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Gyrotactic microorganism and bio-convection during flow of Prandtl-Eyring nanomaterial

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Abstract: Our main intension behind this work is to investigate Prandtl-Eyring nanomaterial in presence of gyrotactic microorganisms. Flow is generated via stretching sheet and is subject to melting heat effect. Radiation and dissipation are addressed. Entropy rate is also reported. Nanofluid effects are explored through Buongiorno model for nanofluid by considering Brownian motion and thermophoresis impacts. Problem related modelling is done by obtaining PDEs and these PDEs are then transmitted into ODEs by using appropriate similarity variables. Homotopic technique has been employed to obtain a convergent series solution of the considered problem. Graphical results have been presented to investigate the impact of different prominent variables over fluid velocity, temperature distribution, nanofluid concentration and on microorganism concentration. Entropy analysis has been discussed and the physical quantities such as surface drag force, Nusselt number, local Sherwood number and microorganism density number for the current problem is obtained. Velocity boost against higher melting and fluid parameters. Temperature of the fluid reduces with an increment in melting and radiation parameters while it intensifies through Prandtl and Eckert number, Brownian motion and thermophoresis parameters. Decay in concentration is noticed against higher values of melting and thermophoresis parameters while it increases for higher Schmidt number and Brownian motion parameter. Microorganism field boosts with higher values of Peclet number and microorganism concentration difference parameter. Moreover entropy generation rate intensifies against higher radiation parameter and Brickman number.

Keywords: bio-convection, melting effect, nanofluid, viscous dissipation, thermal radiation, entropy generation, Brownian motion, thermophoresis diffusion

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1 Introduction

Recently advancements in nanotechnology got much attention from scientists and researchers due to its enormous applications in industrial and technological zones. The introduction of nanofluid brings a lot of its applications to the real world. In the beginning Choi [1] observed that the heat transportation capacity (thermal conductivity) of normal fluids can be improved by adding small nanosize particles with the characteristics of high thermal conductivity. He presented the idea of replacing normal/traditional fluids with these more advance fluids with high thermal efficiency. These advance fluids are known as nanofluids. Nanofluid plays a vital role in technological devices, heat-related equipment, nuclear reactor, space technology, processing of generic drugs, microelectronics, radiators, cooling system, bio-sensors and many more. Buongiorno [2] explained the transportation of mass and heat transfer in nanofluids. He presented the seven slip mechanism and concluded that out of all these thermophoresis and Brownian diffusion is of particular interest. Ahmad *et al.* [3] investigated the nanomaterial flow of magnetohydrodynamic non-Newtonian fluid. Turkyilmazoglu [4] studied the mass and heat transport of unsteady natural convective flow of nanoliquid with radiation past a vertical surface. MHD forced convection in nanofluid is reported to Sheikholeslami and bhatti [5]. Hayat *et al.* [6] considered melting effect in CNTs flow with stagnation point by variable thickness surface. Some recent investigations on nanofluid can be see through Refs. [7–17].

Bio-convection appears due to the up swimming of microorganisms in a particular direction since the density of microorganisms are higher than the base fluid. Therefore,

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the upper surface of the fluid becomes denser due to the gathering of microorganism. Thus upper surface of fluid is unstable and eventually the microorganism falls down causing bioconvection. The up swimming of microorganisms back to the top carry on bioconvection process. The utilization of gyrotactic microorganism in different nanoliquid enhances mass transfer, microscale mixing and also enhances the stability of nanofluids. Khan *et al.* [18] studied bioconvection nanofluid flow considering gyrotactic microorganism. Mutuku and Makinde [19] explored MHD bioconvection flow of nanoliquid past a porous vertical surface with gyrotactic microorganism. Hayat *et al.* [20] examined bioconvection in flow of Walter-B nanomaterials. Kuznetsov [21] examined the bioconvection nanofluid flow with the suspension of both gyrotactic microorganism and nanoparticles.

