Perspective

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Novel non-plasmonic nanolasers empowered by topology and interference effects

<https://doi.org/10.1515/nanoph-2021-0265>

Received May 26, 2021; accepted August 6, 2021;

published online August 19, 2021

Abstract: Historically, nanophotonics deals with a control of light at the nanoscale being closely connected with the rapid advances in plasmonics – the physics of surface plasmon polaritons supported by metal–dielectric interfaces. Properly engineered nanostructures allow the subwavelength propagation of light and its strong confinement in nanowaveguides and nanocavities, making possible the field enhancement and lasing. *Spaser* was suggested as a special type of nanolaser with a very small footprint that can be modulated quickly thus becoming a good candidate for on-chip optical data processing. However, recent developments in the physics of high-index dielectric nanoparticles and resonant dielectric metasurfaces allowed to advance the field of nanophotonics and introduce novel nonplasmonic nanostructures and nanolasers empowered by topology and interference effects. Here we present first some examples of experimentally realized spasers, and then discuss the recent developments in the cutting-edge high-index dielectric nanostructures employed for nonplasmonic nanolasers based on Mie resonances, anapole states, bound states in the continuum, and the physics of topological phases.

Keywords: bound states in the continuum; Mie resonances; nanolaser; spaser; topological photonics.

1 Introduction

In 2003, Bergman and Stockman [1] introduced a novel theoretical concept of surface plasmon amplification by

stimulated emission of radiation (spaser), where surface plasmon polaritons replace the role of photons in classic photonic lasers based on the resonant energy transfer between excitons and surface plasmons. Thus, the plasmonic nanolaser emerged as a special class of lasers with the physical dimensions below the diffraction limit of light. After this theoretical proposal, in 2009 the experimental realizations of spaser [2] and another type of plasmonic nanolasers [3] have been demonstrated, and a novel field of nanolasers with the applications in subwavelength active photonic structures has emerged. A typical nanolaser has a very small footprint that can be modulated quickly, and thus becoming a candidate for on-chip optical computing. Such on-chip light sources are critical for realizing integrated photonic circuits and many other applications [4, 5].

Despite spasers help to reduce radiative losses and sizes of optical devices, they also introduce parasitic Ohmic losses originating from their metallic parts. The inherently high metal loss causes high lasing threshold and makes metal-based nanolasers impractical [6]. Nevertheless, the suggestion of a spaser inspired subsequent efforts to make its dielectric analogues. In general, purely semiconductor-based lasers using photonic band gap or whispering gallery modes, in photonic crystal or microdisk cavities, respectively, have been widely studied [5, 7, 8]. These attempts resulted in nanolasers that were comparable to, or even smaller than, the operating wavelength. But so far, the size of semiconductor lasers has been limited by a few micrometres due to the mode leakage or the challenges in nanofabrication. Further reduction of sizes to the nanoscale is challenging particularly due to radiative losses.

Recently, active all-dielectric metaphotonics has emerged as a new and rapidly expanding field of subwavelength optics that is based on the physics of electric and magnetic optical Mie resonances excited in high-index dielectric nanoparticles and interferences of resonant Mie modes [9]. Like the spaser suggested by Mark Stockman [1], a dielectric nanolaser provides a tight confinement of local electromagnetic fields combined with low losses. Its physics is driven by multipolar interferences available in single dielectric nanoparticles supporting electric and magnetic Mie resonances that provide novel tools to tailor

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the properties of light at the subwavelength scale, often not available with plasmonics. In addition, the new concepts of bound states in the continuum and topological phases allow employing multimode interference effects for creating lasers with superior characteristics, small footprints, and even lower lasing thresholds.

In this perspective, we provide some examples of plasmonic nanolasers, and then discuss the recent developments in the study of cutting-edge nanostructures employed for nonplasmonic nanolasers based on Mie resonances, anapole states, bound states in the continuum, and the recently emerged novel physics of topological phases.

2 Examples of plasmonic nanolasers

One of the first plasmonic lasers was demonstrated by Oulton et al. in 2009 [3], using a cadmium sulphide nanowire on a magnesium fluoride (MgF_2)-coated silver

film (Figure 1a). The plasmonic mode was formed in this hybrid structure of the metal–insulator–emiconductor nanowire with no cutoff frequency. The plasmonic lasing was achieved via optical pumping at cryogenic temperature, exhibiting the mode area as small as $\lambda^2/400$. The lasing peak was observed at wavelengths of 489 nm with a threshold of 100 MW/cm^2 .

