Research article

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Analog signal processing through space-time digital metasurfaces

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Abstract: In the quest to realize analog signal processing using subwavelength metasurfaces, in this paper, we present the first demonstration of programmable time-modulated metasurface processors based on the key properties of spatial Fourier transformation. Exploiting space-time coding strategy enables local, independent, and real-time engineering of not only amplitude but also phase profile of the contributing reflective digital meta-atoms at both central and harmonic frequencies. Several illustrative examples are demonstrated to show that the proposed multifunctional calculus metasurface is capable of implementing a large class of useful mathematical operators, including 1st- and 2nd-order spatial differentiation, 1st-order spatial integration, and integro-differential equation solving accompanied by frequency conversions. Unlike the recent proposals based on the Green's function (GF) method, the designed time-modulated signal processor effectively operates for input signals containing wide spatial frequency bandwidths with an acceptable gain level. Proof-of-principle simulations are also reported to demonstrate the successful realization of image processing functions like edge detection. This time-varying wave-based computing system can set the direction for future developments of programmable metasurfaces with highly promising applications in ultrafast equation solving, real-time and continuous signal processing, and imaging.

1 Introduction

The history of analog computation comes from several electronic and mechanical computing machines developed to implement simple mathematical operations [1]. Due to having large sizes and slow responses, such analog computing devices were then totally overshadowed by the emergence of faster and more efficient digital integrated circuits in the second half of the twentieth century, where a far superior performance for real-time analysis or processing applications was demanded [2]. Relying on a decade of fruitful development and the recent breakthrough in the seemingly unrelated field of metamaterials, Silva et al. [3] brought the analog computations back to the competition as “computational metamaterials” to overcome the speed and energy limitations as well as data conversion loss of digital techniques. In this way, different mathematical operations (spatial differentiation, integration, or convolution) can be realized as electromagnetic (EM) waves propagate through the metamaterial layer. Motivated by the recent renewed interest in wave-based analog signal processing, different proposals within two separate categories have been investigated in one of which the mathematical operators are directly realized in the real-space coordinate using a specially designed structure (Green's function (GF) approach in the literature) [4–14] and in the other group, the transfer functions associated with the operator of choice are realized in the spatial Fourier domain governed by graded-index lenses (metasurface approach in the literature) [15–18]. Although the former case offers great simplicity in accomplishing analog signal processing, it predominantly suffers from three major drawbacks of narrow spatial bandwidths, poor gain levels, and sophistication of access to arbitrary transfer functions [6, 19]. This issue poses great restrictions on the applicability of the architectures achieved by the second method in real-life scenarios. Besides, in a plethora of applications demanding real-time processing of signals and/

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or images, such as medical and satellite applications, reprogrammable and switchable accessing to different mathematical operators at the same time would bring tremendous benefits [20, 21]. Among them, spatial differentiation is a fundamental mathematical operation used in any field of science or engineering. In image processing, spatial differentiation enables image sharpening and edge-based segmentation, with broad applications ranging from microscopy and medical imaging to industrial inspection and object detection [22]. Although numerous efforts have been paid to expand the functionalities of wave-based signal processing systems, to the best of authors' knowledge, no programmable analog computing system has been reported yet. Most importantly, using the previous proposals based on the metasurface approach, an unwanted polarization conversion was imposed on the analog computing system [16]. We believe that the rapid developments in the land of programmable metasurfaces in recent couple of years provide sufficient maturity for them to resolve the unsolved challenges in this research domain.

