Sundarapandian Vaidyanathan*, Aceng Sambas, Sezgin Kacar, and Unal Cavusoglu

A New Finance Chaotic System, its Electronic Circuit Realization, Passivity based Synchronization and an Application to Voice Encryption

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Abstract: Modelling and control applications of dynamical systems in chaos theory arising in several areas are investigated and new control techniques are designed in the chaos literature. We propose a new complex finance chaotic model with states as the interest rate, investment demand, and price index. In this work, after studying the dynamical properties of the new finance model, an electronic chaotic circuit of the model is realized in Multisim. Based on passive control theory, we derive a new controller for globally synchronizing state trajectories of the new finance models. Using the new finance chaotic model developed in this work, a new voice encryption algorithm design is presented. With the proposed algorithm, voice encryption application is performed, and results are described.

Keywords: Chaos, chaotic systems, circuit design, passive control, voice encryption

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***Corresponding Author: Sundarapandian Vaidyanathan**, Research and Development Centre, Vel Tech University, Chennai, Tamil Nadu 600062, India, E-mail: sundarcontrol@gmail.com

Aceng Sambas, Department of Mechanical Engineering, Universitas Muhammadiyah Tasikmalaya, Indonesia, E-mail: acenx.bts@gmail.com

Sezgin Kacar, Department of Electrical and Electronics Engineering, Faculty of Technology, Sakarya University, Sakarya, Turkey, E-mail: skacar@sakarya.edu.tr

Unal Cavusoglu, Department of Computer Engineering, Faculty of Computer and Information Sciences, Sakarya University, Sakarya, Turkey, E-mail: unalc@sakarya.edu.tr

1 Introduction

Chaotic dynamical systems and electronic chaotic circuits have applications in many research fields [1–10]. In [1], Vaidyanathan studied a novel chemical chaotic reactor system and its adaptive control. In [2], Vaidyanathan studied the control and synchronization of a 3-D novel jerk chaotic system with two quadratic nonlinearities. In [3], Uthamacumaran investigated a biophysical approach to cancer dynamics with the help of quantum chaos and energy turbulence. In [4], Senouci *et al.* discussed FPGA implementation of chaotic generators. In [5], Xu *et al.* studied hysteretic chaotic neuron and its applications in communication systems. In [6], Hu *et al.* used DNA insertion and DNA deletion for the study of chaotic image cryptosystem. In [7], Yuan *et al.* studied a 5-D hyperchaotic system and new parallel image cryptosystem. In [8], Zhou *et al.* studied the cluster synchronization on multiple nonlinearly coupled dynamical subnetworks of complex networks with nonidentical nodes and stochastic perturbations. In [9], Li *et al.* proposed a hyperchaos based image encryption algorithm using pixel-level permutation and bit-level permutation. In [10], Hamdi *et al.* proposed a very simple and efficient encryption scheme based on controlled chaotic maps and ADPCM (Adaptive Differential Pulse Code Modulation) coding, to secure the real-time voice communication for operating at 16, 24, 32 or 40 kbps.

In 1990, Pecora and Carroll showed that chaotic systems can be synchronized by using a drive-response configuration of chaotic systems in their seminal paper [11]. In recent years, the idea of synchronization of chaotic systems has received a great deal of interest in research areas such as secure communications, encryption, cryptosystems, automatic control, etc. In [12], Sun *et al.* investigated the synchronization of chaotic PMSM systems for secure communication and parameters identification. In [13], Khorashadizadeh and Majidi proposed a new method for secure communication based on chaos synchronization. In [14], Li *et al.* proposed a novel image

encryption algorithm based on synchronized random bit generated in cascade-coupled chaotic semiconductor ring lasers. In [15], Muthukumar *et al.* studied the sliding mode control design for synchronization of fractional order chaotic systems and its application to a new cryptosystem. In [16], Bhattacharya *et al.* discussed the implementation of chaotic and synchronization properties of logistic maps using artificial neural networks for code generation. In [17], Cavusoglu *et al.* described a chaos based Random Number Generator (RNG) which is designed with the help of a new chaotic system and designed a hybrid RSA algorithm for text and image encryption. In [18], Wang *et al.* described a novel image encryption algorithm that combines the SHA-3 hash function and two chaotic systems, viz. hyperchaotic Lorenz and Chen systems and detailed experimental results of their encryption algorithm. In [19], Jridi and Alfalou deployed chaotic generators to develop a novel chaos-based encryption algorithm. In [20], Nosrati *et al.* discussed cubature Kalman filter-based chaotic synchronization and image encryption.

