2(u, v) - n_1^2}{\pi(1 - n_1^2)}} \quad (4)$$

where n_1 is the refractive index of the dielectric material and p is the periodicity of the subwavelength unitcell. Hence, the hole radius is homogenous for the background material, while spatially variant in the device region. The commercial software, CST Microwave Studio, is used to

carry out the simulations, where the working frequency is 4 GHz and the periodicity of the unit cell is fixed to 5 mm. The field distribution for a TE polarized Gaussian beam illuminated obliquely at the illusion system is displayed in Figure 5B. We can observe that this system effectively eliminates the scattering and creates a virtual reflective plane. The material we choose has small dispersion in microwave regime. Hence, the illusion device can work in a broad wavelength range.

This conformal transformation, as presented in Figure 1 and Figure 2B, can be extended to the application of a wide-illumination polarization-insensitive impedance matched absorber. To demonstrate its size-independent performance, we design two absorbers with different inner defects size as presented in the upper and lower panel in Figure 6. The inner core boundary are chose as the position where the refractive index equals to 3 and 5 for the two absorbers with large and small defects, respectively. Therefore, the permittivity of the large and small ellipse defects is $\varepsilon = 9 - i$ and $\varepsilon = 25 - i$, respectively. Figure 6A–D show the simulated propagation pattern of a TE Gaussian beam projected upon the absorber with large inner defect at different incident angles from 15° to 45° with 10° intervals. The corresponding absorption coefficient for different incident angles are also plotted in Figure 6E, where we can find that the absorber can maintain high absorption efficiency within a wide illumination frequency range. Meanwhile, with the increasing incident angle, the absorption coefficient will be reduced. It is because for larger incident angles, a portion of light will be compressed and focused to the neighbouring periods of the physical space causing some scattering. Instead of employing anisotropic medium [35], the spatially variant isotropic shell we propose makes this absorber insensitive to the polarization. Hence, this system is a perfectly wide-illumination-angle polarization-insensitive absorber that can absorb electromagnetic waves in arbitrary polarization illuminated from a wide-angle range. To a certain extent, it can be regarded as “electromagnetic black body” or an “electromagnetic black hole”. For internal lossy ellipse defects of smaller size with permittivity $\varepsilon = 25 - i$, the light propagation patterns and absorption coefficient under different incident angles are also plotted in Figure 6F–J as a comparison. The absorption coefficient of small internal lossy core system can still exhibit strong absorption performance over the wide incidence angle and frequency range. It is worth noting that, the parameters of the conformal transformation applied to the absorber design is the similar to that of the illusion device shown in Figures S1 and S3. This indicates that through controlling the shape and the refractive index of the internal defect, the illusion

Table 1: Permittivity of different material layers in illusion device practical structure in Figure 5.

Layer	1	2	3	4	5	6	7
Permittivity	600	200	100	40	16	9	4.4

Figure 6: Demonstration of the absorber. Electric field distribution when a TE Gaussian beam is illuminated to the absorber with incident angle (A) 15°, (B) 25°, (C) 35°, (D) 45°. The absorption coefficient from $f = 0.5$ GHz to 3 GHz at different incident angles are plotted in (E). Electric field distribution when a TE Gaussian beam is illuminated to the absorber with incident angle (F) 15°, (G) 25°, (H) 35°, (I) 45°. The absorption coefficient from $f = 0.5$ GHz to 3 GHz at different incident angles are plotted in (J).

device and absorber can be realized by adopting the same conformal transformation.

4 Conclusion

In conclusion, a conformal transformation is utilized to design an illusion device that can make an elliptical PEC defect behave like a flat plane mirror. This illusion device also provides a possible method for solving lateral shift produced by quasi-conformal mapping in carpet cloak design. By drilling holes on the dielectric slab, the practical implementation scheme of illusion device is also demonstrated. Moreover, under the same conformal transformation, an absorber can also be achieved by simply changing the shape and the material property of internal ellipse defect using the same conformal transformation. This conformal transformation we apply contributes to the broad bandwidth, wide-illumination range, size-independent and polarization-insensitive performance of both illusion device and metamaterial absorber. Our design has the potential to provide a guideline for not only the illusion device and absorber but also many other optical devices.

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