Metasurfaces are defined broadly as artificial thin films created from subwavelength arrays of structured elements [23, 24], engineered to manipulate different properties of EM waves like polarization [25], amplitude [26], and phase [27–31]. Recently, a new class of metasurfaces called digital coding, and programmable metasurfaces was pioneered by Cui et al. [32], which is represented in a digital manner with binary codes. The digital description of coding metasurfaces not only facilitates and accelerates the related design and optimization procedures [33–41], drastically but also is inherently compatible with the switchable active elements, such as PIN-diodes [42, 43]. Hence, all coding elements of a digital coding metasurface can be independently controlled by a field-programmable gate array (FPGA). Through altering the coding sequences stored in the FPGA and then modulating the coding meta-atoms, diverse EM functionalities can be switched in a real-time manner, thereby leading to programmable metasurfaces [44, 45]. However, in most of the recent studies, the coding sequences are generally fixed in time [46–49], and changed by the control system only to switch the functionalities whenever needed (called space coding structure). Although several types of research have focused on designing time-modulated metasurfaces, these approaches were based on analog modulations [50–52]. More recently, Zhao et al. [53] introduced the first time-domain digital metasurface that enables efficient manipulation of spectral harmonic distribution. Afterward, different studies have been conducted to extend the arsenal of metasurface-based wave manipulations where a set of

coding sequences is switched cyclically in a predesigned time period, enabling simultaneous manipulations of EM waves in both space and frequency domains [54–58]. It is worth noting that most of the above space-time-modulated metasurfaces serve to modulate only the phase profile of EM waves, leaving the space free for the other functionalities requiring both phase and amplitude modulations such as wave-based signal processing. To boost up the functionalities accompanied with the current analog computing systems, we will analyze the space-time coding metasurfaces from the perspective of reprogrammable analog calculus systems for real-time and parallel continuous data processing to answer this question that “*Are space-time digital metasurfaces useful to perform analog signal processing?*”.

In this paper, we propose a space-time digital metasurface as a reprogrammable versatile signal processor to realize different transfer functions such as 1st- and 2nd-order spatial differentiation, 1st-order spatial integration, and integro-differential equation solving based on the key properties of spatial Fourier transformation (metasurface approach). The space-time coding strategy is exploited to tailor the position-dependent phase and amplitude of the meta-atoms at one specific harmonic frequency so as to imitate the transfer function associated with the operator of choice. To the best of our knowledge, it is the first time that realization of multiple analog signal processing functions by using a single programmable architecture at the microwave frequencies is reported.

2 Analog signal processing framework

The general concept of wave-based signal processing is graphically elucidated in Figure 1. The proposed sketch consists of two cascaded submodules: (1) a single graded-index lens (or focusing metasurface) [59] to perform both Fourier and inverse Fourier transformations and (2) a reflective space-time metasurface to execute signal processing operators. According to the fact that $\text{FT}\{\text{FT}[f(x)]\} \propto f(-x)$, a single bi-functional block is enough to accomplish both FT and inverse FT transformations for the signals propagating along $-z$ and $+z$ directions, respectively, instead of using an additional block for IFT realization in the output [16]. More particularly, the Fourier block can be simply realized with a GRIN dielectric slab with permeability μ_0 and a parabolic variation of permittivity $\epsilon(x) = \epsilon_c[1 - (\pi/2L_g)^2x^2]$ where ϵ_c is the permittivity at the central plane of the GRIN and L_g denotes the

characteristic length [3]. Assuming a linear space invariant system in which $f(x)$ and $g(x)$ indicate the transverse field profiles pertaining to an arbitrary input signal and the corresponding output, respectively, the overall response of the system can be theoretically modeled as:

$$g(x, y) = f(x, y) * h(x, y) \\ = \iint h(x - x', y - y') f(x', y') dx' dy'. \quad (1)$$