Several control techniques such as active control, adaptive control, backstepping control and sliding mode control have been proposed in the literature for the control and synchronization of chaotic systems [21–27].

In recent years, the concept of *passivity* of nonlinear systems has attracted new interest in nonlinear control theory [28–34]. In engineering, passive control is a scheme of energy dissipation devices and seismic base isolation systems applied to structures to reduce the vibrations without using external power. In [28], Wen used passive control technique for controlling chaos in the Lorenz system. In [29], Song *et al.* applied passive control technique to design a passive controller to control the Newton–Leipnik chaotic system with multiple strange attractors. In [30], Wang and Liu used passive control technique to study the passive control of a four-scroll chaotic system. In [31], Wang and Liu designed a passive controller to realize the synchronization between two hyperchaotic Lorenz systems under different initial conditions. In [32], Wang and Liu used passive control technique to realize the chaos synchronization problem for the unified chaotic system. In [33], Wu *et al.* applied the passive control technique to realize the chaos synchronization of Rikitake chaotic attractor. In [34], Kocamaz and Uyaroglu studied the synchronization of Vilnius chaotic oscillators with active and passive control.

Electronic chaotic circuits have important applications in the chaos literature [35–38]. In [35], Pham *et al.* described a chaotic system having an infinite number of equilibrium points on a rounded square loop and designed an experimental circuit implementing their chaotic model.

In [36], Volos *et al.* announced their finding of a new four-dimensional hyperchaotic four-wing system and implemented their work with an experimental circuit realization. In [37], Vaidyanathan *et al.* found a new jerk chaotic circuit and gave an application of their jerk chaotic circuit to voice encryption. In [38], Wang *et al.* announced a new hyperchaotic hyperjerk system with two polynomial nonlinearities and realized it with an electronic circuit model.

Control and synchronization of finance chaotic systems is an important research area in the chaos literature. In [39], Goa and Ma reported chaos in a new finance chaotic system exhibiting Hopf bifurcation. In [40], Vaidyanathan *et al.* studied a new finance model describing a complex economical system displaying chaos. In [41], Jian *et al.* discussed the synchronization of a class of finance chaotic systems. In [42], Cia and Yang studied the globally exponentially attractive set and synchronization of a novel finance chaotic system. In [43], Zhao *et al.* used Lyapunov stability theory and Routh-Hurwitz stability criteria to design effective controllers for the global asymptotic synchronization of the finance chaotic system. In [44], Li and Xie introduced a modified finance chaotic system and discussed its synchronization via a single controller. In [45], Kocamez *et al.* carried out a comparative study of the synchronization of chaos in nonlinear finance system by means of sliding mode and passive control methods. In [46], Zheng *et al.* investigated a simple hybrid synchronization for a class of finance chaotic systems.

In this work, we propose a new finance chaotic model with state variables as the interest rate, investment demand, and price index. The construction of the new finance chaotic system is carried out by combining two known chaotic models of finance systems [39, 40]. In Section 2, we describe the dynamic equations and properties of the new finance model. In Section 3, we build an electronic circuit of the new finance model with the help of MultiSIM. In Section 4, we derive new results for globally synchronizing the state trajectories of the new finance models via passive control method. In Section 5, a new voice encryption application is performed, and analysis results are given. Section 6 contains the conclusions.

