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Fractional variational iteration method for solving time-fractional Newell-Whitehead-Segel equation

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Abstract: In this paper, Newell–Whitehead–Segel equations of fractional order are solved by fractional variational iteration method. Convergence analysis and numerical examples are presented to show the efficiency of the proposed numerical method. Plotted graph demonstrate the mightiness and accurateness of the proposed technique.

Keywords: Fractional variation iteration method; Newell–Whitehead–Segel equation of fractional order; Lagrangian multiplier, Caputo fractional derivative.

1 Introduction

Nonlinear phenomena are always visible in the study of applied Mathematics, Physics, Chemistry and many related fields of science and engineering. Solving nonlinear system is a herculean task in mathematical analysis and applications. In daily life, we come across many real life models of mathematics for solution of nonlinear fractional differential equation.

The advantage of using fractional models of differential equations in physical models is their non-local property. Fractional order derivative is non-local while integer order derivative is local in nature. It shows that the upcoming state of physical system is also dependent on all of its historical states in addition to its present state. Hence the fractional models are more realistic and fractional derivatives are often used to model problems in acoustics, fluid mechanics, diffusion, electromagnetism, signal processing, biology, finance and some more pro-

cesses [1–7]. Recently, various methods have been proposed to solve nonlinear fractional differential equations such as Adomian decomposition method [8, 9], differential transform method [10], homotopy perturbation method [11], variational iteration method [12, 13], homotopy analysis method [14–16], homotopy analysis transform method [17–20], homotopy analysis Sumudu transform method [21], homotopy perturbation transform method [22], fractional variational iteration method [23–26], q-homotopy analysis transform method [27, 28], fractional iteration method [29, 30] etc.

Newell-Whitehead-Segel equation is the well-known amplitude equation which also describes the dynamical behavior near the bifurcation point of the Rayleigh-Benard convection of binary fluid mixtures. Rayleigh-Benard convection is a type of natural convection arising in a plane, horizontal layer of fluid heated from below, in which the fluid develops a regular pattern of convection cells known as Bernard cells. When the heating is ample high, convective motion of the fluid develop spontaneously then the hot fluid moves upwards and the cold fluid moves downwards. Rayleigh-Benard convection is one of the most commonly studied convection phenomena because of its analytical and experimental accessibility. The convection patterns are the most carefully examined examples of self-organizing nonlinear systems. Buoyancy and gravity are responsible for the appearance of convection cells. The initial movement is the upwelling of warmer liquid from the heated bottom layer. There are two types of patterns that are noticed normally. First is the roll pattern in which the fluid stream lines form cylinders. These cylinders may be bent and they may form spirals or target-like patterns. Second pattern is the hexagonal one in which the liquid flow is divided into honeycomb cells. For some fluids, the motion is downwards in the center of each cell and upwards on the border between the cells while for other fluids, the motion is in the opposite direction. The same patterns, stripes and hexagons appear in completely different physical systems and on different spatial scales. For example, stripe patterns are observed in human fingerprints, on Zebra's skin and in the visual cortex. Hexagonal patterns result from the propagation of laser beams through a nonlinear medium and in systems with chemical reaction and diffu-

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sion species. Also, the interaction of the diffusion term affected with the effect of nonlinear reaction term is modeled by it.

The Newell–Whitehead–Segel equation [31, 32] of integer order is

$$\frac{\partial U}{\partial t} = k \frac{\partial^2 U}{\partial x^2} + aU - bU^q, \quad (1)$$

where the constants $a, b, k \in \mathbb{R}$, with q being a positive integer and $k > 0$, $U(x, t)$ is a function of temporal variable t and spatial variable x , where $t \geq 0$, $x \in \mathbb{R}$. Here U can be considered as the nonlinear temperature distribution in a thin as well as infinitely long rod. It may also be seen as the fluid flow velocity in a pipe of infinite length having small diameter. The derivative on the L.H.S. of Eq. (1), $\frac{\partial U}{\partial t}$ denotes the partial rate of change in U w.r.t. temporal variable t at a set position. The derivative on the R.H.S. of Eq. (1), $\frac{\partial^2 U}{\partial x^2}$ shows the partial rate of change in U w.r.t. x at a fixed time. Effect of source is shown by the term $aU - bU^q$ on the R.H.S. of Eq. (1).

