Abstract: Photonic topological insulators (PTIs) bring markedly new opportunities to photonic devices with low dissipation and directional transmission of signal over a wide wavelength range due to the broadband topological protection. However, the maximum gap/mid-gap ratio of PTIs is below 10% and hardly further improved due to the lack of new bandwidth enhancement mechanism. In this paper, a PTI with the gap/mid-gap ratio of 16.25% is proposed. The designed PTI has a honeycomb lattice structure with triangular air holes, and such a wide bandwidth is obtained by optimizing the refractive-index profile of the primitive cell for increasing the energy proportion in the geometric perturbation region. The PTI shows a large topological nontrivial gap (the gap/mid-gap ratio 33.4%) with the bandwidth approaching its theoretical limit. The edge states propagate smoothly around sharp bends within 1430–1683 nm. Due to topological protection, the bandwidth only decreases 1.38% to 1450–1683 nm under 1%-random-bias disorders. The proposed PTI has a potential application in future high-capacity and nonlinear topological photonic devices.

Keywords: broadband photonic crystals; valley photonic crystals; photonic topological insulators.

1 Introduction

Photonic topological insulators (PTIs) [1–5], which utilize photonic crystals rather than electronic lattices [6, 7] to engineer the topological band structure with emergence of topologically protected surface or edge states, have attracted great attentions. Unlike traditional topologically trivial waveguide, the forward and backward edge states of the PTIs have the energy flow vortices along two opposite directions [8]. This effectively suppresses the coupling between these two states, enabling the edges one-way propagation without undergoing any backscattering even in the presence of structural defects that preserve symmetry. These exotic properties of the PTIs bring markedly new opportunities to photonic devices with low dissipation and directional transmission of signal. To date, two dimensional PTIs have been reported in various photonic structures, e.g., the coupled resonator arrays [9, 10], metacrystals superlattices [11, 12], photonic crystals with C₆ᵥ-symmetric unit cells [13–15], and valley photonic crystals (VPC) [16–19]. Furthermore, various novel photonic devices, that are robust to their structural disorders based on these topological structures, have been experimentally demonstrated, including the directional optical waveguide without backscattering [14], the robust
photonic delay line [9, 16], the topological power splitter [14], and the topological photonic router [18].

However, the limited bandwidth of the PTIs makes the topological photonic devices typically operating within a relatively narrow wavelength range, which hinders their potential applications in high-capacity photonic systems. Specifically, the PTIs based on the coupled resonator arrays [10], metacrystals superlattices [12], photonic crystals with $C_{6v}$-symmetric unit cells [15], and VPCs [19] are reported to have the maximum bandwidths with the gap/mid-gap ratios ($\gamma$) of 0.02%, 10%, 6%, and 10%, respectively. Moreover, their bandwidths are hardly further improved. The bandwidth of the coupled resonator structures is inherently limited by the narrow resonant transmission band of high-Q resonators. While, for the metacrystals superlattices, there exists a big challenge to open a large topological nontrivial gap but simultaneously match the permittivity and the permeability of meta-atoms required for realizing the pseudo-spin states. In terms of the photonic crystal with $C_{6v}$-symmetric unit cells, their bandwidths are limited because the $C_{6v}$-symmetric clusters can be only shrunk (or expanded) within a relatively small extent in order to keep the topological nontrivial gap open.

In stark contrast to the above schemes, the valley photonic crystal is usually considered to be a good candidate for achieving a wide bandwidth. The valley photonic crystal [16–19] only requires the units with the $C_3$ symmetry, and can be realized by various materials, flexible element types, and the simpler band structure. The studies have been shown that the topological nontrivial gap of the VPC is proportional to the strength of perturbation that reshapes circle into $C_3$ symmetric geometry [16]. Furthermore, in order to increase the perturbation strength, two methods are usually considered: (1) to increase the refractive-index contrast between the photonic crystal and background; (2) to increase the geometric perturbation area from circle to $C_3$ symmetric shape. By carefully designing to maximize both the refractive-index contrast and geometric perturbation area, the maximum bandwidth with $\gamma = 10\%$ has been reported in the VPC [19]. However, PTIs with wider bandwidth are highly desirable due to their practical applicability. This leads to the natural questions: are there any new initiatives for further increasing the bandwidth? If yes, what is the theoretical limit of the bandwidth of edge states without backscattering?