2 A new finance chaotic model

Gao and Ma finance model [39] is given by the following dynamical system:

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 \\ \dot{x}_2 = 1 - bx_2 - x_1^2 \\ \dot{x}_3 = -x_1 - cx_3 \end{cases} \quad (1)$$

Vaidyanathan finance model [40] is obtained as a modification of the finance model (1) and described by the following dynamical system:

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 \\ \dot{x}_2 = 1 - bx_2 - x_1^4 \\ \dot{x}_3 = -x_1 - cx_3 \end{cases} \quad (2)$$

In the finance dynamical models (1) and (2), the state variables x_1 and x_2 represent the interest rate and investment demand respectively, while the state variable x_3 represents the price exponent. Also, the financial parameter a represents the household savings and b represents the investment cost, while c stands for the demand elasticity of commercial markets. All the three financial parameters are supposed to be positive.

We note that the Gao-Ma finance model (1) has a quadratic nonlinearity x_1x_2 and another quadratic nonlinearity x_1^2 . We also note that the Vaidyanathan finance model (2) has a quadratic nonlinearity x_1x_2 and a quartic nonlinearity x_1^4 .

In [39], Gao and Ma showed that the nonlinear finance chaotic system (1) is chaotic when $(a, b, c) = (6, 0.1, 1)$. The Lyapunov characteristic exponents of the Gao-Ma finance model (1) are obtained in MATLAB for parameter values $(a, b, c) = (6, 0.1, 1)$, initial state $X(0) = (0.5, 3, -0.4)$ and $T = 1E5$ as $L_1 = 0.090439$, $L_2 = 0$ and $L_3 = -0.393061$.

The Kaplan-Yorke dimension of the Gao-Ma finance model (1) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.2301 \quad (3)$$

In [40], Vaidyanathan *et al.* showed that the nonlinear finance chaotic system (2) is chaotic when $(a, b, c) = (7, 0.1, 1)$. The Lyapunov characteristic exponents of the Vaidyanathan finance model (2) are obtained in MATLAB for parameter values $(a, b, c) = (7, 0.1, 1)$, initial state $X(0) = (0.5, 3, -0.4)$ and $T = 1E5$ as $L_1 = 0.125494$, $L_2 = 0$ and $L_3 = -0.421513$.

The Kaplan-Yorke dimension of the Vaidyanathan finance model (2) is derived as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.2977 \quad (4)$$

In this work, we propose a new finance model by combining the Gao-Ma finance model (1) and the Vaidyanathan finance model (2). Specifically, we consider the effect of both quadratic and quartic nonlinearities in the x_2 -dynamics of the system.

The new finance model is described by

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 \\ \dot{x}_2 = 1 - bx_2 - dx_1^2 - x_1^4 \\ \dot{x}_3 = -x_1 - cx_3 \end{cases} \quad (5)$$

In the finance dynamical model (3), the state variables x_1, x_2, x_3 and the parameters a, b, c have the same interpretation as in the finance dynamical models (1) and (2). Furthermore, d is a positive constant. The system (5) has two quadratic nonlinearities and a quartic nonlinearity.

The Lyapunov characteristic exponents of the new finance model (5) are obtained in MATLAB for the financial parameter values taken as $(a, b, c, d) = (7.2, 0.1, 1, 0.1)$, initial state values as $X(0) = (0.5, 3, -0.4)$ and $T = 1E5$ as $L_1 = 0.132893$, $L_2 = 0$ and $L_3 = -0.412008$.

The Lyapunov characteristic exponents of the new finance system (5) show that the new finance system (5) is chaotic. Since the sum of Lyapunov characteristic exponents is negative, the new finance chaotic system (5) is dissipative with a strange chaotic oscillator.

The Kaplan-Yorke dimension of the new finance chaotic system (5) is determined as follows:

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.3225 \quad (6)$$

Table 1 shows a comparison of the Gao-Ma finance system (1), Vaidyanathan finance system (2) and the new finance system (5). From Table 1, we see that the new finance chaotic system (5) is more chaotic than the Gao-Ma finance system (1) and Vaidyanathan finance system (2).

Next, we note that the new finance chaotic system (5) stays invariant under the coordinates transformation

$$(x_1, x_2, x_3) \mapsto (-x_1, x_2, -x_3) \quad (7)$$

for all values of the system parameters (a, b, c, d) . This shows that the new finance chaotic system (5) has rotation symmetry about the x_2 -axis. Hence, for every non-trivial trajectory $(x_1(t), x_2(t), x_3(t))$ of the system (5), $(-x_1(t), x_2(t), -x_3(t))$ is also a trajectory of the system (5).