There is no doubt that the phenomenon of melting heat transfer has been studied extensively due to its ever rising usages in industrial and technological processes. Melting heat transfer plays an essential role in coil exchanger, magma solidification, semiconductor material preparation, welding process, crystal growth, permafrost melting, thermocouple and heat engines. Krishnamurthy *et al.* [22] considered chemical reactions and radiation during nanofluid flow with melting heat. Prasannakumara *et al.* [23] investigated melting heat, thermal radiation, MHD and heat source effects doing flow in region of stagnation point. Both melting heat transfer phenomenon and thermal radiation in magnetohydrodynamic flow past a moving surface is addressed by Das [24]. Further explorations on melting heat transfer phenomenon can be seen through Refs. [25–29].

Very little attention is paid by the researchers towards flow of non-Newtonian fluids in presence of microorganisms. This consideration narrow down when melting heat is considered. Here we have considered melting effect in Prandtl-Eyring nanofluid flow containing gyrotactic microorganism past a stretching sheet. Viscous dissipation and thermal radiation effects are also addressed. Entropy generation is accounted. Appropriate variables are used to transform governing expressions (PDEs) into ODEs. HAM technique is employed for solutions development. Entropy, velocity, concentration, microorganism and temperature under involved physical variables is evaluated graphically.

2 Mathematical formulation

Two dimensional Prandtl-Eyring nanofluid with motile gyrotactic microorganisms is considered by a stretched sheet. Sheet is subject to melting phenomena. Thermal radiation and viscous dissipation elaborates heat transport phenomena. Surface is stretched with the velocity ($u = ax$), with constant (a). Flow geometry is given in Figure 1.

Figure 1: Flow diagram.

After executing aforementioned assumptions and boundary layer approximations we obtain [30, 31]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{A}{\rho_f C^*} \frac{\partial^2 u}{\partial y^2} - \frac{A}{2\rho_f C^{*3}} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2}, \quad (2)$$

$$\left. \begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{k_f}{(\rho c_p)_f} \frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho c_p)_f} \left[\frac{A}{C^*} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{A}{6C^{*3}} \left(\frac{\partial u}{\partial y} \right)^4 \right] \\ &+ \frac{16\sigma^* T_\infty^3}{3k^*(\rho c_p)_f} \frac{\partial^2 T}{\partial y^2} + \tau \left(D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right), \end{aligned} \right\} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right), \quad (4)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + \frac{bW_c}{(C_\infty - C_m)} \left[\frac{\partial}{\partial y} \left(N \frac{\partial C}{\partial y} \right) \right] = D_m \frac{\partial^2 N}{\partial y^2}, \quad (5)$$

With boundary conditions

$$\left. \begin{aligned} u &= ax, \quad T = T_m, \quad C = C_m, \\ N &= N_m, \quad v = 0, \quad \text{at } y = 0, \\ u &\rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad N \rightarrow N_\infty \text{ as } y \rightarrow \infty, \\ k \left(\frac{\partial T}{\partial y} \right)_{y=0} &= \rho_{nf} [\lambda_1 + c_s(T_m - T_0)] v(x, 0). \end{aligned} \right\} \quad (6)$$

Consider the transformations [6, 12]:

$$\left. \begin{aligned} u &= \alpha x f'(\eta), \quad v = -\sqrt{\frac{\alpha}{\nu}} y f(\eta), \quad \eta = \sqrt{\frac{\alpha}{\nu}} y, \\ \theta(\eta) &= \frac{T-T_m}{T_\infty-T_m}, \quad \phi(\eta) = \frac{C-C_m}{C_\infty-C_m}, \quad \chi(\eta) = \frac{N-N_m}{N_\infty-N_m}. \end{aligned} \right\} \quad (7)$$

