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Surface roughness effect on thermohydrodynamic analysis of journal bearings lubricated with couple stress fluids

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Abstract: The paper presents, surface roughness effect for thermo-hydrodynamic analysis of journal bearings extended to couple stress lubricants with high polymer additives. A modified energy equation is simultaneously solved with heat transfer equation as well as modified Reynolds equation by using Multigrid method. The effects of couple stress and surface roughness on the performances of a finite journal bearing are presented in detail. Further, it is shown that lubricants with couple stress and surface roughness, not only increases the load capacity and decreases the friction coefficient, but also generates a lower bearing temperature field. Thus, the lubricant with couple stress improves the performance of journal bearings. The characteristics of bearing are compared with numerical results.

1 Introduction

The performance of journal bearings with thermal effects in lubrication process is analysed by thermal hydrodynamic lubrication (THD). Since it produces peak bearing temperature, bearing failure can be estimated at the design when maximum temperature exceeds a certain limit. Ferron et al. [1] solved Dowson's [2] generalized Reynolds equation simultaneously with energy equation and heat transfer equation and produced excellent results. THD solutions under various boundary conditions, considering the mixing of recirculation of fluid and supply oil is obtained by Khonsari and Beaman [3]. Ott and Paradissiadis [4] applied Jakobsson–Floberg–Olsson (JFO)

boundary conditions, which conserve mass within cavitating region as well as boundary, to determine the cavitation region and used same energy equation for both full film and cavitation regions. In recent years, more comprehensive THD analysis are developed by Paranjpe and Han [5], Khonsari et al. [6], Ramesh et al. [7], and Shi and Wang [8]. However all these studies do not include more complex fluids. The use of complex fluids as lubricants is getting more and more important to fulfil the desires of modern machines. For example, high polymer additives are added in lubricating oils as a kind of viscosity index improver so that viscosity variation with temperature decreases. On the other hand, addition of polymers exhibit non-newtonian behaviour, leading to classical continuum theory which becomes inapplicable.

The classical continuum fluid mechanics neglects size of fluid particles to describe the flow of fluids. The micro-continuum fluid mechanics accounts for intrinsic motion of material constituents in order to extend the range of application, for example, to describe the flow of complex fluids, polymer molecules are used in polymeric suspensions. The classical continuum fluid mechanics is generalized by non-polar theory of fluids, which is characterized by classical Cauchy stress. The micro-continuum fluid mechanics is generalized by polar theory of complex fluids, which is characterized by classical Cauchy stress as well as by couple stress [9]. Recently several studies are carried out to describe the flow of complex fluids. The couple stress theory of fluids was presented by Stokes [10] to examine simplest generalization of classical theory that would allow for polar effects. This model defines the rotation field in terms of velocity field; the rotation vector is equated to one-half of curl of velocity vector. The couple stress fluid model has been widely used in lubrication for its relative mathematical simplicity. A study of a journal bearing lubricated with a couple stress fluids including elasticity of the linear is presented by Mokhiamer et al. [11]. This showed that the influence of couple stress fluid on the bearing characteristics is significantly apparent and that lubricants with couple stresses produce an increase in the load carrying capacity, decrease in attitude angle, the frictional force and

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side leakage. Lin [12] analysed the study of squeeze film characteristics of finite journal bearings assuming the lubricant to be an incompressible fluid with couple stress. The studies showed that couple stress effects provide a longer time to prevent journal bearing contact and resulting in longer bearing life. Das [13] studied the slider bearings lubricated with couple stress fluids in a magnetic field and observed that maximum load capacity increases with an increase in both magnetic and couple stress parameters. Also, the ratio of inlet to outlet film heights for maximum load capacity increases with magnetic parameter and decreases with couple stress parameter.

In most of the theoretical studies of film lubrication it was more or less assumed that the bearing surfaces could be presented by smooth mathematical planes. But this appears to be an unrealistic assumption particularly in bearing, working with small film thickness. However, bearing surfaces particularly after they have received some run-in and wear, seldom exhibit a type of roughness approximated by this model. Christensen and Tonder investigated the hydrodynamic lubrication of rough surfaces having finite width. In 1975 Christensen et al. [17] obtained a generalised Reynolds equation applicable to rough surface by assuming that the film thickness function is governing a stochastic process. Bujurke et al. [14] studied the effect of surface roughness on squeeze film poroelastic bearings with special reference to synovial joints. A simplified mathematical model is developed for understanding combined effect of surface roughness and couple stress. Surface roughness effects in a short porous journal bearing with a couple stress fluid is studied by Naduvanamani et al. [15]. Thermohydrodynamic analysis of journal bearings with couple stress fluids is analysed by Xiao et al. [16]. All the applications of couple stress models are limited to isothermal case as noticed up to now.