Here, $h(x)$ is the desired two-dimensional (2D) impulse response of the system and $*$ stands for the linear convolution operation. Relying on the Fourier-transforming property of the GRIN lenses, the transverse distribution of the input wave will be translated into the spatial Fourier domain once it leaves the lens. Therefore, the mathematical formulation of the problem must be followed in the Fourier domain as:

$$G(k_x, k_y) = F(k_x, k_y) * H(k_x, k_y) \quad (2)$$

in which, $G(k_x, k_y)$, $F(k_x, k_y)$, and $H(k_x, k_y)$ refer to the Fourier version of the output signal, input signal, and the transfer function associated with the desired signal processing operation, respectively. Here, (k_x, k_y) are the 2D spatial frequency variables. Although realizing 2D mathematical operations can be obtained through modulating the metasurface along both x and y directions, without loss

of generality, we limit our discussion to the 1D system, which is symmetric along the y axis. The output field, in this case, is calculated by:

$$g(x) = FT^{-1}[\Gamma(x)F(k_x)] \quad (3)$$

wherein, FT^{-1} denotes the inverse Fourier transform. Note that the real-space coordinates x at the metasurface represent k_x as if the position-dependent reflection coefficient $\Gamma(x)$ implements the desired transfer function $H(k_x)$. Indeed, the real-space coordinate x plays the role of k_x after the input field passes through the Fourier lens. Furthermore, any arbitrary transfer function with specific values over different spatial frequencies, k_x , can be elaborately realized through modulation of both reflection phase and amplitude responses of the proposed metasurface at the corresponding position, x . In comparison to the conventional devices, like standard $4f$ optical processing systems [60], the proposed system reveals reconfigurability, compatibility with the integrated architectures, and the potential of accomplishing parallel computations due to providing different harmonic channels. Besides, by properly engineering the metasurface sub-block, the proposed structure can be adapted to implement more complex mathematical operations, such as local phase control, that are not achievable with standard spatial analog filters. In addition, it provides higher-resolution reconstruction

Figure 1: Schematic demonstration of the proposed analog signal processing scheme. The space-time digital metasurface serves to realize the transfer function associated with the operator of choice in the Fourier domain. The low-frequency digital signals are utilized to modulate the metasurface in order to instantaneously control the equivalent phase and amplitude of the occupying elements at a specific frequency harmonic.

because the space-time digital meta-atoms can be deeply subwavelength. Alignment issues may also be reduced if the entire system can be manufactured in one block [61].

3 Space-time digital metasurface design

3.1 Meta-atom design

In this section, we exploited a two-bit programmable digital particle, as illustrated in Figure 2a, to build the space-time coding metasurface realizing the above-mentioned signal processing concept. The coding element consists of four parts: (1) metallic resonators, (2) an F4B substrate with $\epsilon_r = 2.65$, $\tan\delta = 0.001$, $h_1 = 3$ mm, (3) a copper ground plane with the conductivity of $\sigma = 5.7 \times 10^7$ S/m to make a reflective structure, and (4) the biasing network [62]. The metallic vias connect the rectangular patches to the separate biasing lines and the central strip to the ground plane. The biasing lines are etched on an F4B layer with the thickness of $h_2 = 0.5$ mm beneath the ground plane while being fed by distinct digital AC voltages V_{B1} and V_{B2} (Figure 2a). The spatial periodicity of elements in both horizontal and vertical directions is chosen as $D = 12$ mm so that the size of supercells comprising 7×7 meta-atoms becomes equal to one half-wavelength at 3.5 GHz. As displayed in Figure 2a, two varactors are embedded in the digital meta-atom to connect the metallic strips together through a series RLC circuit. The two AC voltages are

utilized to tune the capacitance level of the varactors, independently [63]. We employed a series RLC model ($R = 0.8 \Omega$, $L = 0.7$ nH, and controllable C) to circuitally represent the varactors in our full-wave simulations around the central frequency. A comprehensive parametric study has been accomplished to seek the best group of four meta-atoms exhibiting constant $\pi/2$ phase differences when different voltage levels have been applied to the varactors. Thus, different capacitances of $(C_1, C_2) = (2.7$ pF, 2.7 pF), $(C_1, C_2) = (2.7$ pF, 0.7 pF), $(C_1, C_2) = (1$ pF, 1 pF), and $(C_1, C_2) = (0.6$ pF, 0.6 pF) have been attained to elucidate the reflection response of “00”, “01”, “10”, and “11” coding states, respectively.