Making use of these transformations in Eqs. (1-6) we get

$$\alpha f'''(\eta) - \alpha \beta f''^2(\eta) f'''(\eta) - f'^2(\eta) + f(\eta) f''(\eta) = 0, \quad (8)$$

$$\theta''(\eta) + Rd \theta''(\eta) + Pr Ec \left(\alpha f''^2(\eta) - \alpha \beta f''^4(\eta) \right) + Pr f(\eta) \theta'(\eta) + Pr N_b \theta'(\eta) \phi'(\eta) + Pr N_t \theta'^2(\eta) = 0, \quad (9)$$

$$\phi''(\eta) + Sc f(\eta) \phi'(\eta) + \frac{N_t}{N_b} \theta''(\eta) = 0, \quad (10)$$

$$\chi''(\eta) + Lb f(\eta) \chi'(\eta) - Pe \left[\phi''(\eta) (\chi(\eta) + \Omega) + \phi'(\eta) \chi'(\eta) \right] = 0, \quad (11)$$

with transformed boundary conditions

$$\left. \begin{aligned} f'(0) &= 1, \quad \theta(0) = 0, \quad \chi(0) = 0, \\ \phi(0) &= 0, \quad f'(\infty) = 0, \quad \phi(\infty) = 1, \\ \theta(\infty) &= 1, \quad \chi(\infty) = 1, \\ Pr f(0) + Me \theta'(0) &= 0. \end{aligned} \right\} \quad (12)$$

Related parameters are

$$\left. \begin{aligned} Rd &= \frac{16\sigma^* T_\infty^3}{3kk^*}, \quad N_b = \frac{\tau D_B (C_\infty - C_m)}{\nu}, \quad \alpha = \frac{A}{\mu C^*}, \\ \beta &= \frac{a^3 x^2}{6C^{*2} \nu}, \quad N_t = \frac{\tau D_T (T_\infty - T_m)}{T_\infty \nu}, \quad Pe = \frac{b W_c}{D_m}, \\ Pr &= \frac{\nu}{\alpha}, \quad Me = \frac{c_p (T_\infty - T_m)}{\lambda_1 + c_s (T_m - T_0)}, \quad Sc = \frac{\nu}{D_B}, \\ Lb &= \frac{\nu}{D_m}, \quad Ec = \frac{(ax)^2}{c_p (T_\infty - T_m)}, \quad \Omega = \frac{N_m}{N_\infty - N_m}. \end{aligned} \right\} \quad (13)$$

3 Quantities of interest

Skin friction, Nusselt, Sherwood and Microorganism density numbers (C_{fx} , Nu_x , Sh_x , Nn_x) are expressed as

$$\left. \begin{aligned} C_{fx} &= \frac{\tau_w}{\frac{1}{2} \rho U_w^2}, \quad Nu_x = \frac{x q_w}{k(T_\infty - T_m)}, \\ Sh_x &= \frac{x J_w}{D_B (C_\infty - C_m)}, \quad Nn_x = \frac{x J_n}{D_m (N_\infty - N_m)}. \end{aligned} \right\} \quad (14)$$

Here (τ_w, q_w, J_w, J_n) are given by

$$\left. \begin{aligned} \tau_w &= \left(\frac{A}{C^*} \left(\frac{\partial u}{\partial y} \right) - \frac{A}{6C^{*3}} \left(\frac{\partial u}{\partial y} \right)^3 \right) \Big|_{y=0}, \\ q_w &= -k_f \left(1 + \frac{16\sigma^* T_\infty^3}{3kk^*} \right) \left(\frac{\partial T}{\partial y} \right) \Big|_{y=0}, \\ J_w &= -D_B \left(\frac{\partial C}{\partial y} \right) \Big|_{y=0}, \\ J_n &= -D_m \left(\frac{\partial N}{\partial y} \right) \Big|_{y=0}. \end{aligned} \right\} \quad (15)$$