Multigrid method is used to find the solution more accurately than the finite difference method which used in earlier study. Hydrodynamic bearings are usually assumed to have a smooth surface. For most practical bearings this is a good approximation since the influence of the roughness on the bearing performance is negligible. Under normal operating conditions the surface roughness heights are small compared to the thickness of the lubrication film. However, when surface roughness amplitudes are of the same range as that of lubrication film thickness, the surface roughness effects can no longer be neglected. Since most of the available literature on micro asperity lubrication is based on the smooth surface assumption, the present work extends the existing literature to cover the effect of random surface roughness on micro asperity hydrodynamic lubrication. The aim of this paper is to present the

THD performance of journal bearings lubricated with couple stress fluids with surface roughness effect. The energy equation together with heat transfer equation and modified Reynolds equation were numerically solved by using multigrid method. The various effects which include temperature field, load carrying capacity, friction force, friction coefficient and end leakage are analysed in detail for different couple stress parameters.

2 Theoretical analysis

On the basis of Micro-continuum theory of Stokes [10], body forces and body couples are neglected; the basic equations of motion of fluids with couple stresses can be expressed as

$$\dot{\rho} + \rho v_{i,i} = 0 \quad (1)$$

$$\rho \dot{v}_i = \sigma_{ji,j} \quad (2)$$

$$M_{ji,j} + e_{ijk}\sigma_{jk} = 0 \quad (3)$$

The constitutive equations for stress tensor and couple stress tensor are given as [10]

$$\sigma_{ij} = (-p + \lambda v_{k,k}) \delta_{ij} + \mu(v_{i,j} + v_{j,i}) - \frac{1}{2} e_{ijk} M_{rk,r} \quad (4)$$

$$M_{rk} = \frac{1}{3} M_{nn} \delta_{rk} + 4\eta \omega_{k,r} + 4\eta' \omega_{r,k} \quad (5)$$

where $\lambda = -2\mu/3$. A finite journal bearing lubricated with an incompressible couple stress fluid is considered and is shown in the Fig. 1. The mathematical formulation for film thickness with surface roughness is

$$H = h + h_s = C(1 + \varepsilon \cos \theta) + h_s(\theta, z, \xi) \quad (6)$$

By the assumptions of hydrodynamic lubrication to thin films, the equations of motion are reduced as

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} \quad (7a)$$

$$\frac{\partial p}{\partial y} = 0 \quad (7b)$$

$$\frac{\partial p}{\partial z} = \mu \frac{\partial^2 w}{\partial y^2} - \eta \frac{\partial^4 w}{\partial y^4} \quad (7c)$$

The velocity boundary conditions at the shaft surface are

$$\begin{aligned} u|_{y=0} &= U, \quad v|_{y=0} = 0, \quad w|_{y=0} = 0, \\ \frac{\partial^2 u}{\partial y^2} \Big|_{y=0} &= 0, \quad \frac{\partial^2 w}{\partial y^2} \Big|_{y=0} = 0 \end{aligned} \quad (7d)$$

Fig. 1: The schematic diagram of journal bearing

The velocity boundary conditions at the bushing surface are

$$\begin{aligned} u|_{y=H} = 0, \quad v|_{y=H} = 0, \quad w|_{y=H} = 0, \\ \frac{\partial^2 u}{\partial y^2} \Big|_{y=H} = 0, \quad \frac{\partial^2 w}{\partial y^2} \Big|_{y=H} = 0 \end{aligned} \quad (7e)$$