The other structural parameters are $L = 10$ mm, $s = 5$ mm, $w = 2$ mm, and $g = 1.2$ mm. The numerical simulations are executed via the frequency domain solver of the commercial software package, CST Microwave Studio, to inspect the performance of the designed coding meta-atom. Periodic boundary conditions are applied in the x and y directions while Floquet ports are assigned to the z -direction. An x -polarized plane wave normally shines on the meta-atoms. For different values of the varactor capacitance, the simulated reflection amplitudes and phases are illustrated in Figure 2b and c, respectively. As can be seen, a constant 90° phase difference with more than 0.85 reflectivity has been successfully achieved between 3.4 and 3.6 GHz. The less than unity reflectivity of the designed meta-atoms originates from the dielectric and ohmic losses. The effect of such an absorption on the overall performance of the designed processors is comprehensively discussed in Supplementary material B.

Figure 2: (a) The designed two-bit coding meta-atom which embeds two varactors with controllable capacitances. (b) The reflection phase, and (c) the reflection amplitude spectra of the coding meta-atoms for different capacitances of $(C_1, C_2) = (2.7$ pF, 2.7 pF), $(C_1, C_2) = (2.7$ pF, 0.7 pF), $(C_1, C_2) = (1$ pF, 1 pF), and $(C_1, C_2) = (0.6$ pF, 0.6 pF), resulting in “00”, “01”, “10”, and “11” coding states, respectively.

3.2 Time-varying coding strategy

To realize the metasurface processor depicted in Figure 1, a space-time digital metasurface including several programmable columns is utilized, each of which is occupied with eight coding elements (Figure 3). As seen, each column's meta-atoms are electrically connected by two biasing lines and, hence, imitate identical digital code by sharing common time-varying control voltages. The overall size of the 1D metasurface processor is $685 \times 342 \text{ mm}$ ($8\lambda_0 \times 4\lambda_0$). As shown in Figure 3, through applying a proper set of time-varying biasing signals $S_1, S_2, \dots$, the digital code of each column cycles arbitrarily between two-bit modes with a certain time periodicity, T_m ($f_m = 1/T_m$). Actually, the digital code of the meta-atoms can be potentially altered L times in each time period in which the order of changes is expressed by a time-coding sequence, e.g., $S = \{10, 11, 11, 00\}$ ($L = 4$). We should remark that the modulation frequency, f_m , is assumed much smaller than the incident wave frequency [64, 65]. According to the time-switched array theory and for a normal monochromatic plane wave, the meta-atoms are subject to a periodic time modulation of the reflection coefficient, which can be mathematically expressed as $E_{\text{ref},i}(t) = \Gamma_i(t) * E_{\text{inc},i}(t)$ in which $\Gamma_i(t) = \sum \Gamma_i^n \Pi^n(t)$. Here, $\Gamma_i^n \in \{e^{j0}, e^{j\pi/2}, e^{j\pi}, e^{j3\pi/2}\}$ refers to the complex reflection coefficient of the digital meta-atoms in the i th column and n th time

interval ($1 \leq n \leq L$), and finally, $\Pi^n(t)$ is a shifted pulse function with the modulation frequency of f_m which possesses a non-zero value only during the n th interval. Fourier representation of the time-modulation reflection coefficients yields $\Gamma_i(t) = \sum a_i^m e^{j2\pi m f_m t}$ and $\Gamma_i(f) = \sum a_i^m \delta[2\pi(f - m f_m)]$ in which [54, 55]

$$a_i^m = \sum_{n=1}^L \frac{\Gamma_i^n}{\pi m} \sin\left(\frac{\pi m}{L}\right) \exp\left[-j(2n-1)\frac{\pi m}{L}\right] \quad (4)$$