In 2012, Lu et al. [10] developed an optimized method of the epitaxial growth of the silver thin film on silicon, which consisted of low-temperature silver deposition followed by room-temperature annealing. The continuous-wave operation of this plasmonic laser was achieved by placing InGaN/GaN core/shell nanorods on the 80-nm-thick atomically smooth silver film (Figure 1b). The lasing peak was observed at wavelengths of 510 nm with thresholds of 2.1 and 3.7 kW/cm^2 for the temperatures of 8 and 78 K, respectively. The lasing action was measured only at temperatures below 120 K. Also, plasmonic lasers were operating in the full-visible spectrum regions by adjusting the composition of InGaN core in the same cavity geometry (Figure 1c) [11]. Subsequently, Zhang et al. [12]

Figure 1: Examples of various plasmonic nanolasers.

(a) Plasmonic nanolaser consisting of a CdS nanowire on top of a silver film separated by a MgF_2 spaser (adopted from Ref. [3]). (b) Plasmonic nanolaser of a GaN nanowire on the epitaxially grown silver film (adopted from Ref. [10]). (c) Lasing images and spectra in the tunable plasmonic lasers of $\text{In}_x\text{Ga}_{1-x}\text{N}$ nanowires (adopted from Ref. [11]). (d) Pseudo-wedge plasmonic nanolaser of a ZnO nanowire on a silver grating covered with 3 nm thick Al_2O_3 (adopted from Ref. [14]). (e) 2D plasmonic nanolaser of a CdS nanoslab with 45 nm thickness and $1 \mu\text{m}$ length, on top of a silver film separated by a MgF_2 spaser (adopted from Ref. [17]). (f) 3D plasmonic laser of an InGaAsP semiconductor nanodisk with silver nanocavity (adopted from Ref. [20]).

demonstrated a room-temperature, single-mode, UV plasmonic laser with threshold of 3.5 MW/cm^2 using a GaN nanowire with a triangular cross-section on an aluminum film separated by a thin layer of silica. The close-contact planar semiconductor–insulator–metal interface supported an efficient exciton–plasmon energy transfer with high Purcell factor and low scattering loss.

In 2018, Chou et al. [13] reported the plasmonic nanolaser consisting of a ZnO nanowire and a pseudowedge plasmonic waveguide. The pseudowedge plasmons were strongly confined at the top edges of the notch in the subwavelength silver grating. An extremely small mode area $A_m = 3.5 \times 10^{-5} \lambda^2$ and mode volume $V_m = 1.5 \times 10^{-5} \lambda^3$ were obtained at the gap region between the ZnO nanowire and silver grating. Subsequently, in 2019 Li et al. [14] demonstrated a ZnO nanowire plasmonic laser, using the graphene–insulator–metal (GIM) structure (Figure 1d). The laser was operated at room temperature in the UV wavelength region. Notably, various GIM structures modulated plasmonic dispersion characteristics and yielded different lasing behaviors.

In addition to the plasmonic lasers using one-dimensional (1D) active materials, two-dimensional (2D) and three-dimensional (3D) plasmonic structures have also been implemented [15, 16]. In 2011, Ma et al. [17] demonstrated a plasmonic laser consisting of a single-crystalline semiconductor CdS nanoslab on the silver substrate, which were separated by a 5-nm-thick MgF_2 layer (see Figure 1e). The plasmonic mode was generated inside a nanogap layer and remained bound by strong feedback arising from total internal reflection of surface plasmons. The Purcell effect resulted in an 18-fold enhancement of the spontaneous emission rate. In the following study in 2017, the plasmonic lasers were developed to be more compact and faster with lower threshold and power consumption, when the cavity size approaches or surpasses the diffraction limit [18].

In 2013, plasmonic lasing action was observed from band-edge lattice plasmons in arrays of plasmonic nanocavities in a homogeneous dielectric environment [19]. While isolated plasmonic cavities suffer from large radiative losses and low values of the quality factor (Q factor), arrays of plasmonic nanoparticles exhibit suppressed radiative losses due to the strong electromagnetic interactions between nanocavities. Also, the spontaneous emission rate of an emitter increases due to a large local density of optical states (LDOS). Two-dimensional arrays of Au nanoparticles were fabricated on a glass substrate and covered by a polymer gain layer of polyurethane and IR-140 dye. Then, lasing behavior was observed, including a narrow linewidth of less than 1.3 nm, a 200-fold enhancement of the spontaneous emission rate, and a directional beam emission of divergence angle $<1.5^\circ$.

A full-3D plasmonic laser was also successfully demonstrated in 2010 by Kwon et al. [20]. The plasmonic cavity consisted of a 235 nm thick InP disk with InAsP quantum wells in the middle of the disk as a gain medium (Figure 1f). The top of the disk was covered by a transparent glass substrate, and the bottom and sidewall of the disk was covered with silver. The numerical simulation showed that a whispering-gallery plasmonic mode was confined at the bottom of the plasmonic cavity, exhibiting a mode volume of $0.56(\lambda/2n)^3$. The plasmonic laser was operated by optical pumping through the glass substrate on top at 8 K. The lasing mode was clearly identified as a whispering-gallery plasmonic mode from the spectrum, mode image, and polarization measurements, as well as agreement with numerical simulations. In addition, the difference in the temperature dependence of the laser threshold between the plasmonic mode and the optical mode was confirmed experimentally.

