Research Article

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Polarization-independent anapole response of a trimer-based dielectric metasurface

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Abstract: The phenomenon of anapole has attracted considerable attention in the field of metamaterials as a possible realization of radiationless objects. We comprehensively study this phenomenon in the cluster-based systems of dielectric particles by considering conditions of anapole manifestation in both single trimers of disk-shaped particles and metamaterial composed on such trimers. Our analytical approach is based on the multipole decomposition method and the secondary multipole decomposition technique. They allow us to associate the anapole with the multipole moments of the trimer and the separate multipole moments of its constitutive particles. The manifestation of anapole in a two-dimensional metamaterial (metasurface) is confirmed by checking the resonant states in the reflected field as well as from the electromagnetic near-field patterns obtained from the full-wave numerical simulation. It is demonstrated that the anapole excitation in trimers results in the polarization-independent suppression of reflection with the resonant enhancement of local electromagnetic fields in the metasurface. Finally, experimental verification of the theoretical results is presented and discussed.

Keywords: metamaterials; Mie scattering; multipole expansion; optical anapole.

1 Introduction

Optical features of metamaterials composed of subwavelength dielectric particles (meta-atoms) can be derived from the single-particle scattering and coupled-mode theory which takes into consideration the interference effects of dynamic multipoles excited in meta-atoms by incoming radiation. In modern nanophotonics, the excitation and interference of multipoles allow manipulation of light at the nanoscale, which is utilized in many practical applications (for a review on dielectric metamaterials and their applications, see Refs. [1–7]).

In the framework of a general scattering theory (like the Mie theory for spherical particles [8]), the total electric field in dielectric particles is expanded in terms of spherical multipoles which correspond to a complete and orthogonal basis for the decomposition of the scattered fields [9]. For the particles whose scale is less than the effective wavelength of light, the multipole long-wavelength approximation (LWA) [10–12] can be applied with inclusion typically of only electric dipole (ED), magnetic dipole (MD), electric quadrupole (EQ), and magnetic quadrupole (MQ) moments. In this case, the multipole moments are the first low-order coefficients of Taylor expansion for the retarded potentials of the electromagnetic field [13, 14]. Nevertheless, when the particles have a complex shape or are arranged into a cluster, the number of multipole terms, which must be taken into account for correct approximation of the scattered field, increases rapidly. For such systems, the multipole terms may include ordinary multipole moments, the so-called mean-square radii [15, 16], and toroidal dipole moments (TDs)¹ [17–22].

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¹ In an alternative approach based on the expansion of the scattered field in the terms of spherical harmonics [11, 12], a spherical electric dipole moment is expressed with inclusion of its toroidal counterpart. In this approach, the toroidal multipoles appear as the higher-order terms of an expansion of the spherical multipolar coefficients of electric parity. However, the direct extraction of toroidal moments sometimes makes it possible to obtain a clearer physical picture of the interaction of light with nanoparticles [18, 23–26].

Although the standard multipole expansions incorporate the fields radiated by a source, the TDs are often not explicit members of these expansions and are excluded from the consideration in classical electromagnetic theory [9, 27]. Physically distinct from a dynamic ED moment, a source with a dynamic TD radiates with the same angular distribution and far-field properties. Consequently, the toroidal and ED moments are indistinguishable for any distant observer [28]. Moreover, the multipole interference can occur between multipoles of the same (e.g., ED and EQ) and different (e.g., ED and MD or TD) types. In particular, this multipole interference in dielectric particles manifests itself in directional scattering [29–33] and nonradiating optical anapole [19, 34–37].

An optical anapole arises as a composition of ED and TD moments, which does not produce any far-field radiation due to complete destructive interference of their similar radiation patterns. In this state, the near fields appear to be strongly localized inside the particles whereas their radiation to the far zone is significantly suppressed. Due to the suppression of scattered fields, the incident wave passes through the meta-atoms without distortion resulting in the total transmission (zero reflection) condition of the metamaterial at the anapole's excitation wavelength. Thereby, the anapole offers a new route to achieve the invisibility (cloaking) effect for lossless dielectric metamaterials based on the suppression of radiation scattering [38–40]. As side effects related to the anapole physics, reciprocity violation and Aharonov–Bohm like phenomena can be also mentioned [41, 42].