The classical Newell–Whitehead–Segel equations have been studied by Laplace Adomian decomposition method [33], Differential transform method [34], Reduced Differential transform method [35], Adomian decomposition method [36, 37], Homotopy perturbation method [38–40], Iterative method [41], Variational iteration method [42], Finite difference scheme [43], etc.

In this article, we consider the fractional model of Newell–Whitehead–Segel equation of the form

$$\frac{\partial^\alpha U}{\partial t^\alpha} = k \frac{\partial^2 U}{\partial x^2} + aU - bU^q, \quad 0 < \alpha \leq 1 \quad (2)$$

where α is a parameter, which describes the order of the time-fractional derivative. The fractional derivative has been taken in Caputo sense. If we take $\alpha = 1$, the fractional Newell–Whitehead–Segel Eq. (2) reduces to the classical Newell–Whitehead–Segel Eq. (1).

In fractional differential equations, the general response expression contains a parameter describing the order of the fractional derivative that can be varied to obtain various responses. The time-fractional Newell–Whitehead–Segel equations describe particle motion with memory in time. Space-fractional derivative arises when variations are heavy-tailed and describes particle motion that accounts for variation in the flow field over the entire system. We are focusing more on motion with memory in time. Also, the fraction in time derivative suggests a modulation or weighting of system memory. Therefore, the study of time-fractional Newell–Whitehead–Segel Eq. (2) is very important.

Recently, Kumar and Sharma [44] provided the numerical approximation of Newell–Whitehead–Segel equation

of fractional order using homotopy analysis Sumudu transform method (HASTM) and found that homotopy perturbation method (HPM), Adomian decomposition method (ADM) and differential transform method (DTM) are particular cases of the solution obtained by HASTM. The fractional model of Newell–Whitehead–Segel equation has not yet been studied by fractional variational iteration method.

Motivated by the above discussions, in this paper, we propose to study the application of fractional variational iteration method (FVIM) to obtain the numerical solution of time-fractional Newell–Whitehead–Segel Eq. (2) and the results are compared with recently developed technique. This paper is organized in the following manner. Section 1, is introductory. Section 2, presents the brief review of preliminary definitions of Caputo fractional derivative and some other results useful in the study of fractional differential equations. In section 3, the solution process of FVIM is proposed by taking the problem under consideration. In section 4, we present the sufficient conditions for the convergence of the proposed method and its error estimate. Section 5, presents some numerical test examples on which FVIM is applied to find the approximate solutions and finally in last section 6, we summarize our results and draw conclusions.

2 Preliminaries

Definition 2.1. Consider a real function $h(\chi)$, $\chi > 0$. It is called in space C_ζ , $\zeta \in \mathbb{R}$ if $\exists$ a real no. $b (> \zeta)$, s.t. $h(\chi) = \chi^b h_1(\chi)$, $h_1 \in C[0, \infty]$. It is clear that $C_\zeta \subset C_\gamma$ if $\gamma \leq \zeta$.

Definition 2.2. Consider a function $h(\chi)$, $\chi > 0$. It is called in space C_ζ^m , $m \in \mathbb{N} \cup \{0\}$ if $h^{(m)} \in C_\zeta$.

Definition 2.3. The left sided Caputo fractional derivative of h , $h \in C_{-1}^m$, $m \in \mathbb{N} \cup \{0\}$ is defined as

$$D_t^\beta h(t) = \begin{cases} \left[I^{m-\beta} h^{(m)}(t) \right], & m-1 < \beta < m, \quad m \in \mathbb{N}, \\ \frac{d^m}{dt^m} h(t), & \beta = m, \end{cases}$$

$$\text{a. } I_t^\zeta h(x, t) = \frac{1}{\Gamma(\zeta)} \int_0^t (t-s)^{\zeta-1} h(x, s) ds; \quad \zeta, t > 0,$$

$$\text{b. } D_\tau^\nu V(x, \tau) = I_\tau^{m-\nu} \frac{\partial^m V(x, \tau)}{\partial \tau^m}, \quad m-1 < \nu \leq m,$$