Figures 1-3 illustrate the two-dimensional phase plots of the new finance chaotic system (5). Figure 4 displays the Lyapunov characteristic exponents of the new finance chaotic system (5) for the parameter d . For these numerical simulations, we have taken the initial state as $X(0) = (0.5, 3, -0.4)$ and the parameter values as $(a, b, c, d) = (7.2, 0.1, 1, 0.1)$ for the new finance chaotic system (5).

The equilibrium points of the new finance chaotic system (5) are obtained by solving the following system of equations.

$$x_3 + (x_2 - a)x_1 = 0 \quad (8a)$$

Table 1: A Comparison of the Finance Chaotic Systems

Chaotic System	Lyapunov Exponents	Maximal Lyapunov Exponent (MLE)	Kaplan-Yorke Dimension
Gao-Ma Finance System [39]	$L_1 = 0.090439, L_2 = 0, L_3 = -0.393061$	$L_1 = 0.090439$	$D_{KY} = 2.2301$
Vaidyanathan Finance System [40]	$L_1 = 0.125494, L_2 = 0, L_3 = -0.421513$	$L_1 = 0.125494$	$D_{KY} = 2.2977$
New Finance System	$L_1 = 0.132893, L_2 = 0, L_3 = -0.412008$	$L_1 = 0.132893$	$D_{KY} = 2.3225$

Fig. 1: Numerical simulations of the 2-D phase plot in (x_1, x_2) -plane of the new finance chaotic system (5) for $X(0) = (0.5, 3, -0.4)$ and $(a, b, c, d) = (7.2, 0.1, 1, 0.1)$ **Fig. 3:** Numerical simulations of the 2-D phase plot in (x_1, x_3) -plane of the new finance model (3) for $X(0) = (0.5, 3, -0.4)$ and $(a, b, c, d) = (7.2, 0.1, 1, 0.1)$ **Fig. 2:** Numerical simulations of the 2-D phase plot in (x_2, x_3) -plane of the new finance model (3) for $X(0) = (0.5, 3, -0.4)$ and $(a, b, c, d) = (7.2, 0.1, 1, 0.1)$ **Fig. 4:** Lyapunov characteristic exponents for the parameter d of the new finance chaotic system (5), where $X(0) = (0.5, 3, -0.4)$ and $(a, b, c) = (7.2, 0.1, 1)$

$$1 - bx_2 - dx_1^2 - x_1^4 = 0 \quad (8b)$$

$$-x_1 - cx_3 = 0 \quad (8c)$$

From Eq. (8c), it follows that

$$x_1 = -cx_3 \quad (9)$$

From (8a) and (9), we get

$$x_3 - cx_3(x_2 - a) = 0 \quad (10)$$

We have two cases to consider: (i) $x_3 = 0$ and (ii) $x_3 \neq 0$.

In Case (i), we let $x_3 = 0$. From (9), $x_1 = 0$. Substituting $x_1 = x_3 = 0$ in Eq. (8b), we get $1 - bx_2 = 0$. Thus, it is easy

to see that the new chaotic system (5) has the equilibrium

$$E_0 = \begin{bmatrix} 0 \\ 1/b \\ 0 \end{bmatrix} \quad (11)$$

In Case (ii), we let $x_3 \neq 0$. From Eq. (10), we must have $1 - c(x_2 - a) = 0$. Thus, we have

$$x_2 = a + \frac{1}{c} = \mu \text{ (say)} \quad (12)$$

Let $z = x_1^2$. Then we can write Eq. (8b) as the quadratic equation

$$z^2 + dz + (b\mu - 1) = 0 \quad (13)$$

Solving (13), we get

$$z = \frac{-d \pm \sqrt{d^2 - 4(b\mu - 1)}}{2} \quad (14)$$

In order that $z > 0$, we must have $b\mu - 1 < 0$ or $\mu < \frac{1}{b}$. In this case, we get