Since the nonradiating anapole state stems from the multipolar interference of scattered fields, an analysis based on the multipole expansion is essential for engineering this state in dielectric metamaterials. Although analytical electromagnetic calculation such as the Mie scattering theory naturally includes the multipole analysis, it allows deriving analytical expressions only for particles with coordinate boundaries (e.g. spheres). For the computation of light scattering by nonspherical particles or particles collected into a cluster, full-field simulation techniques should be used. Among such techniques, the discrete dipole approximation (DDA), which we use for the present study, allows one to perform calculations of the total extinction, scattering cross-sections (SCS) [43], and multipole contributions for particles of an arbitrary shape [44]. In the DDA, the particles are represented as an array of point dipoles in a local domain with dimensions much smaller than the scattered wavelength, whereas the total fields radiated by all these point dipoles are approximately represented as a series of multipole contributions

for localized systems of EDs [44]. When the particles are arranged into a cluster, the secondary multipole decomposition (SMD) technique can be applied to calculate the overall multipole moments based on the characteristics of the cluster's constitutive particles derived with the DDA [23] or other numerical methods.

In the present paper, our goal is to identify the specific features of the anapole occurrence in a dielectric metasurface composed of the trimer-based clusters (meta-atoms). First, we investigate the optical properties of a single standalone trimer composed of dielectric disk-shaped particles and demonstrate the anapole realization that is independent of the polarization of the incident waves. Then, using the SMD technique, we prove that the anapole arises as a resonant excitation associated with the out-of-plane MD moments of the individual disks forming the trimer. The anapole appearance results in the localization of the electromagnetic energy inside the particles. The manifestation of anapole is then detected in the near-field patterns and wave transmission and reflection properties for the trimer-based metamaterial in both full-wave numerical simulations and experiment.

2 Anapole in an isolated trimer

To demonstrate that a standalone trimer composed of disk-shaped particles (see Figure 1A), can support an anapole, we perform a multipole analysis of its SCS characteristics. In general, the multipole decomposition of the scattered fields by a system of dielectric particles can be derived in different multipole representations. It can be formulated in the terms of spherical multipoles expressed in the spherical [11] or Cartesian basis [11, 30], and multipoles obtained in the LWA [18, 19]. For a given problem, the representation based on the LWA is more preferable since it provides a direct way to identify the anapole in scatterers. In this section, at the first stage, the multipole contribution of the standalone trimer is determined, and then the interference of the multipoles of individual disks resulting in the anapole appearance in the whole cluster is studied in details involving the SMD technique.

2.1 Basic multipole expansion

Taking into account several first spherical multipoles in the Cartesian basis (the exact multipoles), the SCS (that is the scattered power divided by the incident wave intensity)

Figure 1: (A) Coordinate frame, schematic of a trimer-based cluster consisting of dielectric disks, and the incident wave condition. (B) Scattering cross-sections and corresponding multipole contributions calculated for the trimer located in medium with $\varepsilon_s = 1$. Parameters of the trimer are $\varepsilon_d = 22$, $h_d/D = 0.44$, $a_d/D = 0.5$ and $a_t/D = 1.125$, and $D = 2a_d$ is the disk's diameter.

can be presented as [30]

$$\sigma_{\text{sca}} \simeq \frac{k_0^4}{6\pi\varepsilon_0^2|\mathbf{E}|^2}|\mathbf{p}|^2 + \frac{k_0^6\varepsilon_s}{720\pi\varepsilon_0^2|\mathbf{E}|^2}\sum_{\alpha\beta}|Q_{\alpha\beta}|^2 + \frac{k_0^4\varepsilon_s\mu_0}{6\pi\varepsilon_0|\mathbf{E}|^2}|\mathbf{m}|^2 + \frac{k_0^6\varepsilon_s^2\mu_0}{80\pi\varepsilon_0|\mathbf{E}|^2}\sum_{\alpha\beta}|M_{\alpha\beta}|^2, \quad (1)$$

where k_0 is the wavenumber in vacuum, ε_0 is the vacuum dielectric constant, ε_s is the relative dielectric constant of the surrounding medium, μ_0 is the vacuum permeability, $\mathbf{E}$ is the electric field amplitude of the incident plane wave, $\mathbf{p}$ and $\mathbf{m}$ are the vectors of the exact ED and MD moments, respectively, and $\hat{Q}$ and $\hat{M}$ are the 3×3 tensors of the exact EQ and MQ moments (the integral expressions determining the exact multipole moments are not presented here and can be found in Reference [23]).