$$\text{c. } D_t^\zeta I_t^\zeta h(t) = h(t), \quad m-1 < \zeta \leq m, \quad m \in \mathbb{N},$$

$$\text{d. } I_t^\zeta D_t^\zeta h(t) = h(t) - \sum_{k=1}^{m-1} h^{(k)}(0+) \frac{t^k}{k!}, \quad m-1 < \zeta \leq m, \quad m \in \mathbb{N},$$

$$\text{e. } I^\nu t^\zeta = \frac{\Gamma(\zeta+1)}{\Gamma(\nu+\zeta+1)} t^{\nu+\zeta}.$$

Definition 2.4. Mittag-Leffler function is demarcated by the given series representation valid in entire complex

plane:

$$E_{\zeta}(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(1+\zeta m)}, \quad \zeta > 0, \quad z \in \mathbb{C}.$$

3 Proposed FVIM for the time-fractional Newell-Whitehead-Segel equation

To illustrate the process of solution by FVIM, we consider the time-fractional Newell-Whitehead-Segel equation

$$\frac{\partial^{\alpha} U}{\partial t^{\alpha}} = k \frac{\partial^2 U}{\partial x^2} + aU - bU^q, \quad 0 < \alpha \leq 1. \quad (2)$$

By FVIM, correction functional is formed as

$$U_{n+1}(x, t) = U_n(x, t) + \lambda \int_0^t \left(\frac{\partial^{\alpha} U_n(x, \tau)}{\partial \tau^{\alpha}} - k \frac{\partial^2 \tilde{U}_n(x, \tau)}{\partial x^2} - a\tilde{U}_n(x, \tau) + b\tilde{U}_n(x, \tau)^q \right) (d\tau)^{\alpha}, \quad (3)$$

where λ is a Lagrangian multiplier.

Now, by variational theory, λ must satisfy $\frac{d^{\alpha} \lambda}{d\tau^{\alpha}}|_{\tau=t} = 0$ and

$$1 + \lambda|_{\tau=t} = 0.$$

Consequently, we obtain $\lambda = -1$ and hence, from equation (3), we get

$$U_{n+1}(x, t) = U_n(x, t) - \int_0^t \left(\frac{\partial^{\alpha} U_n(x, \tau)}{\partial \tau^{\alpha}} - k \frac{\partial^2 U_n(x, \tau)}{\partial x^2} - aU_n(x, \tau) + bU_n(x, \tau)^q \right) (d\tau)^{\alpha}. \quad (4)$$

Now from Eq. (4), we can obtain successive approximations $U_n(x, t)$, $n \geq 0$. The function U_n is restricted variation which means $\delta \tilde{U}_n = 0$. In this way, we get sequences $U_{n+1}(x, t)$, $n \geq 0$.

Finally, the exact solution is obtained as $U(x, t) = \lim_{n \rightarrow \infty} U_n(x, t)$.

4 Convergence analysis

In this section, we focus on the convergence of the proposed fractional variational iteration method applied to equation (2) in section 3. The sufficient conditions for the convergence of the proposed method and its error estimate [46] are presented.

We define the operator B as:

$$B = \int_0^t (-1) \left(\frac{\partial^{\alpha} U_n(x, \tau)}{\partial \tau^{\alpha}} - k \frac{\partial^2 U_n(x, \tau)}{\partial x^2} - aU_n(x, \tau) + bU_n(x, \tau)^q \right) (d\tau)^{\alpha}, \quad (5)$$

and we define the components v_k , $k = 0, 1, 2, \dots$, as,

$$U(x, t) = \lim_{n \rightarrow \infty} U_n(x, t) = \sum_{k=0}^{\infty} v_k. \quad (6)$$

Theorem 1 [47]. Let B , defined in (5), be an operator from a Banach space BS to BS . The series solution $U(x, t) = \lim_{n \rightarrow \infty} U_n(x, t) = \sum_{k=0}^{\infty} v_k$ as defined in (6), converges if $0 < p < 1$ exists such that $\|B[v_0 + v_1 + v_2 + \dots + v_{k+1}]\| \leq p \|B[v_0 + v_1 + v_2 + \dots + v_k]\|$, (i.e. $\|v_{k+1}\| \leq p \|v_k\|$), $\forall k \in \mathbb{N} \cup \{0\}$.