In this Letter, we report a broadband PTI by increasing the energy proportion in the geometric perturbation region, and discuss its theoretical limit bandwidth. Enlightened by the strong evanescent field of nanofiber waveguide [20], we design the primitive cell with resonating rods surrounding hole instead of antiresonant air cavities surrounding rod. The bound modes of the resonating rods can efficiently spread into holes when reducing the size of rods. Therefore, when considering a honeycomb lattice with triangular air holes, its transverse magnetic (TM) modes have more energy contribution to the geometric perturbation region than that of a honeycomb lattice with triangular solid rods, and its topological nontrivial gap is opened with $\gamma = 33.4\%$. A maximum bandwidth of edge bands is achieved with $\gamma = 16.25\%$ at telecommunication wavelengths, and approach to its theoretical limit ($\gamma = 17.58\%$). It should be emphasized that the gap size and the bandwidth are not the same concept in this paper and the gap size can be much greater than the bandwidth, even the theoretical limit bandwidth. The theoretical limit bandwidth is the probable maximum value of bandwidth, which is limited by both the energy gap and the group velocity of edge states. Moreover, the edge states of the designed PTI propagate smoothly around sharp bends. Due to the topological protection, those edge states are very robust to the local perturbation, where the bandwidth only decreases slightly to 14.87% under 1%-random-bias disorders. The proposed broadband PTI may pave the way toward applications of broadband and high-capacity topological photonic devices.

2 Broadband PTI optimization

Using the finite element method (outlined in the Appendix), we consider a generic photonic honeycomb lattice constructed by the circular air holes on the Si slab (the refractive index RI = 3.4757). The lattice constant is $a$, and the side length of the regular hexagon is $a/\sqrt{3}$. This photonic structure preserves $C_{6v}$ symmetry, and supports the Dirac cones at the corner points ($K$ and $K'$) of the first Brillouin zone (BZ), as shown in Figure 1a. Compared with the TE mode, the TM mode has a lower Dirac frequency $\omega_D$ because the lowest Dirac cone spectrum of the TM mode is formed by the first and second bands. So only the TM mode is considered here because the lower Dirac frequency will increase the gap/mid-gap ratio ($\gamma = \Delta \omega/\omega_D \approx \Delta \omega/\omega_0$) of the topological nontrivial gap of further opened Dirac cone, where $\Delta \omega$ and $\omega_0$ are the gap size and the center frequency, respectively. The valley photonic crystal (denoted as VPCI) can be achieved by deforming the geometry of circle air holes into the triangular air holes because of breakdown of inversion symmetry, as shown in Figure 1b. The lower and upper bands at valley $K$ point define the pseudospin up and down states, respectively. Due to the time-reversal symmetry, the lower and upper bands at valley $K'$ point are their time-reversal counterpart. When rotating the
triangular air hole by 180°, the topological index of two bands of new valley photonic crystal (denoted as VPC2) have sign changes, and its pseudospin states are reversed.