Integrating Eqs. (7a) and (7c) with above boundary conditions, the velocity components can be obtained as

$$\begin{aligned} u = U \left(1 - \frac{y}{H} \right) + \frac{1}{2\mu} \frac{\partial p}{\partial x} \left\{ y(y-H) \right. \\ \left. + 2l^2 \left[1 - \left(\cosh \frac{2y-H}{2l} \right) \left(\cosh \frac{H}{2l} \right)^{-1} \right] \right\} \end{aligned} \quad (8)$$

$$\begin{aligned} w = \frac{1}{2\mu} \frac{\partial p}{\partial z} \left\{ y(y-H) \right. \\ \left. + 2l^2 \left[1 - \left(\cosh \frac{2y-H}{2l} \right) \left(\cosh \frac{H}{2l} \right)^{-1} \right] \right\} \end{aligned} \quad (9)$$

where $l = \sqrt{\eta/\mu}$

Substituting Eqs. (7) and (8) into Eq. (1), we get the following modified Reynolds equation in the form

$$\frac{\partial}{\partial x} \left[\frac{f(l, H)}{\mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[\frac{f(l, H)}{\mu} \frac{\partial p}{\partial z} \right] = 6U \frac{\partial H}{\partial x} \quad (10)$$

where

$$f(l, H) = H^3 - 12l^2 H + 24l^3 \tanh \left(\frac{H}{2l} \right). \quad (11)$$

Let $f(h_s)$ be the probability density function of stochastic film thickness h_s . Taking stochastic average of Eq. (10) with respect to $f(h_s)$, we get

$$\begin{aligned} \frac{\partial}{\partial x} \left[E \left(\frac{f(l, H)}{\mu} \right) \frac{\partial E(p)}{\partial x} \right] + \frac{\partial}{\partial z} \left[E \left(\frac{f(l, H)}{\mu} \right) \frac{\partial E(p)}{\partial z} \right] \\ = 6UE \left(\frac{\partial H}{\partial x} \right) \end{aligned} \quad (12)$$

where expectancy operator $E(\bullet)$ is defined by

$$E(\bullet) = \int_{-\infty}^{\infty} (\bullet) f(h_s) dh_s.$$

The probability density function [17] is given as

$$f(h_s) = \begin{cases} \frac{35}{32c^7} (c^2 - h_s^2)^3, & -c < h_s < c \\ 0, & \text{elsewhere} \end{cases}$$

where $c = 3\sigma_1$.

This non dimensional modified Reynolds equation based on a couple stress model [20] can be obtained by introducing following dimensionless variables

$$\theta = \frac{x}{R}, \quad y^* = \frac{y}{H}, \quad z^* = \frac{z}{R}, \quad E(H^*) = \frac{E(H)}{C} = 1 + \varepsilon \cos \theta,$$

$$\mu = \frac{\mu}{\mu_0}, \quad \omega = \frac{U}{R}, \quad E(p^*) = \frac{E(p)C^2}{\mu_0 \omega R^2}, \quad l^* = \frac{l}{C}.$$

The dimensionless form of modified Reynolds equation can be written as

$$\begin{aligned} \frac{\partial}{\partial \theta} \left[E \left(\frac{f^*(l^*, H^*)}{\mu^*_{av}} \right) \frac{\partial E(p^*)}{\partial \theta} \right] + \frac{\partial}{\partial z^*} \left[E \left(\frac{f^*(l^*, H^*)}{\mu^*_{av}} \right) \frac{\partial E(p^*)}{\partial z^*} \right] \\ = -6E(\varepsilon \sin \theta) \end{aligned} \quad (13)$$

where

$$\begin{aligned} E(f^*(l^*, H^*)) = E(H^{*3}) - 12l^{*2} E(H^*) \\ + 24l^{*3} \tanh \left\{ \left(\frac{E(H^*)}{2l^*} \right) \right\}. \end{aligned} \quad (14)$$

The lubricant viscosity μ^*_{av} in Eq. (13) is a cross-film average value. Lubrication in a steady-state and an incompressible couple stress fluid is considered as shown in Fig. 1. According to Tucker and Keogh [18], the viscosity cross film average can offer a close approximation for viscosity in lubrication analysis.

The pressure boundary conditions are:

$$\begin{aligned} \frac{\partial E(p^*)}{\partial z^*} \Big|_{z^*=0} = E(p^*) \Big|_{z^*=\pm B/2R} = 0, \quad E(p^*) \Big|_{\theta=-\delta} = E(p_0^*), \\ \frac{\partial E(p^*)}{\partial \theta} \Big|_{\theta=\theta_{cav}} = E(p^*) \Big|_{\theta=\theta_{cav}} = 0 \end{aligned} \quad (15)$$

where θ_{cav} represents location of starting point for the cavitation zone.