Theorem 1 is a special case of the Banach fixed point theorem, which was used in [48] as a sufficient condition to study the convergence of the FVIM for some partial differential equations.

Theorem 2 [47]. If the series solution $U(x, t) = \sum_{k=0}^{\infty} v_k$ defined in (6) converges, then it is an exact solution of non-linear problem (2).

Theorem 3 [47]. We assume that the series solution $\sum_{k=0}^{\infty} v_k$ defined in (6) is convergent to the solution $U(x, t)$. If the truncated series $\sum_{k=0}^j v_k$ is used as an approximation to the solution $U(x, t)$ of problem (2), then the maximum error, $E_j(x, t)$, is estimated as:

$$E_j(x, t) \leq \frac{1}{1-p} p^{j+1} \|v_0\|$$

If for every $i \in \mathbb{N} \cup \{0\}$, we define the parameters,

$$\chi_i = \begin{cases} \frac{\|v_{i+1}\|}{\|v_i\|}, & \|v_i\| \neq 0 \\ 0, & \|v_i\| = 0 \end{cases}$$

then the series solution $\sum_{k=0}^{\infty} v_k$ of problem (2) converges to an exact solution $U(x, t)$, when

$$0 \leq \chi_i < 1,$$

$\forall i \in \mathbb{N} \cup \{0\}$. Moreover, as stated in theorem 3, the maximum absolute truncation error is estimated as

$$\left\| U(x, t) - \sum_{k=0}^{\infty} v_k \right\| \leq \frac{1}{1-\chi} \chi^{j+1} \|v_0\|,$$

where $\chi = \max \{\chi_i, i = 0, 1, 2, \dots, j\}$.

Remark 1 [47]. If the first finite χ'_i 's, $i = 1, 2, \dots, j$, are not less than one and $\chi_i \leq 1$ for $i > j$, then, of course, the series solution $\sum_{k=0}^{\infty} v_k$ of problem (2) converges to an exact solution. In other words, the first finite terms do not affect the convergence of series solution. In this case, the convergence of FVIM approach depends on χ_i , for $i > j$.

5 Numerical Experiments

In this section, we apply the proposed technique FVIM to some test examples.

Example 1. Consider a linear time-fractional Newell-Whitehead-Segel equation

$$D_t^\alpha U = U_{xx} - 2U, \quad 0 < \alpha \leq 1, \quad (7)$$

with initial condition

$$U(x, 0) = e^x. \quad (8)$$

When $\alpha = 1$, the exact solution of Eqs. (7) – (8) is $U(x, t) = e^{x-t}$.

The initial solution can be taken as $U_0(x, t) = e^x$, then

$$\begin{aligned} U_1(x, t) &= U_0 - \int_0^t \left(\frac{\partial^\alpha U_0}{\partial \tau^\alpha} - \frac{\partial^2 U_0}{\partial x^2} + 2U_0 \right) (d\tau)^\alpha, \\ &= e^x \left(1 - \frac{t^\alpha}{\Gamma(1+\alpha)} \right), \\ U_2(x, t) &= U_1 - \int_0^t \left(\frac{\partial^\alpha U_1}{\partial \tau^\alpha} - \frac{\partial^2 U_1}{\partial x^2} + 2U_1 \right) (d\tau)^\alpha, \\ &= e^x \left(1 - \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} \right), \\ U_3(x, t) &= e^x \left(1 - \frac{t^\alpha}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{t^{3\alpha}}{\Gamma(1+3\alpha)} \right). \end{aligned} \quad (9)$$

Continuing in this way, the next iterations can be computed by using Mathematica software.