Finally, plasmonic lasers using organic materials rather than semiconductors have also been implemented. In 2018, Wu et al. [21] demonstrated a room-temperature plasmonic laser using single-crystalline all-inorganic CsPbBr_3 nanowires on the silver film. A 5–20 nm thick SiO_2 was introduced as a spaser layer between the nanowire and silver film. As the SiO_2 layer was thinner, the plasmonic confinement was stronger in the gap region. The plasmonic lasing was realized with the lasing threshold of $6.5 \mu\text{J/cm}^2$ at room temperature. In addition, Huang et al. reported the plasmonic laser using lead halide perovskite (MAPbX_3) nanosheets on the gold film with SiO_2 spaser [22]. The resonance of the plasmonic laser was controlled by the pattern shape and size of a gold substrate.

3 Mie-resonant nanolasers

As discussed above, on-chip light sources are critical for realizing integrated photonic circuits. The size of typical semiconductor lasers is limited by a few micrometers, and plasmonic nanolasers look promising to reduce radiative losses and device footprint. However, despite successful demonstrations of plasmonic lasers, most of them operated at low temperatures suffering from Ohmic losses produced by their metallic components. Also, a high level of fabrication techniques was necessary to obtain ultrasmooth metal surfaces for the reduced scattering loss. On the other hands, dielectric cavities efficiently support both small mode volumes and high Q factors.

Recently emerged all-dielectric resonant metaphotonics suggests a novel approach for the subwavelength optics employing electric and magnetic multipolar optical

Mie resonances supported by high-index dielectric nanoparticles [9]. A tight confinement of local electromagnetic fields combined with low losses and multipolar interferences that can be realized in isolated dielectric nanoparticles with Mie resonances provide novel tools to tailor the properties of light at the subwavelength scale, often not available with plasmonics.

On the way to miniaturization of nanoscale laser sources, two major problems have to be solved: (i) Optical gain of many materials is not high enough to compensate losses in the nanoscale systems at room temperatures; (ii) radiative losses in subwavelength nanocavities are usually very high. In two recent papers [23, 24], a novel approach to overcome these challenges has been demonstrated. This approach is based on the concept of metaphotonics, and it explores interferences of electric and magnetic Mie resonant modes.

Namely, the far-field engineering allowing to overlap several modes and design the so-called *supercavity mode* [25] being closely connected with a special type of the bound state in the continuum that appears in isolated particles due to the Mie-type mode hybridization. In accord with the Friedrich–Wintgen theory of interfering resonances [26], the radiating tails of leaky modes outside the

resonator can cancel out each other by destructive interference, achieving the field localization. In this way, in a single nanocylinder, the Mie resonances and Fabry–Pérot resonances can interact strongly near the avoided crossing regime [27]. Notably, these modes are approximately orthogonal inside the resonator, and thus they interfere predominantly outside, realizing the so-called supercavity regime (an analogue of quasi-BIC state). The supercavity mode reduces the radiative losses of the system enabling gain/loss compensation in nanoparticles of a small size.

This idea was successfully employed for GaAs cylinders to suppress radiative losses as much as possible for creating subwavelength lasers without any metallic components (Figure 2a) [23]. Using the concept of quasi-BIC, a single GaAs nanocylinder on a quartz substrate reaches the optimal size with 500 nm in diameter and 330 nm in height at a lasing wavelength of ~ 825 nm, which corresponds to a size-to-wavelength ratio as low as 0.6. The optimized geometric parameters of the cylinder lead to a high Q factor of ~ 970 .

The other strategy based on this concept was realized in the visible frequency range and room temperatures with the use of single-crystalline perovskite nanocuboids possessing low-order Mie modes as well as extremely high gain

Figure 2: Two recent demonstrations of the Mie-resonant nanolasers.

(a) Scanning electron image (SEM) of GaAs nanoscale cylinder covered by hydrogen silsesquioxane resist (HSQ) (top left). Calculated electric near-field distribution of the cylinder at the lasing wavelength (top right) and evolution of the emission spectrum of the resonator with diameter 500 nm and height 330 nm at different pumping fluence (bottom) (adopted from Ref. [23]). (b) SEM image of CsPbBr₃ perovskite nanocube (top left), calculated electric near-field distribution at the lasing wavelength (top right), and evolution of the emission spectrum of the perovskite nanocube with the side length of 310 nm at different pumping fluence (bottom) (adopted from Ref. [24]).

even without cooling (Figure 2b) [24]. A halide perovskite (CsPbBr_3) allows the photoexcitation level to be high enough to fill the levels near the excitonic state. In addition, a dielectric nanocube structure designed with a small side length of 310 nm supports Mie resonances in visible/near-IR ranges, enabling single-mode lasing at the wavelengths of 530–540 nm. These approaches are based on the excitation of Mie resonances in nonplasmonic resonators which correspond to a subwavelength regime when the characteristic dimensions of the nanocavity become a fraction of the emitted wavelength of light.