The first law of thermodynamics gives energy equation in fluids and is written as

$$\rho \frac{DE}{Dt} = \nabla \cdot (k \nabla T) + \Phi - p \nabla \cdot \vec{V} \quad (16)$$

where E is the specific internal energy of fluid and Φ is the heat dissipation, it can be written as

$$\Phi = p \nabla \cdot \vec{V} + \vec{\sigma} : \nabla \vec{V}. \quad (17)$$

Substituting Eqs. (4) and (5) in Eq. (17), we get

$$\Phi = \lambda(\nabla_i V_i)^2 + \frac{1}{2}\mu(\nabla_i V_j + \nabla_j V_i)^2 - \eta(\nabla_r \nabla_r e_{kmn} \nabla_m V_n)(e_{kij} \nabla_i V_j) \quad (18)$$

Considering an incompressible couple stress fluid and lubrication in steady state, Eqs. (16) and (18) can be written as

$$\rho c_f \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = K_f \frac{\partial^2 T}{\partial y^2} + \Phi \quad (19)$$

where

$$\Phi = \mu \left\{ \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] - l^2 \left(\frac{\partial u}{\partial y} \frac{\partial^3 u}{\partial y^3} + \frac{\partial w}{\partial y} \frac{\partial^3 w}{\partial y^3} \right) \right\} \quad (20)$$

when l tends to zero, Eq. (20) becomes

$$\Phi = \mu \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \quad (21)$$

The non dimensional modified energy equation can be obtained by introducing following dimensionless variables as

$$u^* = \frac{u}{U}, w^* = \frac{w}{U}, T^* = \frac{T}{T_0}, \rho^* = \frac{\rho}{\rho_0}.$$

and the derivatives of u^* , w^* are given in Appendix A.

The dimensionless form of energy equation becomes

$$P_e \left(\rho^* u^* \frac{\partial E(T^*)}{\partial \theta} + \Gamma^* \frac{\partial E(T^*)}{\partial y^*} + \rho^* w^* \frac{\partial E(T^*)}{\partial z^*} \right) = \frac{1}{E(H^{*2})} \frac{\partial^2 E(T^*)}{\partial y^{*2}} + \Phi^* \quad (22)$$

where $P_e = \frac{\rho_0 c_f U C^2}{K_f R}$

$E(T^*) =$

$$- \frac{1}{E(H^*)} \left[\frac{\partial}{\partial x^*} (E(H^*) \int_0^{y^*} \rho^* u^* dy^*) + \frac{\partial}{\partial z^*} (E(H^*) \int_0^{y^*} \rho^* w^* dy^*) \right]$$

$$\beta = \frac{\mu_0 U^2}{K_f T_0} \quad (23)$$

3 Discretization of modified Reynolds and Energy equations

The modified Reynolds equation and energy equation are solved numerically by using one of the most efficient finite difference method i.e. multigrid method. By using second order finite difference scheme, the derivative terms in

Eqs. (13) and (19) can be approximated as

$$P_{i,j} = A_1 P_{i+1,j} + A_2 P_{i-1,j} + A_3 P_{i,j+1} + A_4 P_{i,j-1} + A_5 \quad (24)$$

where

$$A_0 = r^2(f_{i+0.5,j} + f_{i-0.5,j} + f_{i,j+0.5} + f_{i,j-0.5})$$

$$A_1 = f_{i+0.5,j}/\mu_{i+0.5,j}A_0, A_2 = f_{i-0.5,j}/\mu_{i-0.5,j}A_0,$$

$$A_3 = f_{i,j+0.5}/\mu_{i,j+0.5}A_0, A_4 = f_{i,j-0.5}/\mu_{i,j-0.5}A_0$$

$$A_5 = -6 \Delta z^{*2} \varepsilon_i \cos \theta_i, r = \frac{\Delta z^*}{\Delta \theta}.$$

The discretized boundary conditions becomes

$$p_{i,j} - p_{i-1,j} = 0, p_{i,j} = 0, p_{i,j} = p_{0,i,j}$$

The energy equation can be written in discretized form as

$$P_e \left(\rho^* \mu^* \frac{T_{i,j}^* - T_{i-1,j}^*}{\theta_i - \theta_{i-1}} + \Gamma^* \frac{T_{i,j}^* - T_{i-1,j}^*}{y_i - y_{i-1}} + \rho^* w^* \frac{T_{i,j}^* - T_{i-1,j}^*}{z_i - z_{i-1}} \right) = \frac{1}{H_{i,j}^{*2} - H_{i-1,j}^{*2}} \frac{T_{i,j}^{*2} - T_{i-1,j}^{*2}}{y_i^2 - y_{i-1}^2} + \Phi_i^* \quad (25)$$