Finally, the solution is found as,

$$U(x, t) = \lim_{n \rightarrow \infty} U_n(x, t) = e^x E_\alpha(-t^\alpha). \quad (10)$$

In view of (5) and (6), the iteration formula for problem (7) can be constructed as,

$$\begin{aligned} v_0 &= e^x, \\ v_1 &= -e^x \frac{t^\alpha}{\Gamma(1+\alpha)}, \\ v_2 &= e^x \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}, \\ v_3 &= -e^x \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}, \\ &\vdots \\ v_k &= (-1)^k e^x \frac{t^{k\alpha}}{\Gamma(1+k\alpha)}. \end{aligned}$$

Clearly, we can conclude that the obtained solution, $\sum_{k=0}^{\infty} v_k$ converges to the exact solution $U(x, t) = e^x E_\alpha(-t^\alpha)$, where $E_\alpha(z)$ is the one parameter Mittag-Leffler function defined in [2].

In addition, by computing χ_i 's for this problem, we have,

$$\chi_i = \frac{\|v_{i+1}\|}{\|v_i\|} = \left\| t^\alpha \frac{\Gamma(1+i\alpha)}{\Gamma(1+(i+1)\alpha)} \right\| < 1,$$

when for example, $i > 1$ and $0 < \alpha \leq 1$. This confirms that the variational approach for the problem (7)–(8) gives the positive and bounded solution, which converges to the exact solution.

Remark 2 [47]. The above test problem is considered when $0 < t \leq 1$ in order to discuss the condition of convergence. Of course, we can length the interval and examine the condition of convergence after neglecting the first few terms of series solution. For example, if we consider the time-fractional Newell-Whitehead-Segel equation (2) when $0 < t \leq a$ and $\alpha = 1$, where $a > 0$, then

$$\chi_i = \left\| t \frac{\Gamma(1+i)}{\Gamma(1+(i+1))} \right\| = \left\| t \frac{i!}{(i+1)!} \right\| \leq \frac{a}{i+1} < 1, \text{ for } i > a.$$

Therefore, the series solution $\sum_{k=0}^{\infty} v_k$ is positive and bounded, which converges to the exact solution for every $a > 0$.

Fig. 1: Behavior of exact solution $U(x, t)$ w.r.t. x and t when $\alpha = 1$ for Eqs. (7)–(8).

Abbaoui and Cherruault [45] have proved the convergence of this type of series. It is observed from the results that FVIM works efficiently for this problem, though lower order approximate solution $U_2(x, t)$ is taken. However, if we take $\alpha = 1$, we get the solution of classical Newell-Whitehead-Segel equation as

$$U(x, t) = e^x \left[1 - \frac{t}{1!} + \frac{t^2}{2!} + \dots \right], \quad (11)$$

which converges very fast to the exact solution

$$U(x, t) = e^{x-t}. \quad (12)$$

It is the same solution as obtained by RDTM [35], ADM [36], HPM [39] and HASTM [44]. The numerical results obtained by using FVIM and exact solution are depicted through figs. 1-7. Numerical simulations are carried

Fig. 2: Behavior of second term approximate solution $U(x, t)$ w.r.t. x and t , when $\alpha = 1$ for Eqs. (7)–(8).

Fig. 3: Behavior of second term approximate solution $U(x, t)$ w.r.t. x and t , when $\alpha = 0.3$ for Eqs. (7)–(8).

Fig. 4: Behavior of second term approximate solution $U(x, t)$ w.r.t. x and t , when $\alpha = 0.5$ for Eqs. (7)–(8).

out for $U(x, t)$ at the distinct fractional Brownian motions given by $\alpha = 0.3, 0.5, 0.7$ as shown in figs. 3-5 and the standard motion $\alpha = 1$ as shown in fig. 2. It is observed

Fig. 5: Behavior of second term approximate solution $U(x, t)$ w.r.t. x and t , when $\alpha = 0.7$ for Eqs. (7)–(8).

Fig. 6: Absolute error $E_8(U) = |\text{Exact} - \text{Approximate value}|$ for Eqs. (7)–(8).

Fig. 7: Comparison between exact and approximate solution when $\alpha = 1$ for Eqs. (7)–(8).

from figs. 1-2 that the approximate solution obtained by FVM is almost identical with exact solution at $\alpha = 1$ for different values of x and t . From fig. 7, it is found that exact and approximate solutions are in complete agreement at $\alpha = 1$. It is also to be noted that only eight terms of the

series solution are considered for absolute error in fig. 6. Hence the accuracy of FVIM can be enhanced by increasing the number of iterations.