Making use in Eq. (13) we get

$$\left. \begin{aligned} \frac{1}{2} Re_x^{1/2} C_{fx} &= \alpha f''(0) - \frac{1}{3} \alpha \beta [f''(0)]^3, \\ Re_x^{-1/2} Nu_x &= -(1 + Rd) \theta'(0), \\ Sh_x Re_x^{-1/2} &= -\phi'(0), \\ Nn_x Re_x^{-1/2} &= -\chi'(0). \end{aligned} \right\} \quad (16)$$

3.1 Entropy analysis

Entropy is given by [29]

$$E_G = \frac{k}{T_\infty^2} \left(1 + \frac{16\sigma^*}{3kk^*} T_\infty^3 \right) \left(\frac{\partial T}{\partial y} \right)^2 + \frac{RD}{C_\infty} \left(\frac{\partial C}{\partial y} \right)^2 + \frac{RD}{N_\infty} \left(\frac{\partial N}{\partial y} \right)^2 + \frac{RD}{T_\infty} \left(\frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right) + \frac{1}{T_\infty} \left(\frac{A}{C^*} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{A}{6C^*} \left(\frac{\partial u}{\partial y} \right)^4 \right) + \frac{RD}{T_\infty} \left(\frac{\partial N}{\partial y} \frac{\partial T}{\partial y} \right). \quad (17)$$

In non-dimensionalized form

$$S_G = \alpha_1(1 + Rd)\theta'^2(\eta) + L_1 \frac{\alpha_2}{\alpha_1} \phi'^2(\eta) + L_2 \frac{\alpha_3}{\alpha_1} \chi'^2(\eta) + L_1 \theta'(\eta) \phi'(\eta) + Br [\alpha f''^2(\eta) - \alpha \beta f''^4(\eta)] + L_2 \chi'(\eta) \theta'(\eta). \quad (18)$$

Bejan number is given as

$$\text{Bejannumber} = \frac{\text{Entropy due to mass and heat transfer}}{\text{Total entropy of the system}} \quad (19)$$

or

$$Be = \frac{\alpha_1(1 + Rd)\theta'^2(\eta) + L_1 \frac{\alpha_2}{\alpha_1} \phi'^2(\eta) + L_2 \frac{\alpha_3}{\alpha_1} \chi'^2(\eta) + L_1 \theta'(\eta) \phi'(\eta) + L_2 \chi'(\eta) \theta'(\eta)}{\alpha_1(1 + Rd)\theta'^2(\eta) + L_1 \frac{\alpha_2}{\alpha_1} \phi'^2(\eta) + L_2 \frac{\alpha_3}{\alpha_1} \chi'^2(\eta) + L_1 \theta'(\eta) \phi'(\eta) + Br [\alpha f''^2(\eta) - \alpha \beta f''^4(\eta)] + L_2 \chi'(\eta) \theta'(\eta)} \quad (20)$$

Where

$$L_1 = \frac{RD(C_\infty - C_m)}{k}, \quad Br = \frac{\mu(ax)^2}{k(T_\infty - T_m)}, \quad L_2 = \frac{RD(N_\infty - N_m)}{k}, \quad S_G = \frac{vE_G}{k\alpha_1 a}, \quad \alpha_1 = \frac{T_\infty - T_m}{T_\infty}, \quad \alpha_2 = \frac{C_\infty - C_m}{C_\infty}, \quad \alpha_3 = \frac{N_\infty - N_m}{N_\infty}. \quad (21)$$