The discretized temperature boundary conditions are

$$T_{i,j} = T_{mix,i,j}, T_{i,j} - T_{i,j-1} = 0$$

3.1 Heat transfer equation

The heat transfer equation in bush for a steadily loaded journal bearing is defined as

$$\frac{\partial^2 E(T^*)}{\partial r^{*2}} + \frac{1}{r^*} \frac{\partial E(T^*)}{\partial r^*} + \frac{1}{r^{*2}} \frac{\partial E(T^*)}{\partial \theta^2} + \frac{\partial^2 E(T^*)}{\partial z^{*2}} = 0 \quad (26)$$

The temperature boundary conditions at inlet groove and central plane are respectively given as

$$E(T^*)_{\theta=-\delta} = E(T_{mix}^*), \frac{\partial E(T^*)}{\partial z^*} \Big|_{z^*=0} = 0 \quad (27)$$

where $E(T_{mix}^*)$ is defined as homogeneous mixing temperature of re-circulating oil [3].

At the oil-shaft interface and oil-bush interface, the boundary conditions for THD becomes

$$q^* = \int_0^{2\pi} \frac{1}{H^*} \frac{\partial T^*}{\partial y^*} \Big|_{y^*=y} d\theta = 0 \quad (28)$$

$$\frac{\partial T^*}{\partial r^*} \Big|_{r^*=1} = \frac{K(\theta)R}{K_b H^*} \frac{\partial T^*}{\partial y^*} \Big|_{y^*=1} \quad (29)$$

where $K(\theta)$ is the thermal conductivity of fluid which is equal to K_f in active zone and is variable in inactive zone of the film. In this area the thermal conductivity is given by

$$K(\theta) = K_a - \frac{B(\theta)}{B}(K_a - K_f) \quad (30)$$

In Eq. (30), represents the effective bearing length covered with lubricant. It can be obtained by using continuity equation.

$$B(\theta) = \frac{H^*(\theta_{cav})}{H^*(\theta)} B \quad (31)$$

For the outer surface and lateral surface of bush, the free convection and radiation hypothesis gives

$$\frac{\partial T^*}{\partial r^*} \Big|_{r^*=R_b^*} = -Nu(T^* \Big|_{r^*=R_b^*} - T_a^*) \quad (32)$$

$$\frac{\partial T^*}{\partial z^*} \Big|_{z^*=B/2R} = -Nu(T^* \Big|_{z^*=B/2R} - T_a^*) \quad (33)$$

where $Nu = h_b R / K_b$.

3.2 Viscosity- temperature relation

The variation of lubricant viscosity with temperature can be represented by Walther equation and written as

$$\log \left[\log \frac{\mu}{\rho} + 0.6 \right] = k_1 - k_2 \log(T) \quad (34)$$

where k_1 and k_2 are constants.

3.3 Load carrying capacity

The fluid film force and its attitude angle are given as:

$$W = \sqrt{W_1^2 + W_2^2}, \phi = \arcsin \left(\frac{W_2}{W} \right) \quad (35)$$

where $W_1 = -\frac{2R^4\mu_0\omega}{C^2} \int_0^{2R} \int_0^{B/2R} E(p) \cos \theta dz d\theta$, $W_2 = -\frac{2R^4\mu_0\omega}{C^2} \int_0^{2R} \int_0^{B/2R} E(p) \sin \theta dz d\theta$.

3.4 Friction force and friction coefficient

The shear stress along journal surface is expressed as [11]

$$\tau = \mu \frac{\partial u}{\partial y} \Big|_{y=0} - \eta \frac{\partial^3 u}{\partial y^3} \Big|_{y=0} \quad (36)$$

The friction force can be obtained by integrating shear stress around journal surface and it is written as

$$F_r = \frac{2\omega\mu_0R^3}{C} \int_0^{2\pi} \int_0^{B/2R} \left[\mu \left(\frac{1}{H} + \frac{h}{2R} \frac{\partial P}{\partial \theta} \right) \right] dz d\theta. \quad (37)$$

The coefficient of friction can be obtained by dividing friction force by film force

$$C_f = \frac{F_f R}{WC} \quad (38)$$

3.5 Side leakage flow

The side leakage flow for a journal bearing is given as

$$Q = \frac{-CR^2\omega}{6} \int_0^{2\pi} \frac{1}{\mu} \frac{\partial E(p)}{\partial z} \Big|_{z=B/2R} \left\{ H^{*3} - 12H^*l^2 + 24l^3 \tanh \left(\frac{H^*}{2l} \right) \right\} d\theta. \quad (39)$$