Further assembly of such particles into 1D and 2D structures opens additional possibilities for creating highly directional nanoscale lasers with tailorable directivity. Based on these concepts, Hoang et al. [28] experimentally demonstrated a directional GaAs nanolaser at cryogenic temperatures with well-defined, in plane emission, which can be controlled by selective excitation. The lasing threshold is shown to be significantly reduced by optimizing the interparticle gap such that the optimal near-field confinement is achieved at a resonant wavelength corresponding to the highest gain of GaAs. The lasing performance of this nanolaser is orders of magnitude better than a nanowire-based laser of the same dimensions.

4 Anapole metasurface lasers

Anapole, which means “without poles”, comes from the Greek word “ana” (meaning “without”), and this term was first introduced by Yakov Zel’dovich in 1957 [29]. In electrodynamics, the anapole state could be generated by an interference of the fields produced by an electric dipole moment and toroidal dipole moment which have the same radiation patterns in the far-field region [30, 31]. This configuration leads to a radiationless state and enhancements in near-field energy, and thus can be a promising candidate for nonlinear optics and lasing. In 2013, the theoretical and experimental study of the anapoles was based on a metallic metamaterial composed of thin plates with specific perforations generated toroidal modes inside the structure [32].

The anapole state in silicon nanodisks was suggested in 2015 [33]. The authors used an analytical multipole expansion to investigate the unique features of the anapole state. A strong suppression of the total scattering was observed because of the destructive interference of electric and toroidal dipole radiations. The theoretical predictions were confirmed by experiments using silicon nanodisks with 310 nm diameter and 50 nm thickness. Experimental dark-field scattering spectra and near-field enhancement

in the NSOM measurement demonstrated the existence of the anapole regime [33].

Very recently, room-temperature lasing from the anapole lattices of engineered active metasurfaces was demonstrated in Ref. [34]. Strongly localized, high- Q anapole state was excited in the split-nanodisk resonator, which originates from a superposition of the electric dipole and toroidal dipole Mie-resonant optical modes (Figure 3a). The Q factor of the anapole mode depends on the geometric parameters, especially the split-gap size. In the calculation of the Q factor as a function of the gap size, G (Figure 3b, top), we notice the inverse square dependence proportional to $(G - G_0)^2$, where the maximum Q at $G_0 = 228$ nm exceeds 4×10^8 . A significant increase of the Q factor indicates that the anapole mode is related to the resonance in a single element rather than to the symmetry mismatch between the collective mode and radiation modes of the free space. In addition, a multipole decomposition of the field in the unit cell was performed to investigate the contributions of the electric dipole and magnetic quadrupole moments to the stored electromagnetic energy (Figure 3b, bottom).

The experimental realization of the anapole-mode laser was demonstrated in the same paper [34]. Two-dimensional arrays of split-nanodisk resonators were fabricated in InGaAsP membranes with embedded quantum wells (Figure 3c). In the lasing experiments, the samples were optically pumped, and the emission intensity was examined as a function of the pump intensity (Figure 3d). A rapid increase of the output signal and decrease of the linewidth were observed at the wavelength of ~ 1503 nm, above the threshold of ~ 10 $\mu\text{J}/\text{cm}^2$ at room temperature. This result provides a novel approach to engineer emission of light in coupled anapole resonators and active metasurfaces.

5 Topological lasers

Topology, a branch of mathematics related to quantities preserved under continuous deformations, has been emerging and opening a new way to find the exotic properties in various realms of study [35, 36]. In condensed-matter physics, the new phases of material termed as topological insulators were discovered, which have insulating states in the bulk but conducting states on their surfaces. Such conducting states in topological insulators are not affected by the presence of large impurities thus there is no dissipation or backscattering of electricity. These extraordinary features were successfully implanted to electromagnetic regime in 2008 by Haldane and Raghu

Figure 3: Anapole-based nanolaser.

(a) Numerically calculated electric (top) and magnetic (bottom) field cross-sections in the vicinity of the anapole resonance for the split-gap size of 230 nm. Black arrows are guides for an eye. (b) Top, simulated Q factor of the anapole-type mode as a function of the split-gap size. The inset shows a schematic of top view of the metasurface. Bottom, multipole decomposition at the anapole regime and energy stored by the system. Spectra of the electric dipole moment, the magnetic quadrupole moment, and the stored energy are indicated by blue crosses, red open circles, and black solid curve, respectively. The gap size is 230 nm. (c) SEM image of the fabricated metasurface. A cross-sectional view is shown in the inset. (d) Evolution of the normalized output power as a function of the pump fluence and wavelength (adopted from Ref. [34]).

[37, 38], showing unidirectional propagation without backscattering and robustness against structural defects.