4 Solution procedure

Obtained non-linear ODEs with BCs are tackled analytically through homotopy analysis method (HAM). This technique is very helpful for solving highly non-linear ODEs. This method works on basis of defining appropriate initial guesses and linear operators. Thus initial guesses and linear operators are [32–34]:

$$\left. \begin{aligned} f_0(\eta) &= (1 - e^{-\eta}) - \frac{Me}{Pr}, \\ \theta_0(\eta) &= 1 - e^{-\eta}, \\ \phi_0(\eta) &= 1 - e^{-\eta}, \\ \chi_0(\eta) &= 1 - e^{-\eta}, \end{aligned} \right\} \quad (22)$$

$$\left. \begin{aligned} L_f &= \frac{\partial^3}{\partial \eta^3} - \frac{\partial}{\partial \eta}, \\ L_\theta &= \frac{\partial^2}{\partial \eta^2} - 1, \\ L_\phi &= \frac{\partial^2}{\partial \eta^2} - 1, \\ L_\chi &= \frac{\partial^2}{\partial \eta^2} - 1, \end{aligned} \right\} \quad (23)$$

which has the property

$$\left. \begin{aligned} L_f &= [c_0 + c_1 e^\eta + c_2 e^{-\eta}], \quad L_\theta = [c_3 e^\eta + c_4 e^{-\eta}], \\ L_\phi &= [c_5 e^\eta + c_6 e^{-\eta}], \quad L_\chi = [c_7 e^\eta + c_8 e^{-\eta}]. \end{aligned} \right\} \quad (24)$$

Here c_i ($i = 1, 2, 3, \dots, 8$) are all positive constants. where ($c_1 = c_3 = c_5 = c_7 = 0$)

Zeroth order deformation equation is defined as

$$L_f [F(\eta; p) - f_0(\eta)] (1 - p) = p \hbar_f N_f [F(\eta; p)], \quad (25)$$

$$L_\theta [\theta(\eta; p) - \theta_0(\eta)] (1 - p) = p \hbar_\theta N_\theta [\theta(\eta; p)], \quad (26)$$

$$L_{\phi} [\phi(\eta; p) - \phi_0(\eta)] (1 - p) = p \bar{h}_{\phi} N_{\phi} [\phi(\eta; p)], \quad (27)$$

$$L_{\chi} [\chi(\eta; p) - \chi_0(\eta)] (1 - p) = p \bar{h}_{\chi} N_{\chi} [\chi(\eta; p)], \quad (28)$$

$$\left. \begin{aligned} \frac{\partial F(0; p)}{\partial \eta} &= 1, \quad \theta(0; p) = 0, \quad \chi(\infty; p) = 1, \quad \chi(0; p) = 0, \\ \theta(\infty; p) &= 1, \quad \phi(\infty; p) = 1, \quad \frac{\partial F(\infty; p)}{\partial \eta} = 0, \\ \phi(0; p) &= 0, \quad \text{Pr} F(0; p) + \text{Me} \frac{\partial \theta(0; p)}{\partial \eta} = 0. \end{aligned} \right\} \quad (29)$$

Where $p \in [0, 1]$ is embedding parameter while $(\bar{h}_f, \bar{h}_{\theta}, \bar{h}_{\phi}, \bar{h}_{\chi})$ are convergence control parameters.

The non-linear operators are written as

$$N_f = \alpha \frac{\partial^3 F(\eta; p)}{\partial \eta^3} - \left(\frac{\partial F(\eta; p)}{\partial \eta} \right)^2 + F(\eta; p) \frac{\partial^2 F(\eta; p)}{\partial \eta^2} - \alpha \beta \left(\frac{\partial^2 F(\eta; p)}{\partial \eta^2} \right)^2 \frac{\partial^3 F(\eta; p)}{\partial \eta^3}, \quad (30)$$

$$N_{\theta} = \text{Pr} N_b \left(\frac{\partial \theta(\eta; p)}{\partial \eta} \frac{\partial \phi(\eta; p)}{\partial \eta} \right) + \frac{\partial^2 \theta(\eta; p)}{\partial \eta^2} + \text{Pr} N_t \left(\frac{\partial \theta(\eta; p)}{\partial \eta} \right)^2 + \text{Pr} \left(F(\eta; p) \frac{\partial \theta(\eta; p)}{\partial \eta} \right) + R d \frac{\partial^2 \theta(\eta; p)}{\partial \eta^2} + \alpha \text{Pr} E c \left(\frac{\partial^2 F(\eta; p)}{\partial \eta^2} \right)^2 - \alpha \beta \text{Pr} E c \left(\frac{\partial^2 F(\eta; p)}{\partial \eta^2} \right)^4, \quad (31)$$