4 Multigrid method

The fluid velocity components and their derivatives are evaluated for the assumed initial pressure distribution. Also, energy equation and heat transfer equation are solved simultaneously, thus producing temperature field in oil and bushing. Further, the cross film average temperature can be obtained as

$$T_{av}^*(\theta, z^*) = \int_0^1 T^*(\theta, y^*, z^*) dy^* \quad (40)$$

The modified Reynolds equation is solved by Multigrid method then new pressure distribution is yielded. The discretized equation can be written in more general form as

$$A^h u^h = f^h$$

where A is a linear operator, u is the solution, f is the right hand side and h is the mesh size.

The algorithm for V cycle multigrid method as discussed in [25–29] and is given as

- Relaxing $A^h u^h = f^h$, 2 times with initial guess u_0^h
- Compute the residual $r^h = f^h - A^h u^h$
- Restrict $f^{2h} = I^{2h} r^h$, to zeros to get u^{2h}
- Relaxing $A^{2h} u^{2h} = f^{2h}$, 2 times with guess u^{2h}
- Compute the residual, $r^{2h} = f^{2h} - A^{2h} u^{2h}$
- Restrict, $f^{4h} = I^{4h} r^{2h}$, to zeros to get u^{4h}
- .
- .
- .
- Solve the error equation, $e^H = (A^H)^{-1} f^H$

- Interpolate $e^{2h} \leftarrow I_{4h}^{2h} u^{4h}$.
- Correcting the fine grid approximation $u^{2h} \leftarrow u^{2h} + e^{2h}$.
- Interpolate $e^h \leftarrow I_{2h}^h u^{2h}$.
- Correcting $u^h \leftarrow u^h + e^h$.

The initial approximations are taken on finest grid and two iterations of Gauss-Seidel method is applied on finest grid for smoothing errors. To transfer the calculated residual to next coarse grid level, the half weighting restriction is used. Repeating the procedure till coarsest grid level reaches to single point. The solution is obtained at coarsest level. The bilinear interpolation is applied to prolongation of solution obtained from coarsest level to the next fine grid level and then applying two time Gauss-Seidel iterations. Repeating the procedure until original finest grid level is reached. This is referred to as one V-cycle. The iterations are repeated until it reaches to convergence.

Flowchart

Table 1: Bearing variables used in the analysis

Bearing length, B (m)	0.028
Radial clearance, C (m)	5.2×10^{-5}
Bush specific heat, c_b (J kg ⁻¹ K ⁻¹)	465
Lubricant specific heat, c_f (J Kg ⁻¹ K ⁻¹)	2022
Shaft radius, R (m)	0.04
Bush outside diameter, R_b (m)	0.043
Heat transfer coefficient, h_b (W m ⁻² K ⁻¹)	230
Air thermal conductivity, K_a (W m ⁻¹ K ⁻¹)	0.025
Bush thermal conductivity, K_b (W m ⁻¹ K ⁻¹)	49
Lubricant thermal conductivity, K_f (W m ⁻¹ K ⁻¹)	0.141
Rotational speed, n (rpm)	3300
Lubricant inlet pressure, P_0 (Pa)	2×10^4
Lubricant inlet temperature, T_0 (K)	333
Bush density, ρ_b (kg m ⁻³)	7800
Inlet attitude angle, (°)	24
k_1	7.8212
k_2	3.0145

5 Results and discussion

The effect of surface roughness for thermo-hydrodynamic analysis of journal bearings extended to couple stress lubricants with high polymer additives is presented. The modified Reynolds and energy equations are solved by finite difference method i.e. multigrid method.

Couple stress mainly depends on the polar additives in the lubricant. The presence of micro structure in the fluid film influences the couple stress of the fluid which in turn increases the fluid pressure. The increase in film pressure results in an increase in the load carrying capacity. It is observed that film pressure is high for a low eccentricity ratio. This shows that load carrying capacity is high for a low eccentricity ratio. Minimum fluid film thickness increases for the bearing operate under couple stress lubricants. Stiffness and damping coefficients are larger than the couple stress fluids lubricated bearings and threshold speed improves when the bearings are lubricated by couple stress lubricants.