If the principal geometrical parameter of the scatterer (in our problem, this parameter can be related to the disk's diameter, D) is enough smaller than the incident wavelength λ , so that $k_0\sqrt{\varepsilon_s}D \lesssim 1$, the expressions for the exact multipoles can be expanded in this parameter resulting in

the LWA multipole representations [11]. With the inclusion of the first few terms of these expansions in the multipole definitions, the SCS in Eq. (1) is transformed to another presentation [19]:

$$\sigma_{\text{sca}} \simeq \frac{k_0^4}{6\pi\varepsilon_0^2|\mathbf{E}|^2}\left|\mathbf{p}_0 + \frac{ik_0}{c}\varepsilon_s\mathbf{T} + \frac{ik_0^3}{c}\varepsilon_s^2\mathbf{T}^{(R)}\right|^2 + \frac{k_0^4\varepsilon_s\mu_0}{6\pi\varepsilon_0|\mathbf{E}|^2}\left|\mathbf{m}_0 + \frac{ik_0}{c}\varepsilon_s\mathbf{T}^m\right|^2 + \frac{k_0^6\varepsilon_s}{720\pi\varepsilon_0^2|\mathbf{E}|^2}\sum_{\alpha\beta}\left|Q_{\alpha\beta}^0 + \frac{ik_0}{c}\varepsilon_s T_{\alpha\beta}^Q\right|^2 + \frac{k_0^6\varepsilon_s^2\mu_0}{80\pi\varepsilon_0|\mathbf{E}|^2}\sum_{\alpha\beta}\left|M_{\alpha\beta}^0 + \frac{ik_0}{c}\varepsilon_s T_{\alpha\beta}^M\right|^2, \quad (2)$$

where c is the vacuum light velocity, $\mathbf{p}_0$, $\mathbf{T}$, $\mathbf{m}_0$, $\hat{Q}^0$, and $\hat{M}^0$ are the ED, TD, MD, EQ, and MQ moments, respectively, given in the LWA, $\mathbf{T}^{(R)}$, $\mathbf{T}^m$, $\hat{T}^Q$, and $\hat{T}^M$ are the mean-square radii of the toroidal dipole (TDR), magnetic dipole (MDR), electric quadrupole (EQR), and magnetic quadrupole (MQR), respectively. Explicit definitions of these multipoles entering in Eq. (2) are collected in Table 1. Using Eq. (2) one can analyze the individual contributions of various multipole terms to the SCS and, as a result, reveal their role in the scattering process. Note that in this paper we use the definitions of the EQ tensors $\hat{Q}^0$ and $\hat{T}^Q$ (see Table 1) which differ from their definitions given in Ref. [19] by a factor of 3. However, this difference is not reflected on their contributions to the SCS expressed by Eq. (2).

In our simulation procedure we, firstly, calculate the induced displacement current density $\mathbf{j}(\mathbf{r})$ in the disks and then the total SCS without application of the multipole decomposition method, and, secondly, using obtained $\mathbf{j}(\mathbf{r})$, the multipole moments (see Table 1) and their contributions to the SCS are determined according to Eq. (2). All multipole moments considered above are located at the coordinate-system origin coincident with the mass center of the trimer (details of the DDA numerical approach can be found in Ref. [23]).

The SCS and corresponding multipole contributions for the trimer are presented in Figure 1B for the frontal irradiation condition ($\mathbf{k} = \{0, 0, k_z\}$). One can see that the 'Total' SCS, calculated without multipole decomposition procedure, is well approximated by the 'Mult' SCS calculated with the use of Eq. (2). This indicates the correctness of the multipole approximation. To carry out a multipole analysis, Figure 1B includes also separate contributions of different multipoles and their combinations in the SCS. Note, since the EQ contribution to the SCS in the wavelength range of our interest is negligibly small, this is

Table 1: LWA multipole moments determining the scattering cross-sections expressed by Eq. (2).