$$z = x_1^2 = \frac{-d + \sqrt{d^2 - 4(b\mu - 1)}}{2} \quad (15)$$

Thus, when $\mu = a + \frac{1}{c} < \frac{1}{b}$, the new chaotic system (5) has two more equilibrium points

$$E_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \left[-d + \sqrt{d^2 - 4(b\mu - 1)} \right]^{0.5} \\ \mu \\ \frac{-1}{c\sqrt{2}} \left[-d + \sqrt{d^2 - 4(b\mu - 1)} \right]^{0.5} \end{bmatrix}$$

and

$$E_2 = \begin{bmatrix} \frac{-1}{\sqrt{2}} \left[-d + \sqrt{d^2 - 4(b\mu - 1)} \right]^{0.5} \\ \mu \\ \frac{1}{c\sqrt{2}} \left[-d + \sqrt{d^2 - 4(b\mu - 1)} \right]^{0.5} \end{bmatrix} \quad (16)$$

Combining the cases (i) and (ii), we obtain three equilibrium points of the new finance chaotic system E_0 , E_1 and E_2 , which are defined in the equations (11) and (16).

For the parameter values $(a, b, c, d) = (7.2, 0.1, 1, 0.1)$, the new finance chaotic system (5) has three equilibrium points, which are derived as follows:

$$E_0 = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.6142 \\ 8.2 \\ -0.6142 \end{bmatrix}$$

$$\text{and } E_2 = \begin{bmatrix} -0.6142 \\ 8.2 \\ 0.6142 \end{bmatrix} \quad (17)$$

We can express the new finance chaotic system (5) in vector notation as follows:

$$\dot{X} = f(X), \text{ where } f(X) = \begin{bmatrix} x_3 + (x_2 - a)x_1 \\ 1 - bx_2 - dx_1^2 - x_1^4 \\ -x_1 - cx_3 \end{bmatrix} \quad (18)$$

We find that

$$J_0 = Df(E_0) = \begin{bmatrix} 2.8 & 0 & 1 \\ 0 & -0.1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \quad (19)$$

which has the spectral values

$$\lambda_1 = -0.1, \lambda_2 = -0.7155, \lambda_3 = 2.5155 \quad (20)$$

Thus, the equilibrium E_0 is a saddle point and unstable.

A simple calculation shows that

$$J_1 = Df(E_1) = \begin{bmatrix} 1.0000 & 0.6142 & 1 \\ -1.0496 & -0.1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \quad (21)$$

which has the spectral values

$$\lambda_1 = 0.2133, \lambda_{2,3} = 0.8434 \pm 1.1720i \quad (22)$$

Thus, the equilibrium E_1 is a saddle-focus and unstable. A similar calculation also shows that E_2 is a saddle-focus and unstable.

Figures 5, 6 and 7 describe the calculation of Lyapunov characteristic exponents and the bifurcation diagrams for the new system (5) with respect to the three system parameters a , b and c , respectively.

Our bifurcation analysis is described as follows.

Case (A): Effect of varying a

Here, we fix $b = 0.1$, $c = 1$, $d = 0.1$ and vary a . When $7.20 \leq a \leq 8.20$, the new system (5) displays chaotic behavior. Near $a = 8.21$, the system (5) displays periodic behavior. For the region $8.22 \leq a \leq 8.30$, the system (5) displays chaotic behavior. Also, when $a > 8.30$, the system (5) displays periodic behavior. The bifurcation analysis for the case of varying a is described by Figure 5.

Case (B): Effect of varying b

Here, we fix $a = 7.2$, $c = 1$, $d = 0.1$ and vary b . When $0 \leq b \leq 0.067$, the new system (5) exhibits periodic behaviour. When $0.068 \leq b \leq 0.113$, the system (5) displays chaotic behaviour. Also, when $b \geq 0.114$, the system (5) displays a fixed point. The bifurcation analysis for the case of varying b is described by Figure 6.

Case (C): Effect of varying c

Here, we fix $a = 7.2$, $b = 0.1$, $d = 0.1$ and vary c . When $0 \leq c \leq 0.90$, the new system (5) displays periodic behavior. When $0.91 \leq c \leq 1.56$, the system (5) displays chaotic behaviour. Also, when $c \geq 1.57$, the system (5) displays periodic behaviour. The bifurcation analysis for the case of varying c is described by Figure 7.