In photonics regime, the topological properties can be described by the band dispersion in 2D Brillouin zone [5, 39, 40]. Like the electronic band structure of insulators, optical mirrors have frequency gaps where optical states cannot exist. The Chern number, the topological invariant of a 2D dispersion band, is a quantity that can categorize the quantized behavior of the wavefunctions on the band, and the sum of the Chern numbers of the dispersion bands below the frequency gap represents the topological properties. For example, topologically nontrivial mirrors have a nonzero Chern number.

Once topologically inequivalent two mirrors with different Chern numbers face together, a topological phase transition occurs at the interface, which requires closing the frequency gap, neutralizing the Chern numbers, and reopening the gap [39]. The gapless frequency states at the interface define the topologically protected edge states. The number of edge states equals the difference of the bulk topological invariants across the interface, which is known as the bulk-edge correspondence. The topological features of the edge states enable directional transport and built-in immunity to disorder.

Wang et al. [41] demonstrated the first topological edge-state waveguide in 2009, which was operated at the microwave regime. The waveguide comprised a gyromagnetic periodic photonic crystal of a square lattice of ferrite

rods that were bonded on one side by a nonmagnetic metallic cladding. In the designed structure, a photonic chiral edge state existed at the interface between the photonic crystal and the metal cladding. The experiment showed that the edge states travelled in only one direction and were robust against scattering from disorder.

The photonic topological concept has been extended to a new feedback mechanism of laser. In 2017, Bahari et al. [42] realized the first nonreciprocal single-mode lasing in a topological cavity that was composed of an InGaAsP photonic crystal slab bonded on yttrium iron garnet (YIG) grown on gadolinium gallium garnet (GGG) (Figure 4a, left). The time-reversal symmetry was broken by applying static magnetic field to the cavity. Depending on the direction of the external magnetic field, one-way topological edge states were excited in the topological cavity. To show the robustness against backscattering, an arbitrarily shaped closed contour cavity was examined. Despite the various shape and sharp corners, unidirectional lasing was demonstrated with the external magnetic field. In addition, in 2018, Bandres et al. [43] demonstrated the topological edge-state laser without an external magnetic field (Figure 4a, right). The cavity structure consisted of an aperiodic array of 10-by-10-unit cell coupled ring resonators on an InGaAsP quantum wells. The ring resonators were linked by a set of hopping phases, which provided a synthetic magnetic field and established two topologically nontrivial band gaps. When the perimeter of the ring

resonator array was optically pumped, the topological edge-state lasing mode was observed with a higher slope efficiency than the topologically trivial one, exhibiting the robustness against fabrication defects.

After then, various types of topological cavities were proposed [44–48]. In 2020, Smirnova et al. reported the triade lasing mode in a 2D valley-Hall topological cavity (Figure 4b) [44]. The topological cavity was designed based on the closed valley-Hall domain wall created by the inversion of a staggered sublattice potential in a honeycomb periodic photonic lattice. The triangle-shaped cavity was implemented in an InGaAsP membrane, and two triangular holes in each unit cell of the bipartite lattice have different sizes that lead to the opening of band gaps near the Dirac points of the corresponding Brillouin zone. Room-temperature lasing with a narrow spectrum, high

coherence, and threshold behavior was observed by optical pumping. The emitted beam hosted a singularity encoded by a triade cavity mode that resides in the band gap of two interfaced valley-Hall periodic photonic lattices with opposite parity breaking.

In 2020, Yang et al. [45] realized spin-momentum-locked edge mode in non Hermitian topological photonic system (Figure 4c). A high-performance topological vortex laser was demonstrated in X-shaped photonic crystal cavity which was fabricated in an InGaAsP multiple quantum wells membrane. Two photonic crystals formed a topological interface that were distinct in topology but with a common bulk band gap. Then, the two edge states lying in the bulk band gap were with dispersion of a Dirac cone centered at the Γ point in the Brillouin zone. Lasing action was achieved at a wavelength of 1510.5 nm by optical

Figure 4: Representative topological lasers.

(a) Left, one-way topological edge-state laser by applying static magnetic field B_0 to the cavity (adopted from Ref. [42]). Right, topological edge-state laser without an external magnetic field (adopted from Ref. [43]). (b) Lasing from a 2D valley-Hall topological cavity. SEM image of the fabricated topological cavity (left) and measurement of emission energy versus the pump energy (right). The inset of the right panel shows the spatial distribution of the above-threshold emission (adopted from Ref. [44]). (c) Topological vortex laser from spin-momentum-locked edge mode. The corresponding spin direction of the cavity mode is indicated by the out-of-plane arrows (adopted from Ref. [45]). (d) Topological corner-state laser. Left, SEM image of the fabricated topological cavity. Scale bar, 2 μm . Right, measured (top) and calculated (bottom) lasing images of the quadrupole, dipole (d_1), dipole (d_2), and monopole topological modes (from left to right). Scale bar, 10 μm (adopted from Ref. [46]).

pumping. The threshold power density of the device was $\sim 120 \text{ kW/cm}^2$. The directional surface emitting feature was observed with a donut-shaped pattern with a radius of $\sim 3.39^\circ$. The side-mode suppression with a ratio of 42 dB was also measured in the topological edge-state laser as a result of the uniform intensity distribution along the resonance loop of the traveling mode lasing.