ED: $\mathbf{p}_0 = \frac{i}{\omega} \int_V \mathbf{j} \, d\mathbf{r}$	TD: $\mathbf{T} = \frac{1}{10} \int_V [(\mathbf{r} \cdot \mathbf{j})\mathbf{r} - 2r^2\mathbf{j}] \, d\mathbf{r}$
MD: $\mathbf{m}_0 = \frac{1}{2} \int_V [\mathbf{r} \times \mathbf{j}] \, d\mathbf{r}$	TDR: $\mathbf{T}^{(R)} = \frac{1}{280} \int_V [3r^4\mathbf{j} - 2r^2(\mathbf{r} \cdot \mathbf{j})\mathbf{r}] \, d\mathbf{r}$
MQ: $\hat{M}^0 = \frac{1}{3} \int_V [(\mathbf{r} \times \mathbf{j}) \otimes \mathbf{r} + \mathbf{r} \otimes (\mathbf{r} \times \mathbf{j})] \, d\mathbf{r}$	MDR: $\mathbf{T}^m = \frac{i\omega}{20} \int_V r^2 [\mathbf{r} \times \mathbf{j}] \, d\mathbf{r}$
EQ: $\hat{Q}^0 = \frac{3i}{\omega} \int_V (\mathbf{j} \otimes \mathbf{r} + \mathbf{r} \otimes \mathbf{j} - \frac{2}{3}(\mathbf{r} \cdot \mathbf{j})\hat{U}) \, d\mathbf{r}$	MQR: $\hat{T}^M = \frac{i\omega}{42} \int_V r^2 [(\mathbf{r} \times \mathbf{j}) \otimes \mathbf{r} + \mathbf{r} \otimes (\mathbf{r} \times \mathbf{j})] \, d\mathbf{r}$
	EQR: $\hat{T}^Q = \frac{3}{42} \int_V [4(\mathbf{r} \cdot \mathbf{j})\mathbf{r} \otimes \mathbf{r} + 2(\mathbf{r} \cdot \mathbf{j})r^2\hat{U} - 5r^2(\mathbf{j} \otimes \mathbf{r} + \mathbf{r} \otimes \mathbf{j})] \, d\mathbf{r}$

The multipole moments are calculated with respect to the origin of the Cartesian coordinate system, V is the total volume of the particles in the system, ω is the angular frequency of the incident wave [the time dependence $\exp(-i\omega t)$ is assumed]. For the system composed of three disks (trimer) $V = \sum_{n=1}^3 V_n$, where n is the disk number.

not shown in Figure 1B. The total SCS has a global minimum at $\lambda/D \approx 4.35$ corresponding to the suppression of the scattering due to the *anapole* excitation. This anapole wavelength is marked out in Figure 1B by a vertical dashed line.

Similar to the mechanism discussed in Ref. [34], the suppression of total ED contribution (ED + TD + TDR) appears due to destructive interference between scattered waves separately generated by the LWA ED and TD. From Figure 1B one can conclude that at the anapole wavelength the contribution of the TDR to the total ED is very weak, and the separate contributions of the ED and TD are larger than the SCS minimum. However, the combination of ED + TD results in the suppression of the ED scattering so that $|\mathbf{p}_0 + (ik_0\epsilon_s/c)\mathbf{T}^p + (ik_0^3\epsilon_s^2/c)\mathbf{T}^{(R)}| \rightarrow 0$ in Eq. (2). Figure 1B also demonstrates that both TD and MQ provide a resonant contribution to the anapole (see the maximal values of corresponding TD and MQ curves at the anapole wavelength, $\lambda/D \approx 4.35$). Since the resonant contribution of TD in the scattering is canceled due to the destructive interference with ED, the scattering pattern (scattering directivity) of the anapole basically corresponds to the MQ scattering. Remarkably, the SCS, corresponding multipole contributions, and the spectral position of the anapole shown in Figure 1B do not depend on the polarization state of the plane wave incident under the frontal irradiation conditions.

In what follows, we show that the anapole is associated with the resonant excitation of the out-of-plane MD moments of the individual disks forming the trimer, and, therefore, is accompanied by localization and increasing of electromagnetic energy in the cluster.

2.2 Secondary multipole analysis

The main purpose of involving the SMD technique is to obtain a representation of the multipole moments of an

entire cluster via the multipole moments of its individual constitutive particles. It allows elucidating physical conditions of multipole resonances and their interference resulting in the anapole manifestation. Here, we use this technique to explain a somewhat paradoxical situation related to the anapole – why the minimal value of the trimer SCS corresponds to the maximal values of the TD and MQ moments.