The topological nontrivial gap of the VPC with triangular air holes can be increased by optimizing the size and the shape of the triangular air hole in unit cell. The triangular air hole has the round corner with radius \( r = \eta \cdot \rho \cdot (a/\sqrt{3}) \), and the length between the center of corner circle and the hexagonal cell is \( h = \eta \cdot (1-2\eta/\sqrt{3}) \cdot (a/\sqrt{3}) \), where \( \eta \) is the hole size control parameter and \( \rho \) is the hole shape control parameter. To be specific, when \( \rho = \sqrt{3}/2 \), the air hole becomes the circle one. If \( \rho \) is less than \( \sqrt{3}/2 \), the structural inversion symmetry is broken, the degeneracy of two Dirac bands at \( K(K') \) point is lifted, and thus the band gap is opened. As shown in Figure 1c, the gap/mid-gap ratio \( \gamma_{\text{hole}} \) of the VPC with triangular air holes increases as \( \eta \) increases, because holes with larger size results in the larger geometric perturbation area (see Figure 2a) in the primitive cell when the circle air hole is deformed into the triangular air hole. Theoretically, when \( \eta \) is fixed, the geometric perturbation area increases as \( \rho \) decreases. However, when \( \rho \) increases, the gap/mid-gap ratio \( \gamma_{\text{hole}} \) shows a trend of increase before decrease, as shown in Figure 1d. This anomalous relationship between \( \gamma_{\text{hole}} \) and \( \rho \) indicates that there is another factor beside the geometric perturbation area and the refractive-index contrast that regulates the energy gap, which is explained as the energy proportion \( \zeta = \int \frac{|\mathbf{e}|^2 \text{d}V}{\int |\mathbf{e}|^2 \text{d}A} \), where \( \mathbf{e} \) is the electric field of pseudospin states, \( V \) and \( A \) are the geometric perturbation area (Figure 2a) and the unit area, respectively. According to Figure 1c, the gap/mid-gap ratio of the VPC with triangular air holes has a maximum value of \( \gamma_{\text{hole}} = 33.4\% \) for \( \eta = 0.99 \) and \( \rho = 0.45 \). In contrast, when considering the VPC with a honeycomb lattice constructed by the triangular solid Si scatters, the bands contributed from the TE mode form a larger gap than the TM mode. The theoretical maximum gap/mid-gap ratio is \( \gamma_{\text{solid}} = 10\% \) for \( \eta = 0.85 \) and \( \rho = 0.01 \).

In order to explain that the gap/mid-gap ratio of the VPC with air hole arrays can be much larger than the VPC with solid scatter arrays, we consider the following low-energy Hamiltonian at Dirac points \([16]\):

\[
\mathcal{H} = v_D \left( \delta k_x \tau_z \sigma_x + \delta k_y \tau_0 \sigma_y \right) + \omega_D \Delta P \tau_0 \sigma_z
\]  

where \( v_D \) is the Dirac velocity, \( \left( \delta k_x, \delta k_y \right) \) is the reciprocal vector measured from the Dirac points, \( \tau_{x,y,z} \) and \( \sigma_{x,y,z} \) are the Pauli matrices for valley and spin degrees of freedom, \( \omega_D \) is the Dirac frequency, and \( \Delta P \) is the perturbation strength. According to Hamiltonian \( \mathcal{H} \) in Eq. (1), the energy gap between lower and upper bands can be written as \( \Delta \epsilon = 2\omega_D|\Delta P| \approx 2\omega_D|\Delta P| \). Thus \( \gamma = \Delta \epsilon/\omega_D \approx 2|\Delta P| \), which is proportional to the perturbation strength. According to the first-order perturbation theory, we have

\[
2\Delta P = \int_V \Delta \epsilon \cdot |\mathbf{e}_R(K)|^2 - |\mathbf{e}_L(K)|^2| \text{d}V
\]  

where \( V \) is the geometric perturbation area in the primitive cell when the circle air hole is deformed into the triangular air hole. It is composed of two parts: the side area \( V_i \) and

![Figure 1:](image-url)
the angle area $V_2$, as shown in Figure 2a. $\Delta \varepsilon = \pm (\varepsilon_{Si} - 1)$ is the variation of permittivity between the primitive cell with the circle hole and the triangular hole, and the side area $V_1$ (angle area $V_2$) has the positive (negative) sign. $e_{L,R}(K)$ is the electric field of counterclockwise and clockwise pseudospin states, associated with pseudospin up and down states, at $K$ point. Figure 2b plots the electric field distributions of the air array at $K$ point. In this structure, the unit cell is composed of the triangular air hole surrounded by six solid cylinders, and each TM mode of the array consists of the bound modes of three solid cylinders. When the area of the solid cylinder is reduced, the bound mode is no longer localized in the cylinder, but leaks into the triangular air as an evanescent field. Therefore, the side area $V_1$ has the electric field $e_{L}(K)$, and the angle area $V_2$ has the electric field $e_{R}(K)$. Then, the net perturbation strength is

$$2 \Delta P_{hole}(K) = (\varepsilon_{Si} - 1) \left( \int_{V_1} |e_{L}(K)|^2 dV_1 + \int_{V_2} |e_{R}(K)|^2 dV_2 \right)$$