has been obtained after several mathematical manipulations. Theoretically speaking, a_i^m discloses the equivalent reflection coefficient of the i th column at the m th harmonic frequency, i.e., $f + m f_m$. Being as a weighted average of the reflection coefficients in the static case, a_i^m values drastically expand the range of our choices where arbitrary equivalent amplitudes and phases with high quantization levels can be artificially synthesized by using proper sets of time-coding sequences in spite of exploiting unitary-amplitude physical meta-atoms with only two-bit phase modulation. This feature can be thought of as the inflection point of our study, which elaborately enables analog signal processing with the digital programmable metasurfaces, an apparent paradox that can be resolved using a space-time coding strategy. A larger time-coding sequence (larger L) results in more steps of available equivalent phase and amplitude in each harmonic.

Figure 3: The overall scheme of the proposed space-time coding metasurface and its occupying supercells including 7×7 connected elements with identical digital states to implement the desired amplitudes/phases distribution dictated by the transfer function of interest, where different time-varying sequences lead to a specific harmonic amplitude/phase distribution.

4 Results and discussion

For our proposed system illustrated in Figures 1 and 3, two 17×8 and 25×8 reprogrammable metasurface processors with the dimension of $P_x \times P_y$ ($-P_x/2 < x < P_x/2$ and $-P_y/2 < y < P_y/2$) are responsible for realizing the position-dependent amplitudes and phases corresponding to the operators of choice. In this way, the reflection coefficient of the time-modulated digital meta-atoms should spatially mimic the k_x -dependency of the transfer function associated with the desired mathematical operation. Indeed, the main idea of this paper is to transform the energy of the incident signal into the high-order harmonics so that the required spatial amplitude variation dictated by the transfer function of choice is imitated by the position-dependent equivalent amplitude of the meta-atoms at one specific harmonic. To show the versatility of the proposed signal processing system, the 1st-order and 2nd-order spatial differentiation, 1st-order spatial integration, and integro-differential equation solving are demonstrated in this paper. According to the Fourier transform principles, the transfer function accompanied by the above-mentioned 1D operators are

$$H(k_x) = j2k_x/P_x \quad (5)$$

$$H(k_x) = -4k_x^2/P_x^2 \quad (6)$$

$$H(k_x) = \begin{cases} 1 & |k_x| \leq d \\ d/jk_x & |k_x| > d \end{cases} \quad (7)$$

$$H(k_x) = \frac{jk_x}{\beta + jak_x - k_x^2} \quad (8)$$

respectively. The last transfer function belongs to the solving operator of a constant-coefficient ordinary integro-differential equation (CIDE), i.e., $dg(x)/dx + ag(x) + \beta \int g(x)dx = f(x)$. Since the metasurface processor is inherently passive with a finite lateral dimension, we have normalized all transfer functions to guarantee that the maximum reflection coefficient of digital meta-atoms across the space-time metasurface is below unity and avoid gain requirements. Meanwhile, in the case of 1D integration, the singularity of the ideal transfer function at $k_x = 0$ has been circumvented by truncating it within a spatial bandwidth of $2d = P_x/8$ around $k_x = 0$. In an attempt to realize Eqs. (5)–(8) with space-time digital metasurfaces at one specific frequency harmonic, the 2D space-time coding matrix of each metasurface column is optimized so as to produce the desired equivalent reflection phase and amplitude in the real-space coordinate, x , playing the role of the transfer function of choice in the spatial Fourier

space, k_x . Although Eqs. (5)–(8) give seemingly continuous reflection profiles, the proposed calculus metasurfaces can tailor the scattered fields in the integer steps of P , corresponding to the positions of the digital meta-atoms. The optimization procedure is as follows: we seek for those time-coding sequences for which the difference between the equivalent amplitude/phase of each metasurface column at the zeroth harmonic and that of the desired transfer function at the same position is minimized. The error function is defined as the sum of squares of the differences at a finite number of columns for both real and imaginary parts:

$$\text{Error} = w_a \sum_i (|a_0(x_i)| - |H(x_i)|)^2 + w_p \sum_i (Ra_0(x_i) - RH(x_i))^2 \quad (9)$$