The SMD for a trimer-based cluster was firstly introduced in Ref. [23]. It has been shown that using the replacement $\mathbf{r} = \mathbf{r}_n + \mathbf{r}'_n$ of the radius-vector $\mathbf{r}$ defined with respect to the trimer center of mass, where $\mathbf{r}_n$ ($n = 1, 2, 3$) is the radius-vectors of disk's centers, $\mathbf{r}'_n$ is the radius-vector of any point inside corresponding disk with respect to its center (see Figure 9 in Ref. [23]), the vector of trimer TD moment can be represented as:

$$\mathbf{T} = \sum_{n=1}^3 \left\{ \mathbf{T}_0(\mathbf{r}_n) + \mathbf{T}_n + \frac{4}{5}[\mathbf{r}_n \times \mathbf{m}_n] + \mathbf{I}(\mathbf{r}_n) \right\}, \quad (3)$$

where

$$\mathbf{T}_0(\mathbf{r}_n) = \frac{\omega}{10i} [(\mathbf{r}_n \cdot \mathbf{p}_n)\mathbf{r}_n - 2r_n^2\mathbf{p}_n] \quad (4)$$

is the TD moment at the trimer mass center associated with the LWA ED moment

$$\mathbf{p}_n = \frac{i}{\omega} \int_{V_n} \mathbf{j} \, d\mathbf{r}_n$$

of the disk with number n ,

$$\mathbf{T}_n = \frac{1}{10} \int_{V_n} [(\mathbf{r}'_n \cdot \mathbf{j})\mathbf{r}'_n - 2(r'_n)^2\mathbf{j}] \, d\mathbf{r}'_n$$

and

$$\mathbf{m}_n = \frac{1}{2} \int_{V_n} [\mathbf{r}'_n \times \mathbf{j}] \, d\mathbf{r}'_n$$

are the TD moment and the LWA MD moment of the disk with number n calculated with respect to its mass center,

respectively, and

$$\mathbf{I}(\mathbf{r}_n) = \frac{1}{10} \int_{V_n} [(\mathbf{r}'_n \cdot \mathbf{j}) \mathbf{r}_n - 3(\mathbf{r}_n \cdot \mathbf{j}) \mathbf{r}'_n] d\mathbf{r}'_n \quad (5)$$

is an additional integral term which accounts for the offset of the n th particle from the cluster's mass center.

Application of the similar procedure to the LWA MQ presented in Table 1 results in

$$\hat{M}^0 = \sum_{n=1}^3 \left\{ \hat{M}_0(\mathbf{r}_n) + \hat{M}_n + \frac{2}{3}(\mathbf{m}_n \otimes \mathbf{r}_n + \mathbf{r}_n \otimes \mathbf{m}_n) + \hat{I}'(\mathbf{r}_n) \right\}, \quad (6)$$

where

$$\hat{M}_0(\mathbf{r}_n) = \frac{\omega}{31} [(\mathbf{r}_n \times \mathbf{p}_n) \otimes \mathbf{r}_n + \mathbf{r}_n \otimes (\mathbf{r}_n \times \mathbf{p}_n)]$$

is the MQ moment at the trimer mass center associated with the LWA ED moment $\mathbf{p}_n$ of the disk with number n ,

$$\hat{M}_n = \frac{1}{3} \int_{V_n} [(\mathbf{r}'_n \times \mathbf{j}) \otimes \mathbf{r}'_n + \mathbf{r}'_n \otimes (\mathbf{r}'_n \times \mathbf{j})] d\mathbf{r}'_n$$

and $\mathbf{m}_n$ are the LWA MQ and MD moments of the disk with number n calculated with respect to its mass center, and

$$\hat{I}'(\mathbf{r}_n) = \frac{1}{3} \int_{V_n} [(\mathbf{r}_n \times \mathbf{j}) \otimes \mathbf{r}'_n + \mathbf{r}'_n \otimes (\mathbf{r}_n \times \mathbf{j})] d\mathbf{r}'_n.$$

The results of the SMD for the trimer TD moment expressed by Eq. (3) and excited for the frontal irradiation condition (Figure 1A) are summarized in Figure 2. One can see that at the anapole wavelength, the TD moment $\mathbf{T}$ of the trimer is primarily composed of the contributions of the MD moments $\mathbf{m}_n$ induced in the individual disks. Since the TD moment $\mathbf{T}$ of the trimer and the radius-vectors $\mathbf{r}_n$ have only in-plane components, the only out-of-plane z -components of $\mathbf{m}_n$ can enter in the SMD expressed by Eq. (3) and support the TD resonance. Such feature is also associated with the MQ moment $\hat{M}^0$ because its resonant contribution to the anapole is associated with equality of the M_{yz} and M_{zy} components which, as follows from Eq. (6), can be connected only with the out-of-plane z -components of $\mathbf{m}_n$. In this context, we conclude that the TD and MQ resonances at the anapole wavelength originate from the resonant excitation of the out-of-plane MD moments $\mathbf{m}_n$ of the individual disks forming the trimer.