The temperature contour in the centre plane at the eccentricity ratio 0.3 and roughness parameter 0.1 is shown in Fig. 2. The journal bearing variables used in computer simulations to the flow with eccentricity ratio for different couple stress parameters are listed in Table 1. The shaft temperature and the maximum oil temperature are affected due to couple stress in fluids for different eccentricity ratios are shown in Figs. 3 and 4. It is observed that maximum oil temperature and shaft temperature decreases on increasing I^* , and it increases with the addition of roughness parameter 0.1, the influence of couple stress is more significant at a higher eccentricity ratio is

Fig. 2: Temperature contour in the centre plane at eccentricity ratio 0.3 and roughness parameter 0.1

Fig. 4: Maximum temperature versus eccentricity ratio for different values of the couple stress parameter with roughness parameter 0.1

Fig. 3: Shaft temperature versus eccentricity ratio for different values of the couple stress parameter with roughness parameter 0.1

Fig. 5: Load capacity versus eccentricity ratio for different values of the couple stress parameter with roughness parameter 0.1

predicted. Fig. 5 shows that for any eccentricity ratio, load capacity increases with increasing l^* and roughness parameter 0.1 load capacity increases for all values of l^* .

The friction force remains in increasing trend with increasing l^* and it increases with roughness parameter 0.1 is depicted in Fig. 6, while Fig. 7 shows that friction coefficient decreases with increase in l^* but it is seen that friction coefficient increases for the roughness parameter 0.1. The variation in the side leakage flow with eccentricity ratio for different couple is showed in Fig. 8. It shows that the side leakage flow of couple stress with roughness parameter 0.1 is almost same as that for different couple stress parameter l^* .

To verify the accuracy of the present model, the circumferential pressure distribution of the lubricant film with smooth case is compared with that of surface rough-

Fig. 6: Friction force versus eccentricity ratio for different values of the couple stress parameter with roughness parameter 0.1

Fig. 7: Friction coefficient versus eccentricity ratio for different values of the couple stress parameter with roughness parameter 0.1

Fig. 8: Side leakage versus eccentricity ratio for different values of the couple stress parameter with roughness parameter 0.1

ness. Multigrid method is used to minimize the errors and find the solution in fast and efficient way. It is observed that, the shape of temperature contour for is same as for $l^* = 0.3$ but in case of temperature field for $l^* = 0.3$ it is lower than that for $l^* = 0.0$. This shows that couple stress fluids yields less heat dissipation than that of Newtonian fluids.

6 Conclusion

The following conclusions are made from this study.

1. The bearing temperature field is affected due to the application of polymeric additives and the presence of surface roughness. The maximum oil temperature and the shaft temperature become lower at higher eccentricity ratios.

2. The couple stress effects produce a higher load capacity, higher friction force and a lower friction coefficient but the presence of surface roughness enhances the coefficient of friction. The side leakage flow almost remains constant for different values of couple stress parameter and surface roughness.
3. The rate of convergence of the method used is more compared to the other finite difference methods.
4. The surface roughness effect case is comparable with the earlier smooth case.

A Appendix

Velocity components and derivatives

Introducing the dimensionless variables $y^* = \frac{y}{H}$, $H^* = \frac{H}{C} = 1\epsilon \cos \theta$ and $l^* = \frac{l}{C}$ in Eqs. (8) and (9), it becomes

$$u^* = (1 - y^*) + \frac{1}{2\mu^*} \frac{\partial p^*}{\partial \theta} \left\{ H^{*2} y^* (y^* - 1) + 2l^* \left[1 - \left(\cosh \frac{2H^* y^* - H^*}{2l^*} \right) \left(\cosh \frac{H^*}{2l^*} \right)^{-1} \right] \right\} \quad (i)$$

$$\frac{\partial u^*}{\partial y^*} = -1 + \frac{1}{2\mu^*} \frac{\partial p^*}{\partial \theta} \left\{ H^{*2} (2y^* - 1) - 2l^* H^* \left(\sinh \frac{2H^* y^* - H^*}{2l^*} \right) \left(\cosh \frac{H^*}{2l^*} \right)^{-1} \right\}, \quad (ii)$$

$$\frac{\partial^3 u^*}{\partial y^{*3}} = -\frac{H^{*3}}{\mu^* l^*} \frac{\partial p^*}{\partial \theta} \left(\sinh \frac{2H^* y^* - H^*}{2l^*} \right) \left(\cosh \frac{H^*}{2l^*} \right)^{-1} \quad (iii)$$