Here, w_p and w_a represent the weight coefficients used to balance the phase and amplitude contributions, respectively, and x_i refers to the position of each metasurface column. It should be noted that the optimization procedure of Eq. (9) can be performed for each of the harmonics. In this way, four different space-time coding metasurfaces are programmed to implement the operators pertaining to the 1st-order and 2nd-order spatial differentiation, 1st-order spatial integration, and integro-differential equation solving. To present sufficient degrees of freedom, each column of the metasurface processors is modulated by suitable time-varying sequences with the length $L = 16$. The modulation frequency is chosen as 500 kHz [54] throughout the paper, which is much smaller than the center frequency. The corresponding space-time coding matrices set for the zeroth harmonic frequency are shown in Supplementary Figure S1. A full-wave analysis is carried out in CST Microwave Studio to authenticate the transfer function accompanied with each digital metasurface processor wherein an x -polarized plane wave normally impinges on the structure and the reflected electric fields along the transverse plane are plotted in Figures 4, 5, 6, and 7. The results have been obtained by the time-domain solver of the CST Microwave Studio. Without loss of generality, the parameters of α and β are chosen as 0.9 and 0.012, respectively. The amplitude and phase of the transfer functions are plotted in the panels (a) and (c) of these figures, respectively. All results are presented for the zeroth harmonic at 3.5 GHz. The detailed information about the other harmonics can be found in Supplementary material C. The excellent agreement between the numerical simulations and the ideal transfer functions proves the validity and versatility of the proposed signal processing approach. The linear and

quadratic behaviors of the 1st-order and 2nd-order differentiation operations, respectively, are quite observable. Also, the required phase profiles are successfully provided by the space-time digital metasurface. As mentioned before, the equivalent amplitude of the space-time metasurface at each position can freely vary between 0 and 1, which does not basically arise from the dielectric and ohmic losses. In fact, the rest of the power is coupled to the other frequency harmonics. The detailed information is given in Supplementary material C. We emphasize that the space-time coding metasurface can be programmed to realize the transfer functions associated with different signal processing operators at the same time without adopting any change in the shape and geometry of the structure. Moreover, we should remark that the acceptable agreement between the results can be observed across the entire spatial frequency bandwidth, i.e., $|k_x| \leq k_0$, meaning that unlike analog computing platforms based on GF approach [4–7, 66], the space-time signal processors in this paper operate effectively even for those input signals containing high spatial frequencies. Albeit, it should be noted that the overall performance of the system is surely affected by the properties of the Fourier lens. Designing the Fourier lenses with large angle-of-views has attracted great attention in recent years, from microwave to optical frequencies [59, 67, 68]. To the best of our knowledge [59, 68], provide worthy solutions for Fourier lenses with wide

spatial bandwidths which show acceptable efficiencies with more than 120° ($0.866k_0$) and 170° ($0.9962k_0$) angle-of-views, allowing us to assume an ideal response for the Fourier lens of our proposed system.

In order to assess the quality of the output signals, the electric fields $f(x) = \exp(-x^2/c_1^2)$, $f(x) = \text{sinc}(x/c_2)$, $f(x) = -x \exp(-x^2/c_3^2)$, and $f(x) = x \exp(-x^2/c_3^2)$ are considered as the input functions of the system corresponding to Figures 4, 5, 6, and 7, respectively, in which $c_1 = 0.001$, $c_2 = 0.01$, and $c_3 = 0.01$. The input field profiles, the simulated output signals, and the ideal analytical ones are demonstrated in Figures 4, 5, 6, and 7 (b), (d). We should remark that for ease of presentation, suitable phase factors $\exp(i\varphi)$ have been involved in the output profiles as the multiplicative coefficients to ensure a pure real field at the observation plane. As can be seen, the proposed space-time coding processing system reveals a good performance by exposing the reflected fields whose spatial variations are in perfect agreement with the exact responses. In the end, we intend to demonstrate the potential applications of the proposed 1D differentiator in image processing, where edge detection is known as a fundamental step [22]. This terminology enables the extraction of boundaries between two regions with different texture characteristics. In the 1D differentiated image being modulated along the x -direction, edges along the vertical direction will have higher pixel intensity levels than those surrounding them,