To confirm the latter conclusion, in Figure 3 we demonstrate the spectral dependences of the total trimer MD moment and individual MD moments of each disk in the

cluster calculated in the LWA for two different polarization states of the incident wave for the frontal irradiation conditions (see corresponding insets in Figure 3A and B).

One can see that the total trimer MD moment (the black dashed lines in Figure 3A and B) is basically approximated by the vector sum of the MD moments of the separate disks $\mathbf{m}_1 + \mathbf{m}_2 + \mathbf{m}_3$ and does not have any resonances around the anapole wavelength independently on the polarization state of the incident wave. However, the magnitudes of the disk MD moments $m_n = |\mathbf{m}_n|$ ($n = 1, 2, 3$) can have maximal (resonant) values at the anapole wavelength. From the component presentation of the MD moments shown in Figure 3C and D, it is revealed that, first, the resonances of $\mathbf{m}_n$ (with the components $m_x^{(n)}, m_y^{(n)}, m_z^{(n)}$) are associated only with their out-of-plane z -components (see the curves for $m_z^{(1)}, m_z^{(2)},$ and $m_z^{(3)}$) and, second, the z -component of the trimer MD moment $m_z \simeq m_z^{(1)} + m_z^{(2)} + m_z^{(3)}$ is negligibly small. This indicates that at the anapole wavelength, the resonant MD $\mathbf{m}_n$ are basically oriented along the z -axis with different phases, so that $m_z^{(1)} + m_z^{(2)} + m_z^{(3)} = 0$.

The circular displacement current flow in the disks at the anapole wavelength, shown in Figure 3E and F for two corresponding polarization states of the incident wave confirms the above discussed feature of $\mathbf{m}_n$ (these patterns are calculated with the use of the COMSOL Multiphysics finite-element electromagnetic solver). In particular, for irradiation with the x -polarized wave (Figure 3E), the MD moments of disks denoted as ① and ② are excited in phase, while the MD moment of the disk ③ is excited out of phase, so that $m_z^{(1)} + m_z^{(2)} = -m_z^{(3)}$. In the case of the irradiation with the y -polarized wave (Figure 3F), only the MD moments of disks denoted as ① and ② are

Figure 2: Absolute values of all terms included in Eq. (3) which contribute to the TD moment of the trimer. They are $C_m = (4/5) \left| \sum_{n=1}^3 [\mathbf{r}_n \times \mathbf{m}_n] \right|$, $C_{T_0} = \left| \sum_{n=1}^3 \mathbf{T}_0(\mathbf{r}_n) \right|$, $C_T = \left| \sum_{n=1}^3 \mathbf{T}_n \right|$, and $C_I = \left| \sum_{n=1}^3 \mathbf{I}(\mathbf{r}_n) \right|$. All the material and geometrical parameters of trimers are the same as in Figure 1.

Figure 3: (A) and (B) Absolute values of the total trimer LWA MD moment and the magnetic dipole moments of separate disks $\mathbf{m}_1$, $\mathbf{m}_2$, $\mathbf{m}_3$ and their vector sums $\mathbf{m}_1 + \mathbf{m}_2$ and $\mathbf{m}_1 + \mathbf{m}_2 + \mathbf{m}_3$, and (C) and (D) the absolute values of the Cartesian components of the total trimer MD moment $\mathbf{m} = (m_x, m_y, m_z)$ and the disks $\mathbf{m}_n = (m_x^{(n)}, m_y^{(n)}, m_z^{(n)})$, where $n = 1, 2, 3$, calculated for two polarization states of the incident wave shown in the insets, and (E) and (F) electric near-field distribution and displacement current flow in the trimer at the anapole wavelength, and corresponding far-field scattering patterns. The vertical orientation of MD moments is presented schematically by blue arrows. All the material and geometrical parameters of trimers are the same as in Figure 1.

resonantly excited with inverse directions, so that $m_z^{(1)} = 0$ and $m_z^{(2)} = -m_z^{(3)}$. Importantly to note that the appearance of scattering patterns shown in Figure 3E and F are equivalent, but their space orientation is determined by the incident wave polarization. The equivalence of these scattering patterns is ensured by the specific symmetry (C_{3v}) of the trimer based on the equilateral triangle [24]. This feature results in the polarization independence of the trimer SCS (for a fixed incident direction) and predicts the appearance of polarization-independent anapole properties in a metasurface composed of such trimer-based clusters.