While the above topological systems show the topological states with dimensionality one lower than that of the system, the higher-order topological insulators support topological states two or more dimensions lower than the system [46, 47]. Notably, the topological corner state, as a higher-order topological state, is useful to further reduce the cavity size by confining light at the corner of the structure. Small mode volumes as well as robustness against the defects can improve the laser performance in planar landscapes. Very recently, Kim et al. [46] demonstrated the corner-state lasers in nanophotonic topological structures (Figure 4d). The cavity was designed based on 2D Su–Schrieffer–Heeger model on a square lattice, in which the unit cell represented a symmetric quadrumer composed of four elements. A topologically nontrivial domain of 6×6 -unit cells, constituted by two types of square air holes, was surrounded by a trivial one with the inverted order of the square air holes. Theoretical analysis, given coupling of the corner states, predicted four multipolar corner modes, namely the quadrupole, two degenerate dipoles, and monopole modes. With 980-nm pulsed laser pumping, these four corner-state lasing modes were clearly observed via hyperspectral imaging. The mode identification was performed by measuring the mode images, lasing spectra, and light-in light-out curves. In particular, the corner-state lasing modes were observed even when a defect is introduced at the interface between the trivial and nontrivial domains, which is not feasible in typical photonic-crystal defect lasers. These results open novel prospects for the topologically controlled generation of light in active nanophotonic structures with topological phases and the exploration of radiatively coupled topological states in non Hermitian systems.

6 BIC lasers

Open systems generally lose their energies due to the inevitable coupling to external radiation states. However, optical bound states in the continuum (BICs) completely decouple from the incoming radiation and are prohibited to radiate into an open photonic system, even if the states are embedded in the continuous spectrum of the environment [49–52]. Thus, BICs can possess infinite Q factors

theoretically. The extremely high Q factor is particularly useful for the implementation of a low-threshold laser source, with the integration with optical gain materials.

One representative example of the optical BICs is a symmetry-protected BIC, which was demonstrated in periodic structures such as gratings, photonic crystals, and metasurfaces [49, 50, 53]. The periodicity of the structure causes not only the coupling of guided modes to the continuum of propagating modes in the free space but also backscattering of leaky modes and their strong coupling in high-symmetry points of the momentum space. The coupling of these resonances to leaky modes is forbidden by symmetry or separability if the symmetry is preserved. The suppression of radiation into the normal direction based on symmetry-protected BICs at the Γ point led to the implementation of many surface-emitting lasers with a low lasing threshold and large-area high-power emission [49]. In addition, an accidental BIC is another example of BICs, which occurs with additional symmetries at the nonzero wavevectors of photonic bands [49–51]. When leaky modes interfere destructively and turns into the accidental BIC with nonzero group velocity, the radiation loss disappears only at a specific wavevector. Accidental BIC does not occur at high symmetry points in the momentum space, and thus it is different from the symmetry-protected BIC.

The control of topological charges in BICs is a new approach to manipulate light confinement. A new kind of BIC mode has been proposed more recently, through merging of several BIC charges in the momentum space [54–56]. The topological nature of the BICs in reciprocal lattice appears as several vortices with integer topological charges of ± 1 . At the specific value of the lattice constant, all BIC charges are merged into a single BIC. Then, the Q factor in the merging-BIC exhibits higher than that in symmetry-protected BIC along all directions in momentum space. The engineering of Q factors in the merging-BIC regime enables the development of lasers that facilitate strong light confinement and large boosts to its amplitude.

In 2017, Kodigala et al. [57] demonstrated room-temperature lasing action from an optically pumped BIC cavity (Figure 5a). The BIC cavity was designed with a periodic array of InGaAsP cylindrical nanoresonators connected by supporting bridges for a stable freestanding membrane. There exist three BIC modes at Γ point: One is a symmetry-protected BIC, and the other two are doubly degenerate resonance-trapped BIC modes. The resonance-trapped BICs are less sensitive to symmetry-breaking perturbations because a variation in radius of the cylindrical nanoresonator only induces its displacement in momentum space. Using the resonance-trapped BIC modes, persistent room-temperature single-mode lasing was

Figure 5: Representative BIC lasers.