(3)

However, for the VPC with the solid scatters (see Figure 2c), the unit cell consists of the triangular solid scatter surrounded by six air resonators (the antiresonant reflecting optical waveguides). Each TE mode of the array is the antiresonant mode of three air cavities. The antiresonant mode is formed by the reflection of the solid scatter, and is hardly penetrated into the solid scatter even when the size of air resonantor is reduced. Therefore, only the side area $V_1$ has the electric field of $e_{R}(K)$. Then the perturbation strength is

$$2 \Delta P_{Si}(K) = - (\varepsilon_{Si} - 1) \left( \int_{V_1} |e_{R}(K)|^2 dV_1 \right)$$

(4)

According to Eqs. (3) and (4), the air hole array has larger perturbation strength than the solid scatter array, because it can support much more electric field distribution in the geometric perturbation area. Therefore, the air hole array has larger gap/mid gap ratio. Particularly noteworthy here is the above results are only suit for the dielectric VPCs. For the metallic VPCs, the net perturbation strength can not be expressed by (3) because the mode fields are always localized in air and hard to leak into the metal.

Besides the topological nontrivial gap, the bandwidth of the VPC is also inherently limited by the group velocity of edge states $v_{edge}$. Here, the bandwidth of the VPC is defined as the frequency range for light smoothly propagating along the zigzag path with negligible backscattering, which is the overlapped band dispersions of edge states at two different interfaces (see Figure 4). Due to the crossover bands structure with near linear dispersion around $K/K'$ valley, the maximum bandwidth of the VPC with a large energy gap has a theoretical limit $\Delta \omega_{max} = v_{edge} \cdot (\pi/a)$, where $\pi/a$ denotes the half period in the wave vector space. Lu et al. [21] proved that the group velocity of edge states is the same as the Dirac velocity of the corresponding
photonic graphene with circular air holes, i.e., $v_{\text{edge}} = v_D$. Thus, we have

$$\Delta \omega_{\text{max}} = v_{\text{edge}} \cdot (\pi/a) = v_D \cdot 2|KM| \approx \Delta \omega_M$$

(5)

where $2|KM|$ is the distance between the high symmetry $K$ and $M$ points in the first Brillouin zone of photonic graphene, and $\Delta \omega_M$ is the frequency difference between the two bands of Dirac cone at $M$ point. Therefore, we achieve a theoretical limit value of $\gamma_{\text{max}} = \Delta \omega_M / \omega_D$. Figure 1c, d respectively show $\gamma_{\text{max}}$ versus $\eta$ and $\rho$, where the corresponding photonic graphene is reconstructed by transforming the triangular air holes of the VPC into the circular air holes when the area of hole remains constant. For the VPC with $\eta = 0.99, \rho = 0.15$, and $\gamma_{\text{hole}} = 21.95%$, $\gamma_{\text{max}}$ achieves a maximum value of 17.87%. It is worth noting that the topological nontrivial gap of this VPC does not support its actual bandwidth approaching to the theoretical limit. Because the crossover frequency of two edge states bands is seriously deviate from the gap center frequency. By the trade-off between the gap/mid-gap ratio and the theoretical limit, the VPC with the triangular holes ($\eta = 0.99$ and $\rho = 0.35$) achieves a maximum bandwidth value of 16.86%.