3 Anapole in a trimer-based metasurface

In this section, we study the appearance of the anapole in a metasurface composed of an array of trimers. We fix all geometrical and material parameters of the disks and trimer as in the previous section and arrange the trimers into square unit cells with the period p . The resulting metasurface appears as an infinitely large two-dimensional array of disks disposed of in the x - y plane and supported by

Figure 4: (A) Schematic view of a dielectric metasurface composed of trimer-based clusters and (B) the reflection coefficient magnitude of the metasurface as a function of wavelength and period. For an illustrative purpose, the minimal values of the reflection coefficient magnitude corresponding to the anapole manifestation are projected into points at the bottom. All the material and geometrical parameters of trimers are the same as in Figure 1.

a low-permittivity ($\epsilon_s \approx 1$) dielectric substrate (see Figure 4A).

We suppose that the metasurface is irradiated by a normally incident ($\mathbf{k} = \{0, 0, k_z\}$) plane electromagnetic wave, whose electromagnetic field vectors $\mathbf{E}$ and $\mathbf{H}$ lie in the x - y plane. With such a problem statement, one can expect that the above-discussed anapole of trimer should manifest itself in the spectra of the given metasurface at a wavelength close to the anapole wavelength of the standalone trimer, whereas its possible resonant shift should be dependent on the value of p . The occurrence of anapole implies nonradiating interaction of the incident wave with the metasurface which can be found in the total transmission (zero reflection) resonant state of the metasurface. Since the given metasurface is predominantly transparent in the wavelength range of interest, it is preferable to perform the anapole detection by checking

the resonant states in the reflected field. The specified conditions of the anapole are indeed detected in the spectrum of the reflected wave, as shown in Figure 4B. This set of curves corresponds to the reflection coefficient magnitude $|R|$ calculated for different values of the parameter p in the particular case of irradiation of the metasurface with the x -polarized wave (for our numerical simulations in this section, we use the COMSOL Multiphysics finite-element electromagnetic solver). In what follows our aim is to verify the polarization-independent feature and other related characteristics of this state.

3.1 Polarization-independent feature

To verify the polarization-independent feature of the anapole in the given dielectric metasurface, the polarization characteristics of the reflected field are further elucidated by performing calculation of the anapole resonant conditions for all polarization states of the incident wave.

In particular, the response of the metasurface for different polarization states is mapped onto the surface of the full Poincaré sphere presented in Figure 5A. For this study, the electric field vector $\mathbf{E}$ of the incident wave is defined by components $E_x = [0.5(P + 1)]^{1/2}$ and $E_y = [0.5(P - 1)]^{1/2} \exp(i\beta)$, where $P \in [-1, 1]$ and $\beta \in [-\pi, \pi]$. The sphere is related to the reflection characteristics of the metasurface at the anapole resonant wavelength. The axes of the Poincaré sphere are labeled in terms of the Stokes parameters. Linearly polarized states are arranged along the equator of the sphere, while right and left circularly polarized waves are located at its north and south poles, respectively. All other points of the sphere depict elliptically polarized states of the incident wave. The uniformly colored appearance of the sphere indicates that the structure exhibits zero reflection for all possible polarization states of the incident wave which confirms the polarization-independent feature of the resonance at the anapole wavelength.

The calculated electric near-field patterns within the metasurface unit cell are presented in Figure 5B for five different polarization states. They correspond to the meta-material irradiation by the waves with linear x -, diagonal-, y -, right-handed circular, and left-handed circular polarizations. One can conclude that the appearance of the anapole resonant patterns and displacement current flow for the cases of incidence of the x - and y -polarized waves corresponds to the characteristics presented above in Figure 3E and F for the standalone trimer, respectively, while for the other polarization states, the general picture of the field distribution is preserved, where the clockwise rotation of

Figure 5: (A) Magnitude of the reflection coefficient $|R|$ of the metasurface mapped on the surface of a Poincaré sphere related to the polarization states of the incident wave and (B) normalized electric near-field patterns and displacement current flow within the metasurface unit cell at the anapole wavelength. The states (i)–(iii) correspond to linear x -, diagonal-, and y -polarization, and (iv) and (v) are right-handed and left-handed circular polarizations, respectively. All the material and geometrical parameters of trimers are the same as in Figure 1 and $p/D = 2.9$.

the displacement current in one disk is accompanied by its anticlockwise rotation in one or two other disks. Such a displacement current flow is fully compensated, which ensures the resonant transparency of the metasurface at the anapole wavelength.