(a) Schematic of BIC laser composed of a periodic array of InGaAsP nanodisks with supporting bridges (adopted from Ref. [57]). (b) Schematic (left) and measured lasing spectrum (right) of the BIC laser consisting of an array of TiO_2 cylinders covered with CdSe/CdZnS core-shell nanoplatelets. The inset of the right panel shows the back focal plane image at the onset of lasing (adopted from Ref. [58]). (c) Ultrafast control of vortex microlaser. Two beams are spatially induced with a delay time. The right images show the far-field image patterns under both symmetric and asymmetric excitations (adopted from Ref. [62]). (d) Left, schematic of the merging-BIC structure. Right, calculated Q factors before merging (grey), at merging (red), and after merging (blue) (adopted from Ref. [55]). (e) Super-BIC laser. Left, measured lasing wavelengths of the symmetry-protected BIC lasers (black dots; right inset) and accidental BIC lasers (red dots; left inset) as a function of the lattice constant. The bottom inset shows the SEM image of the fabricated laser structure with $a = 563$ nm. Right, measured threshold (top) and Q factor (bottom) as a function of the lattice constant. The threshold values are divided by the pump area (~ 5.4 μm in size). The Q factor is obtained using $\lambda/\Delta\lambda$, where λ is the lasing wavelength and $\Delta\lambda$ is the linewidth at the transparent pumping condition (adopted from Ref. [65]).

achieved for various resonator radii and array sizes (8×8 , 10×10 , 16×16 , and 20×20). The threshold average power was 56 μW , which exhibited the inverse relationship with Q factor. The robustness and scalability of the optical system were demonstrated in the BIC laser.

More BIC lasers were developed using different gain materials [58–61]. In 2020, Wu et al. [58] demonstrated optically pumped room-temperature lasing at visible wavelength (Figure 5b). A film of colloidal CdSe/CdZnS

core-shell nanoplatelets was combined with square arrays of titanium dioxide (TiO_2) nanocylinders to achieve BIC lasers. The BIC mode originated from the lattice-mediated Mie resonances can provide theoretically infinite Q factors. In experiment, by adjusting the diameter of the TiO_2 nanocylinders, the lasing wavelength was tuned across the gain bandwidth of the nanoplatelets. The lasing mode was situated optimally in the middle of the nanoplatelet gain bandwidth for the diameter 300 nm at a central wavelength

of 647.7 nm. The lasing wavelength was approximately 30 nm red-shifted when the diameter varied from 290 to 310 nm. The ability to tune the lasing wavelength offers unprecedented freedom and convenience on the design and implementation of lasing devices based on dielectric Mie resonances.

In addition to the ultrahigh Q factor and low lasing threshold, the BIC modes can possess vortex behaviors with different topological charges, which are important for vector beams [62–64]. Optical vortex beams with spiral wavefronts and screw phases are attractive due to their excellent flexibility via resonant tuning or polarization manipulation. In 2020, BIC-based perovskite vortex microlasers have been reported, showing the ability of ultrafast optical switching at room temperature (Figure 5c) [62]. This laser structure consisted of the metasurface of 220-nm lead bromide perovskite (MAPbBr_3) film patterned with a square array of circular holes, which was placed on a glass substrate. The laser properties were successfully controlled by two optical pump beams. When the structure was pumped only by the first beam, the output was a uniform donut. Once the second beam overlapped temporally with the first beam, the optical symmetry was broken. The vortex beam lasing can be switched to linearly polarized beam lasing, or vice versa, with switching times of 1–1.5 ps, which is useful for the high-speed optical communication.

Moreover, merging several BICs was demonstrated experimentally in a passive optical device. In 2019, Jin et al. [55] observed ultrahigh- Q resonances in a 600-nm-thick silicon photonic crystal slab with $\sim 250 \times 250 \mu\text{m}^2$ in size when multiple BICs merge in the momentum space (Figure 5d). The lattice constants varied from 530 to 580 nm to excite merging and isolated BICs. The Q values of resonances at different k points were characterized using iso-frequency contours. Then, the merging-BIC by multiple topological charges exhibited a record-high Q factor of 4.9×10^5 . In comparison, the highest Q observed in the isolated BIC was limited to 4×10^4 , which was more than an order of magnitude lower. These ultrahigh- Q resonances became robust even against fabrication imperfections and potentially useful in improving the performance of optoelectronic devices.

Furthermore, the merging-BIC mode can also show unique optical features such as high Q factor even in the finite-size cavity, by controlling the topological charges in the reciprocal space and engineering the radiation condition [55, 56, 65]. This new type of BIC mode, termed as a super-BIC (BIC in the supercavity regime with extremely high values of the Q factor), is particularly useful for the demonstration of active devices with high performance. In

2021, Hwang et al. demonstrated the super-BIC laser which is a combination of the symmetry-protected and accidental BICs in a finite periodic photonic structure (Figure 5e) [65]. The theoretical analysis showed that in the vicinity of the super-BIC mode, the finite-size cavity can possess a high radiative Q factor, in contrast to the other BIC modes. After the transition to the super-BIC regime at the lattice constant of ≥ 574 nm, the measurements showed the far-field laser image with strong angular confinement, the reduced threshold to $\sim 1.47 \text{ kW cm}^{-2}$, and the increased Q factor up to ~ 7300 . The measured threshold peak power was approximately 50 to 10 million times lower than that of earlier demonstrated BIC nanolasers [65].