Example 2. Consider the nonlinear time-fractional Newell-Whitehead-Segel equation

$$D_t^\alpha U = U_{xx} + 2U - 3U^2, \quad 0 < \alpha \leq 1, \quad (13)$$

with the initial condition

$$U(x, 0) = \lambda. \quad (14)$$

When $\alpha = 1$, the exact solution of Eqs. (13)-(14) is $U(x, t) = \frac{-2}{3} \lambda e^{2t}$.

The initial solution can be taken as $U_0(x, t) = \lambda$, then

$$\begin{aligned} U_1(x, t) &= U_0 - \int_0^t \left[\frac{\partial^\alpha U_0}{\partial \tau^\alpha} - \frac{\partial^2 U_0}{\partial x^2} - 2U_0 + 3U_0^2 \right] (d\tau)^\alpha, \\ &= \lambda + (2\lambda - 3\lambda^2) \frac{t^\alpha}{\Gamma(1+\alpha)}, \end{aligned}$$

$$\begin{aligned} U_2(x, t) &= U_1 - \int_0^t \left[\frac{\partial^\alpha U_1}{\partial \tau^\alpha} - \frac{\partial^2 U_1}{\partial x^2} - 2U_1 + 3U_1^2 \right] (d\tau)^\alpha, \\ &= \lambda + \frac{(2\lambda - 3\lambda^2)}{\Gamma(1+\alpha)} t^\alpha + \frac{2(1-3\lambda)(2\lambda - 3\lambda^2)}{\Gamma(1+2\alpha)} t^{2\alpha} \\ &\quad - \frac{3\lambda(2-3\lambda)^2 \Gamma(1+2\alpha)}{(\Gamma(1+\alpha))^2 \Gamma(1+3\alpha)} t^{3\alpha}. \end{aligned}$$

Proceeding in this manner, the enduring components can also be obtained using Mathematica software. Hence, we find the solution as $U(x, t) = \lim_{n \rightarrow \infty} U_n(x, t)$.

Now, by taking $\alpha = 1$, we get the solution of classical nonlinear Newell-Whitehead-Segel equation as

$$\begin{aligned} U(x, t) &= \lambda + (2\lambda - 3\lambda^2) \frac{t}{1!} + 2(1-3\lambda)(2\lambda - 3\lambda^2) \frac{t^2}{2!} \\ &\quad + 6\lambda(2-3\lambda)^2 \frac{t^3}{3!} + \dots, \end{aligned} \quad (15)$$

which converge very fast to the exact solution

$$U(x, t) = \frac{-2}{3} \lambda e^{2t}. \quad (16)$$

It is the same solution as obtained by RDTM [35], ADM [36], HPM [39] and HASTM [44]. The numerical results obtained by using FVIM and exact solution are presented through fig. 8 at the distinct fractional Brownian motions given by $\alpha = 0.25, 0.5, 0.75$ and the standard motion $\alpha = 1$ at $\lambda = 1$ for different values of t . It is observed that for $\alpha = 0.25$, as t increases, U first decreases but then sharply increases afterwards. For $\alpha = 0.5$ also,

U first decreases and then increases with increasing t . It is interesting to note that U decreases with increasing t for values of $\alpha \geq 0.7$ and finally approaches the exact solution at $\alpha = 1$. fig. 8 shows that the results obtained with the help of FVIM are approximately same as the exact solution at $\alpha = 1$. The numerical results indicate that FVIM works very well, even if lower order approximate solution $U_2(x, t)$ is used. However, the accuracy can be improved by using higher order approximations in solution.

Fig. 8: Plots of $U(x, t)$ vs. t at $\alpha = 0.25, 0.5, 0.75$ and $\alpha = 1$ at $\lambda = 1$ for exact solution and approximate solution for Eqs. (13)–(14).

6 Conclusion

In this article, FVIM is successfully applied to solve fractional model of Newell–Whitehead–Segel equation. It is apparently seen from illustrative example that FVIM is easy to implement, powerful and efficient numerical method to find an approximate solution. It is also to be noted that FVIM is used directly without using linearization, perturbation, Adomian polynomial or any restrictive assumptions. Hence, FVIM is more convenient and also easier than other existing methods.

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