3.2 Experimental validation

We further validate the identified polarization-independent characteristics of the anapole in our dielectric metasurface by performing a direct quasi-optical experiment. To this end, we construct a metasurface prototype based on trimers by assembling the array of disks made of a low-loss, high-permittivity microwave ceramic. The geometric and material parameters of dielectric particles and trimers correspond to those introduced in the theoretical part of our study when the diameter of disks is fixed at $D = 8.0$ mm. In this case, the operating frequency range is 7.0–9.0 GHz.

For our experiments, we use a setup that was described in detail earlier [45, 46] (see, Figure 6A). The results of measurements of the reflected spectra for two orthogonal linear polarizations of the incident wave are summarized in Figure 6B and C. They are supplemented by simulated

data where the actual material losses ($\tan \delta$) existing in the ceramic material of disks are taken into account. Foremost, one can conclude that our experimental data are in good agreement with the simulated ones. The measurements performed confirm the existence of anapole state in the reflected fields which is identified as the minimum of the reflection coefficient magnitude. The corresponding resonant feature arises in the reflected spectra at the same frequency for the incident waves of both polarizations which additionally confirms the polarization-independent nature of the anapole.

According to the results of our theoretical study, when the anapole arises, the electric near-field appears to be mostly concentrated in-plane of the structure and inside the particles, whereas the magnetic near-field is penetrating out of the particles. Moreover, the signature of anapole under consideration is the feature that the magnetic moments of the disks in trimer particles are oriented out-of-plane acquiring positions ‘up’ and ‘down’ and demonstrating different patterns with respect to the polarization states of the incident wave. Thus, the measurement of the real part of the magnetic near-field can provide unambiguous identification of the anapole in the given metasurface.

Figure 6: (A) Experimental setup with a metasurface prototype, (B) and (C) simulated and measured reflected spectra of the metasurface, (D) magnetic near-field probe, and (E) and (F) patterns of the real part of the z-component of the magnetic near-field mapped for four metasurface unit cells at the anapole frequency. The array is irradiated by the (B) and (E) x-polarized and (C) and (F) y-polarized waves. The inset demonstrates a photo of the metamaterial prototype. The overall size of the sample is approximately 300×300 mm, and it consists of an array of 12×12 trimers (432 particles in total). Parameters of the prototype are: $\epsilon_d = 22$, $\tan \delta \approx 1 \times 10^{-3}$, $a_d = 4$, $h_d = 3.5$, $\epsilon_s = 1.1$, $h_s = 10$, $a_t = 9$, and $p = 23$. All geometrical parameters are given in millimeters.

To measure the corresponding component of the magnetic near-field (H_z), we use a small magnetic probe (loop) placed in the x - y plane at the distance of 1 mm above the metasurface plane (see Figure 6D). Four unit cells in the central part of the actual metasurface are included in the scan area. Over this area, the near-field imaging system performs the movement of the probe with a 1 mm step in both horizontal directions. The results of our near-field mapping presented in Figure 6E and F confirm that the anapole exists in the trimers with the magnetic-near field distributions as predicted in the theoretical part of our study.

4 Conclusions

In this paper, we provide a systematic study and present our theoretical and experimental results for the metasurface composed of dielectric trimer-based clusters that

maintain a nonradiating anapole state. The analytical basis for our study includes the multipole decomposition method and the SMD technique. The conditions were derived for the configuration of the displacement currents and near-fields leading to the appearance of a nonradiating state of the anapole type. The appearance of anapole is derived from the characteristics of multipoles and their interference. It is shown that the anapole in the trimer is independent of the polarization of the irradiating wave.

The effect of the polarization-independent resonant total transmission (zero reflection) in the dielectric metasurface is presented and associated with the anapole conditions by performing the multipole analysis.

As a proof of concept, a quasi-optical (microwave) experiment has been conducted to confirm the appearance of polarization-independent anapole in an actual dielectric metasurface composed of ceramic particles (disks).

Along with the traditional physical-chemistry methods of synthesizing new materials [47–49], the

development of artificial metamaterials with predetermined properties is a fundamentally important area of modern research. In this context, we expect that the future integration of dielectric metasurfaces maintaining anapole states into more complex nanophotonic systems can shape a wide range of exciting applications, including optical sensing, nonlinear active components, and many new types of near-field lasers, nanoantennas, and optical switches.

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