3 Wideband topological edge states

The interface between two valley photonic crystals with different Chern numbers can support the topologically protected edge states. The valley Chern number of bulk bands at $K/K'$ valley is $C_{K/K'} = \frac{1}{2} \text{sgn}(|P(K/K')|)$ [14]. VPC1 and VPC2 have the valley-Chern indices $C_{\text{VPC1}} = C_{\text{VPC2}}$ because of their opposite Dirac masses, i.e., $\Delta P_{\text{VPC1}}(K) = \Delta P_{\text{VPC2}}(K')$. Figure 3a schematically show two kinds of interfaces (denoted as IFS12 and IFS21), which are formed by the VPC1/VPC2/VPC1 sandwich structure. The bands structure of edge states at IFS12 and IFS21 are shown in Figure 3b, which are indicated as the red and green dotted lines, respectively. The edge state bands of both interfaces do not extend across the whole gap. This is because the topological nontrivial gap is larger than the theoretical limit bandwidth in the designed VPC. Due to bulk-boundary correspondence, the number of edge states at each interface around $K$ valley equals to 1, which is corresponding to the differences of the topological indices at $K$ point of two valley photonic crystals, i.e., $\Delta C_K = C_{\text{VPC1}} - C_{\text{VPC2}} = 1$.

The sectionalized break lines at high frequency band $\left(\frac{\omega}{2\pi a} = (0.293, 0.33)\right)$ and low frequency band $\left(\frac{\omega}{2\pi a} = (0.240, 0.244)\right)$ in Figure 3b are the projection of the bands of edge states outside the first period ($k_x = (-\pi/a, \pi/a)$) due to the periodicity of band theory. To illustrate this, the electric field distribution of the edge states (correspond to the inset labels in Figure 3b) are shown in Figure 3c–f. As an example, the edge states of dark red lines, in both the frequency range $\left(\frac{\omega}{2\pi a} = (0.293, 0.33)\right)$ with $k_x > 0$ and the frequency range $\left(\frac{\omega}{2\pi a} = (0.240, 0.293)\right)$ with $k_x < 0$, have the Poynting vectors forwardly (backwardly) (c–f).
Figure 4: (a) Schematic of the IFS12 zigzag path formed by VPC1 (blue) and VPC2 (red) with $a = 407$ nm, $\eta = 0.99$, and $\rho = 0.35$. (b) Electric field distribution of the edge state traveling at 1550 nm along the IFS12 zigzag path. (c) Transmission spectra of the edge state of the IFS12 zigzag path and the IFS12 straight path. The yellow shaded indicate the bandwidth calculated by the energy spectrum shown in Figure 3b. (d) Schematic of the IFS21 zigzag path with $a = 407$ nm, $\eta = 0.99$, and $\rho = 0.35$. (e) Electric field distribution of the edge state traveling at 1550 nm along the IFS21 zigzag path. (f) Transmission spectra of the edge state of the IFS21 zigzag path and the IFS21 straight path. The red circles in (a) and (d) highlight the same structure between the sharp corner part of the IFS12 zigzag path and the IFS21 straight path. The purple star indicates the dipole source. S1 and S2 are two power monitors used for calculating the transmission.

Figure 5: Bandwidth of the VPC with the defective triangular air holes. (a) Offset of triangular hole center with $dl = L_{O1}$; (b) rotation of triangular hole with $d\theta$; (c) Scaling of integral triangular hole with $d\eta$; (d) deformation by offset three corners of triangular hole with $L_{O1}$, $L_{O2}$, $s \cdot dl$, and $L_{O3}$, $s \cdot dl$. The insets are the sketches of four disorders and the red dashed lines indicate the defective triangular air holes. The yellow shaded indicate the bandwidth of the VPC without defect. All data are the average values of ten stochastic computations.
with the same direction, which means they are belong to the same band.

To demonstrate the topological robustness of the wideband edge modes, we simulate (see Appendix) one-way edge mode propagation along a zigzag path with 120° bends (see Figure 4a, d). As shown in Figure 4b, e, the edge modes for the interfaces IFS12 and IFS21 can smoothly propagate along the zigzag path. Figure 4c, f show the transmission spectra of the edge state propagating along the zigzag paths and straight paths. For the mid-gap modes from 1430 to 1683 nm, the edge states can smoothly pass through the zigzag path with negligible backscattering. Therefore, the designed PTI has a gap/mid-gap ratio of 16.25%. The bandwidth is slightly smaller than the results from 1430 to 1683 nm, the edge states can smoothly pass through the zigzag path with negligible backscattering. Therefore, the designed PTI has a gap/mid-gap ratio of 16.25%. The bandwidth is slightly smaller than the results of band structure because the edge states near \( k_x = 0 \) are more easily reflected. It is interesting that the low-loss window of IFS12 zigzag path is the overlapped operating band of edge states at IFS12 and IFS21 straight paths. The reason may be that the sharp corner part of IFS12 zigzag path owes the same structure as IFS21 straight path (highlighted by the red circles in the insets of Figure 4a, d), which inhibits the light beyond the bandwidth of IFS21 straight path.