Fig. 5: Diagram of the Lyapunov characteristic exponents versus a and the bifurcation diagram of $X_{3\max}$ versus a for $b = 0.1$, $c = 1$ and $d = 0.1$

Fig. 6: Diagram of the Lyapunov characteristic exponents versus b and the bifurcation diagram of $X_{3\max}$ versus b for $a = 7.2$, $c = 1$ and $d = 0.1$

Fig. 7: Diagram of the Lyapunov characteristic exponents versus c and the bifurcation diagram of $X_{3\max}$ versus c for $a = 7.2$, $b = 0.1$ and $d = 0.1$

3 Circuit realization of the new finance chaotic model

In this section, the electronic circuit design of the new finance model (5) is constructed. In this study, a linear scaling is considered as follows:

$$\begin{cases} \dot{x}_1 = x_3 + (4x_2 - a)x_1 \\ \dot{x}_2 = \frac{1}{4} - bx_2 - 4dx_1^2 - 64x_1^4 \\ \dot{x}_3 = -x_1 - cx_3 \end{cases} \quad (23)$$

By applying Kirchhoff's laws to this circuit, we get the following circuital equations:

$$\begin{cases} \frac{dV_{C_1}}{dt} = \frac{1}{C_1R_1}V_{C_3} + \frac{1}{C_1R_2}V_{C_1}V_{C_2} - \frac{1}{C_1R_3}V_{C_1} \\ \frac{dV_{C_2}}{dt} = \frac{1}{C_2R_4}V_1 - \frac{1}{C_2R_5}V_{C_2} - \frac{1}{C_2R_6}V_{C_1}^2 - \frac{1}{C_2R_7}V_{C_1}^4 \\ \frac{dV_{C_3}}{dt} = -\frac{1}{C_3R_8}V_{C_1} - \frac{1}{C_3R_9}V_{C_3} \end{cases} \quad (24)$$

In Eq. (7), C_1, C_2, C_3 are the capacitors and $V_{C_1}, V_{C_2}, V_{C_3}$ are the voltages across them. Also, each state of the finance model (5), i.e. x_1, x_2, x_3 is implemented as the voltage across the corresponding capacitors C_1, C_2 and C_3 , respectively.

We choose the values of the circuital elements as: $R_1 = R_8 = R_9 = 400 \text{ k}\Omega$, $R_3 = 55.55 \text{ k}\Omega$, $R_6 = 1 \text{ M}\Omega$, $R_5 = 4 \text{ M}\Omega$, $R_7 = 6.25 \text{ k}\Omega$, $R_4 = 1.6 \text{ k}\Omega$, $V_1 = -1 \text{ V}_{DC}$, $R_2 = R_{10} = R_{11} = R_{12} = R_{13} = 100 \text{ k}\Omega$, $C_1 = C_2 = C_3 = 1 \text{ nF}$.

The supplies of all active devices are $\pm 15 \text{ Volt}$. Using the circuit design approach based on the operational amplifiers [47–53], we have designed the electronic chaotic circuit as shown in Figure 8. The circuit simulations of the phase plots are displayed in Figures 9–11. Clearly, the Multisim output results in Figures 9–11 show a good match with the MATLAB simulations in Figures 1–3.

4 Passive control design for the complete synchronization of the new finance chaotic model

This section describes our new results for the complete synchronization of a pair of identical new finance chaotic models (taken as the master and slave systems) via passive control.