$$w^* = \frac{1}{2\mu^*} \frac{\partial p^*}{\partial z^*} \left\{ H^{*2} y^* (y^* - 1) + 2l^* \left[1 - \left(\cosh \frac{2H^* y^* - H^*}{2l^*} \right) \left(\cosh \frac{H^*}{2l^*} \right) \right] \right\} \quad (iv)$$

$$\frac{\partial w^*}{\partial y^*} = \frac{1}{2\mu^*} \frac{\partial p^*}{\partial z^*} \left\{ H^{*2} (2y^* - 1) - 2l^* H^* \left(\sinh \frac{2H^* y^* - h^*}{2l^*} \right) \left(\cosh \frac{H^*}{2l^*} \right)^{-1} \right\} \quad (v)$$

$$\frac{\partial^3 w^*}{\partial y^{*3}} = -\frac{H^{*3}}{\mu^* l^*} \frac{\partial p^*}{\partial z^*} \left(\sinh \frac{2H^* y^* - H^*}{2l^*} \right) \left(\cosh \frac{H^*}{2l^*} \right)^{-1} \quad (vi)$$

Nomenclature

B Bearing width (m)
 c_b Bush specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
 c_f Lubricant specific heat ($\text{J kg}^{-1} \text{K}^{-1}$)
 C Radius clearance (m)
 e_{ijk} Permutation tensor
 F_f Friction force (N)
 h Dimensional film thickness (m)
 h^* Dimensionless film thickness, $h^*=1+\epsilon \cos\theta$
 h_b Heat transfer coefficient ($\text{Wm}^{-2} \text{K}^{-1}$)
 K_a Air thermal conductivity ($\text{Wm}^{-1} \text{K}^{-1}$)
 K_b Bush thermal conductivity ($\text{Wm}^{-1} \text{K}^{-1}$)
 K_f Lubricant thermal conductivity ($\text{Wm}^{-1} \text{K}^{-1}$)
 l Characteristics length of the additives, (m)
 l^* Couple stress parameter, $l^*=l/C$
 M_{ij} Couple stress tensor in fluid
 n Rotational speed (rpm)
 p Hydrodynamic pressure (Pa)
 p^* Dimensionless hydrodynamic pressure, $p^*=pC^2/R^2$
 p_0 Lubricant inlet pressure (Pa)
 Q Side leakage flow ($\text{m}^3 \text{s}^{-1}$)
 R Journal radius (m)
 R_b^* Dimensionless bush outside radius, $R_b^*=R_b/R$
 T Temperature (K)
 T^* Dimensionless temperature, $T^*=T/T_0$
 T_0 Lubricant inlet temperature (K)
 T_a^* Dimensionless ambient temperature, $T_a^*=T_a/T_0$
 $T_{\max}^*$ Dimensionless maximum bearing temperature, $T_{\max}^*=T_{\max}/T_0$
 T_{mix}^* Dimensionless mixing temperature of recirculating fluid and supply oil, $T_{\text{mix}}^*=T_{\text{mix}}/T_0$
 U Shaft linear speed (m s^{-1})
 u^*, v^*, w^* Dimensionless components of the lubricant velocity in the x, y and z directions, respectively,
 $u^*=u/U, v^*=v/U, w^*=w/U$
 $W1, W2$ Fluid film forces in the x and z directions (N)
 x, y, z Coordinates (m)
 ϵ Eccentricity ratio
 $\text{Inlet attitude angle}$
 θ, y^*, z^* Dimensionless coordinates, $q=x/R, y^*=y/h, z^*=z/R$
 θ, z^*, r Dimensionless coordinates, $q=x/R, z^*=z/R, r^*=r/R$
 θ_{cav} Location of the starting of cavitation zone
 η, η' Material constant responsible for the couple stress property (N s)
 μ Lubricant viscosity (Pa s)
 μ_0 Inlet lubricant viscosity (Pa s)
 μ^* Dimensionless lubricant viscosity ($\mu^*=\mu/\mu_0$)
 ρ Lubricant density (kg m^{-3})
 ρ_b Bush density (kg m^{-3})
 ρ_0 Inlet lubricant density (kg m^{-3})

ρ^* Dimensionless lubricant density, $\rho^*=\rho/\rho_0$
 τ Viscous shear stress (Pa)
 σ_{ij} Stress tensor in fluid
 ω Rotation vector (rad s^{-1})

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