The bandwidth of the designed PTI can be immune to the disorder. In Figure 5, we consider four kinds of structural defects: the offset, the rotation, the scaling, and the deformation of the triangular air hole, respectively. All cells of IFS21 zigzag path have the same kind of defect with the different random value. The coupling effect between \( K \) valley and \( K' \) valley will increase with increasing the size of defect and the robustness of the edge states is broken. Due to the larger group velocity \( v_{\text{edge}} \), the edge states near \( k_x = 0 \) have the stronger topological protection than the edge states near \( k_x = 0 \). So the bandwidth is reduced under all four kinds of defects. As shown in Figure 5, the bandwidth only decreases slightly as 1450–1683 nm (\( \gamma = 14.87\% \)) under the 1%, 5%, 3%, and 3% random-bias of the offset, the rotation, the scaling, and the deformation, respectively. This preparation tolerance (about 4 nm) can be realized by the current nanofabrication technology, such as the aberration-corrected electron-beam lithography [22].

4 Conclusion

In conclusion, we demonstrate a broadband PTI achieved by a honeycomb lattice with triangular air holes. The triangular air hole configuration distributes more energy in the geometric perturbation region and opened the topological nontrivial gap up to 33.4%. We also show that the theoretical limit bandwidth of edge states without backscattering is limited by both the energy gap and the group velocity of edge states. The topological nontrivial gap size of the designed VPC is larger than its bandwidth and the edge state bands of both interfaces do not extend across the whole gap. Based on the designed VPCs, broadband transmission of the edge states without backscattering is achieved within 1430–1683 nm (\( \gamma = 16.25\% \)). The bandwidth of defective PTI with 1% random-bias still maintains a broad wavelength band as 1450–1683 nm. The triangular-hole array approach may find applications in broadband optical data networks and nonlinear topological photonics.

Note added—After the submission of this paper, we found the independent discovery of the VPC with the quasi-hexagonal holes array, which was used for the realization of the electrically pumped topological laser [23].

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References

Appendix Numerical simulation

The band structures are calculated by the COMSOL RF module. The model equation is \( \nabla \times \nabla \times E - k_0^2 \varepsilon E = 0 \), where \( E \) is the electric field, \( k_0 \) is the wave vector in vacuum, \( \varepsilon \) is the permittivity distributions of simulation model. The Floquet periodic boundary conditions are used. The eigen frequencies and the modal field distributions with different \( k \)-vector can be calculated and the whole band structures are formed by sweeping the \( k \)-space.

The electric field distributions of the edge state traveling along interface are calculated by the Numerical FDTD Solutions. The model equation is the Maxwell equations \( \nabla \times E = -\mu_0 \partial H/\partial t \) and \( \nabla \times H = -\varepsilon_0 \partial E/\partial t \), where \( H \) is the magnetic field, \( \mu_0 \) and \( \varepsilon_0 \) are the vacuum permeability and the vacuum permittivity, respectively. A two-dimensional FDTD model consists of 20 \( \times \) 30 units and the perfectly matched layer boundary conditions are used. The simulations time were performed for 5000 fs. As shown in Figure 4a, b, the dipole source \( H = H_x + iH_y \) is placed at the center of a specific unit near the interface and two power monitors \( S_1 \) and \( S_2 \) are used for calculating the transmission \( (T = P_{S1}/P_{S2}) \), where \( P_{S1} \) and \( P_{S2} \) are the power at two monitors.