As the master system, we take the new finance model with the following dynamics:

$$\begin{cases} \dot{\xi}_1 = \xi_3 + (\xi_2 - a)\xi_1 \\ \dot{\xi}_2 = 1 - b\xi_2 - d\xi_1^2 - \xi_1^4 \\ \dot{\xi}_3 = -\xi_1 - c\xi_3 \end{cases} \quad (25)$$

As the slave system, we take the new controlled finance model on with the following dynamics:

$$\begin{cases} \dot{\eta}_1 = \eta_3 + (\eta_2 - a)\eta_1 + u \\ \dot{\eta}_2 = 1 - b\eta_2 - d\eta_1^2 - \eta_1^4 \\ \dot{\eta}_3 = -\eta_1 - c\eta_3 \end{cases} \quad (26)$$

The synchronization between the finance models (25) and (26) is quantified by the synchronization error defined by the following equations:

$$\begin{cases} e_1 = \eta_1 - \xi_1 \\ e_2 = \eta_2 - \xi_2 \\ e_3 = \eta_3 - \xi_3 \end{cases} \quad (27)$$

The time-evolution of the error trajectories is governed by the following differential equation:

$$\begin{cases} \dot{e}_1 = e_3 - ae_1 + \eta_1\eta_2 - \xi_1\xi_2 + u \\ \dot{e}_2 = -be_2 - e_1(\xi_1 + \eta_1)(d + \xi_1^2 + \eta_1^2) \\ \dot{e}_3 = -e_1 - ce_3 \end{cases} \quad (28)$$

Suppose that $y = e_1$ is the output of the error dynamics system (28).

We also define new states as $z_1 = e_2$ and $z_2 = e_3$. Then it is possible to express the error dynamical system (28) in normal form as follows.

$$\begin{cases} \dot{z}_1 = -bz_1 - (\xi_1 + \eta_1)(d + \xi_1^2 + \eta_1^2)y \\ \dot{z}_2 = -y - cz_2 \\ \dot{y} = z_2 - ay + \eta_1\eta_2 - \xi_1\xi_2 + u \end{cases} \quad (29)$$

We express the system (29) in the standard form of passive control theory [54] as follows:

$$\begin{cases} \dot{Z} = f_0(Z) + p(Z, y)y \\ \dot{y} = \beta(Z, y) + \alpha(Z, y)u \end{cases} \quad (30)$$

where

$$\begin{cases} f_0(Z) = \begin{bmatrix} -bz_1 \\ -cz_2 \end{bmatrix}, p(Z, y) = \begin{bmatrix} -(\xi_1 + \eta_1)(d + \xi_1^2 + \eta_1^2) \\ -1 \end{bmatrix} \\ \beta(Z, y) = z_2 - ay + \eta_1\eta_2 - \xi_1\xi_2, \alpha(Z, y) = 1 \end{cases} \quad (31)$$

We consider

$$W(Z) = \frac{1}{2} (z_1^2 + z_2^2) \quad (32)$$

We find the time-derivative of $W(Z)$ along the dynamics

$$\dot{Z} = f_0(Z) \quad (33)$$

We find that

$$\dot{W} = \frac{\partial W}{\partial Z} f_0(Z) = -bz_1^2 - cz_2^2 \quad (34)$$

Fig. 8: Circuit realization of the new finance model (5)

We choose the storage function by the relation

$$V(Z, y) = W(Z) + \frac{1}{2}y^2 \quad (35)$$

Clearly, V is positive definite on R^3 .

The time-derivative of V along the error dynamics (14) is found as follows:

$$\dot{V} = -bz_1^2 - cz_2^2 + y[A + u] \quad (36)$$

where

$$A = -(\xi_1 + \eta_1)(d + \xi_1^2 + \eta_1^2)z_1 - ay + \eta_1\eta_2 - \xi_1\xi_2 \quad (37)$$

As the passive controller u , we choose

$$u = (\xi_1 + \eta_1)(d + \xi_1^2 + \eta_1^2)z_1 + (a - k)y - \eta_1\eta_2 + \xi_1\xi_2 \quad (38)$$

where $k > 0$ is a positive gain constant.

Putting (38) into (36), we find that

$$\dot{V} = -bz_1^2 - cz_2^2 - ky^2, \quad (39)$$

which is a negative definite function on R^3 .

By Lyapunov stability theory [55], it is deduced that the synchronization error dynamics (29) is globally asymptotically stable for all initial conditions.

Hence, we have proved the following control result.

Theorem 1. The passive control (38) with $k > 0$ achieves global and complete asymptotic synchronization of the new finance chaotic models (25) and (26) for all $\xi(0), \eta(0) \in R^3$.