Figure 4: A comparison between the simulated and ideal results for (a) amplitude and (c) phase of the transfer function belonging to the 1st-order spatial differentiation and (d) the output signal corresponding to (b) a Gaussian input signal. The results are presented for zeroth harmonic.

Figure 5: A comparison between the simulated and ideal results for (a) amplitude and (c) phase of the transfer function belonging to the 2nd-order spatial differentiation and (d) the output signal corresponding to (b) a sinc-shape input signal. The results are presented for zeroth harmonic.

Figure 6: A comparison between the simulated and ideal results for (a) amplitude and (c) phase of the transfer function belonging to the 1st-order spatial integration and (d) the output signal corresponding to the (b) input signal $f(x) = -x \exp(-x^2/c_3^2)$. The results are presented for zeroth harmonic.

yielding a directional selectivity. The simulated transfer functions are exploited to perform 1D spatial filtering on “Iran China” words along with three rectangular shapes (Figure 8a) and “Butterfly” picture (Figure 8c) as two different image fields shining the space-time

metasurface. The normalized output fields are achieved, as shown in Figure 8b and d, which successfully expose all outlines of the incident images in the horizontal orientations with the same intensity. Although the proof-of-concept simulations are related to 1D space-time

Figure 7: A comparison between the simulated and ideal results for (a) amplitude and (c) phase of the transfer function belonging to the integro-differential equation solving operator and (d) the output signal corresponding to the (b) input signal $f(x) = x \exp(-x^2/c_3^2)$. The results are presented for zeroth harmonic.

Figure 8: Illustration of the edge detection capability of the proposed 1D space-time metasurface processor upon accomplishing the 1st-order spatial differentiation. (a), (b) the input and (b), (d) output images. As can be seen, all exterior boundaries along the horizontal direction have been successfully extracted. The results are presented for zeroth harmonic.

metasurface configurations, we can simply extend them into 2D signal processing schemes by programming each meta-atom in an arbitrary row and column, independently, to implement the required spatial Fourier content of the transfer function along either single or both directions. Also, we should remark that this paper focused on realizing the desired transfer functions at only one specific harmonic. Theoretically speaking, the

amplitude and phase distributions of any arbitrary harmonic can be obtained with meticulously designed $\Gamma_i(t)$. Following the approach presented in [69], one may achieve a flexible control over dual harmonics by adding an extra initial phase ψ_0 and a time delay t_0 , simultaneously, i.e., $\Gamma_i(t - t_0)\exp(i\psi_0)$. This ability would be very promising in the realization of parallel analog signal processing [70].

5 Conclusion

In this work, a reprogrammable space-time digital metasurface processor was elaborately designed to be utilized in an analog computing platform based on the metasurface approach. Through applying proper time-domain coding signals, the space-time digital metasurface is empowered to dynamically realize the required phases and amplitudes of the transfer function associated with the operator of choice at one specific harmonic frequency. Several illustrative examples were presented to demonstrate the versatility of the proposed metasurface processor in performing diverse mathematical operations and functionalities such as spatial differentiation, spatial integration, solving integro-differential equations, and edge detection in a real-time manner. The proposed space-time coding strategy takes a great step forward in developing reprogrammable wave-based signal processors with offering great versatility in the operations.

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