7 Conclusion and outlook

Since Mark Stockman proposed the concept of a spaser, various plasmonic nanolasers have been demonstrated at the subwavelength scale. After that, new physical mechanisms were suggested in all-dielectric resonant nanostructures, maintaining both small mode volumes and high Q factors. In this Perspective, we have summarized the unique optical features of dielectric subwavelength lasers, mainly focusing on the demonstrations of Mie-resonant lasers, anapole lasers, topological lasers, and BIC lasers.

The key metrics of the most representative nanolasers are summarized in Table 1. Plasmonic lasers typically show relatively high lasing thresholds at low temperatures, whereas most of dielectric lasers show lower thresholds at room temperature (RT). Lasing from Mie-resonant modes at the subwavelength scale can lead to stable lasing operation at room temperature, through the interference of multipole resonances. For topological lasers, the higher-order corner-state lasers show lower thresholds compared with the edge-state topological lasers. In addition, for the BIC lasers, controlling topological charges can increase Q factors in a broad range of the momentum space and reduce the lasing thresholds substantially. Although BIC-based lasers seem to have higher values of the lasing threshold compared with other types of lasers (see Table 1), they can exhibit or have the potential to show reduced thresholds through the optimization of the structural parameters satisfying the supercavity condition. For example, the recent BIC lasers demonstrated by Kodigala et al. [57] and Hwang et al. [65] took full advantage of the unique physics of BICs and successfully demonstrated much lower thresholds.

We believe that the nonplasmonic nanostructures based on Mie resonances, bound states in the continuum, and the physics of topological phases, became very useful

Table 1: A summary of the properties of some plasmonic and nonplasmonic nanolasers.

	Wavelength	Threshold peak power	Threshold power density	Temperature
Plasmonic lasers				
Nature 460, 27 (2009) [2]	531 nm	8.1×10^5 mW	18 kW/cm ²	RT
Nature 461, 629 (2009) [3]	489 nm		100 MW/cm ²	<10 K
Nano Lett. 10, 3679 (2010) [20]	1308 nm		120 kW/cm ²	8 K
Science 337, 45 (2012) [10]	510 nm		3.7 kW/cm ² (CW)	78 K
Nat. Commun. 5, 4953 (2014) [12]	375 nm		3.5 MW/cm ²	RT
Nat. Commun. 8, 1889 (2017) [18]	698 nm		10 kW/cm ²	RT
Nano Lett. 18, 747 (2018) [13]	371.8 nm		13 MW/cm ²	77 K
Mie-resonant lasers				
ACS Nano 14, 7338 (2020) [23]	825 nm		1500 MW/cm ²	77 K
ACS Nano 14, 8149 (2020) [24]	530–540 nm		470–2000 MW/cm ²	RT
Topological lasers				
Science 359, eaar4005 (2018) [43]	1550 nm	11,000 mW	16 kW/cm ²	RT
Nat. Nanotechnol. 15, 67 (2020) [48]	~1600 nm		4.5 kW/cm ²	RT
Phy. Rev. Lett. 125, 013903 (2020) [45]	1510.5 nm		120 kW/cm ²	RT
Light Sci. Appl. 9, 127 (2020) [44]	~1410 nm		8.1 kW/cm ²	RT
Nat. Commun. 11, 5758 (2020) [46]	1549.9 nm	400–460 μ W		RT
BIC lasers				
Nature 541, 196 (2017) [57]	1551.4 nm	15.6 mW	4 kW/cm ²	RT
Nat. Nanotechnol. 13, 1042 (2018) [60]	825 nm	8.80×10^5 mW	70 MW/cm ²	RT
Science 367, 1018 (2020) [62]	552 nm	5.28×10^5 mW	42 MW/cm ²	RT
Nano Lett. 20, 6005 (2020) [58]	647.7 nm	5.09×10^6 mW	180 MW/cm ²	RT
Nat. Commun. 12, 4135 (2021) [65]	~1600 nm	340 μ W	1.47 kW/cm ²	RT

in the recent years for demonstrating nanolasers with superior characteristics such as small footprints and low lasing thresholds, and they can be employed as ultimate light sources in the future photonic integrated circuitry.

Acknowledgements: The authors thank R.-M. Ma for useful suggestions.

Author contributions: All the authors have accepted responsibility for the entire content of this submitted manuscript and approved submission.

Research funding: This work was supported by the Australian Research Council (grant DP200101168) and the National Research Foundation of Korea (NRF) funded by the Korean Government (MSIT) (2021R1A2C3006781).

Conflict of interest statement: The authors declare no conflicts of interest regarding this article.

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