For MATLAB simulations, we take the financial parameters as in the chaotic case, viz. $(a, b, c, d) = (7.2, 0.1, 1, 0.1)$. Also, as gain constant, we choose $k = 0.5$.

Fig. 9: MultiSIM plot of the new finance model (5) in x_1 - x_2 plane

Fig. 10: MultiSIM plot of the new finance model (5) in x_2 - x_3 plane

Fig. 11: MultiSIM plot of the new finance chaotic system (5) in x_1 - x_3 plane

The initial state of the master system (25) is taken as $\xi(0) = (1.2, 0.8, 1.7)$ and that of the slave system (26) is taken as $\eta(0) = (0.6, 1.4, 0.9)$. Figure 12 displays the asymptotic synchronization of the new finance models (25) and (26). Figure 13 displays the time-history of the synchronization errors between the finance models (25) and (26).

Fig. 12: MATLAB simulation demonstrating the asymptotic synchronization of the new finance models (9) and (10) for $\xi(0) = (1.2, 0.8, 1.7)$ and $\eta(0) = (0.6, 1.4, 0.9)$

Fig. 13: MATLAB simulation depicting the time-history of the synchronization errors between the new finance models (9) and (10) for $\xi(0) = (1.2, 0.8, 1.7)$ and $\eta(0) = (0.6, 1.4, 0.9)$

5 Voice encryption application and its analysis

5.1 Voice encryption algorithm design

Using the new finance model (5) developed in this work, voice encryption is performed. The generation of random numbers to be used in the voice encryption process has been performed. After entering the initial conditions of the chaotic model and system parameters in the random number generation, the chaotic model is solved by the classical fourth-order Runge-Kutta (RK-4) method and the float number values are obtained. With the conversion of the float numbers obtained from all three phases to the binary system, random bit sequences are generated. The length of the generated random bit array depends on the length of the voice file. For voice file encryption, it is converted to binary form from binary. In a binary format voice file is subjected to XOR processing with the bit sequence obtained from the random number generator. After the encryption process, the data is converted to a float form and an encrypted voice file is obtained. On the receiving side, the same random number is obtained with the identical random number generator, and the original voice file is obtained by decrypting the encrypted voice file. Figure 14 depicts the block diagram of the voice encryption algorithm.

Fig. 14: The voice encryption block diagram

5.2 Voice encryption application and its analysis

After the design of the encryption algorithm, voice encryption application is implemented. Figure 15 shows the original, encrypted and decrypted voice files used in the en-

Fig. 15: Original, encrypted and decrypted voice files

ryption process. When we analyse at the graph of the encrypted voice file, it appears that a completely different voice file is obtained from the original file in Figure 15(a). When Figure 15(a) and (c) are examined, it is seen that the voice file has been successfully decrypted and the original voice file has been obtained. Figure 16 shows frequency spectrum analysis results of original, encrypted and decrypted voice files. When Figure 16(a) and (b) are examined, it is seen that the spectrum graph of the encrypted voice file has much wider frequency values than the original voice file of spectrum analysis results and has almost

equal spectrum values for each sampling value. In Figures 16(a) and (c), it is seen that the spectrum analyses of the original and the decrypted voice file are equal. According to the results of frequency spectrum analysis, it has been found that the proposed encryption algorithm successfully performs voice encryption and decryption.

(a)

(b)

(c)

Fig. 16: The frequency spectrum analysis results of the original, encrypted and decrypted voice files

6 Conclusions

This work reported the finding a new complex finance chaotic model with state variables consisting of the interest rate, investment demand, and price index. The dynamical properties of the new finance chaotic model were studied in detail. An electronic chaotic circuit of the new finance model was designed and analyzed in Multisim. Using passive control theory, we developed a new controller for the global and asymptotic synchronization of new finance chaotic models. Numerical simulations were displayed to elucidate all the main results presented in this work. In this study, a new voice encryption algorithm is presented using random numbers generated by a random number generator designed with the new finance chaotic model. Voice encryption application is performed the new algorithm and frequency spectrum analysis of encryption process is performed. According to the analysis results, it has been determined that the proposed encryption algorithm successfully performs the encryption process.

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