$$N_{\phi} = \frac{\partial^2 \phi(\eta; p)}{\partial \eta^2} + \frac{N_t}{N_b} \frac{\partial^2 \theta(\eta; p)}{\partial \eta^2} + S c F(\eta; p) \frac{\partial \phi(\eta; p)}{\partial \eta}, \quad (32)$$

$$N_{\chi} = \frac{\partial^2 \chi(\eta; p)}{\partial \eta^2} + L b F(\eta; p) \frac{\partial \chi(\eta; p)}{\partial \eta} - P e \left[\frac{\partial^2 \phi(\eta; p)}{\partial \eta^2} (\chi(\eta; p) + \Omega) + \frac{\partial \phi(\eta; p)}{\partial \eta} \frac{\partial \chi(\eta; p)}{\partial \eta} \right]. \quad (33)$$

The solution in series form is written as

$$\left. \begin{aligned} f(\eta) &= f_0(\eta) + \sum f_m(\eta), \\ \theta(\eta) &= \theta_0(\eta) + \sum \theta_m(\eta), \\ \phi(\eta) &= \phi_0(\eta) + \sum \phi_m(\eta), \\ \chi(\eta) &= \chi_0(\eta) + \sum \chi_m(\eta). \end{aligned} \right\} \quad (34)$$

5 Convergence analysis

Convergence of solution is highly dependent on auxiliary variables $(\bar{h}_f, \bar{h}_{\theta}, \bar{h}_{\phi}, \bar{h}_{\chi})$. These variables have great role in obtaining a convergent series solution. For this sketches of $\bar{h}$ -curves have been presented in Figure 2. Convergence regions are $-1.4 \leq \bar{h}_f \leq -0.2$, $-1.5 \leq \bar{h}_{\theta} \leq -0.5$, $-1.5 \leq \bar{h}_{\phi} \leq -0.1$, and $-1.5 \leq \bar{h}_{\chi} \leq -0.2$.

Figure 2: Combine $\bar{h}$ -curves.

6 Discussion

The investigations of different influential parameters over velocity field, temperature distribution, mass concentration, microorganism concentration and on entropy rate is the main aim behind this section

6.1 Velocity

Variations of melting and fluid parameters (Me, α) for velocity field ($f'(\eta)$) are depicted in Figures 3-4. Figure 3 displays outcomes of melting parameter for velocity. Velocity enhances against higher melting variable (Me). Physically with an increment in melting (Me) the heat transfer enhances due to high convection from hot fluid to cold melting surface and as a result velocity boosts. Figure 4 elucidate impact of material parameter (α) for velocity ($f'(\eta)$). Clearly an augmentation in velocity occurs against higher material parameter (α). It is quite clear that absolute viscosity and fluid parameter (α) are related inversely. Hence higher fluid parameter (α) leads to intensification in velocity of fluid ($f'(\eta)$).

Figure 3: $f'(\eta)$ versus Me .

6.2 Temperature field

Influences of different prominent variables on temperature are displayed in Figures 5-10). Figure 5 depicts melting parameter (Me) variations for temperature distribution. Higher melting parameter decays temperature distribution. Physically for higher melting (Me) convective flow enhances which cause more heat transfer from hot fluid towards melting surface and as a results the tempera-

Figure 4: $f'(\eta)$ versus α .

ture decays. Figure 6 illustrates variation of Brownian motion parameter against temperature. Temperature rises for higher (N_b). Outcomes of radiation (Rd) for temperature is displayed in Figure 7. A reduction in temperature occurs against larger radiation parameter. Figure 8 elucidate temperature variations for thermophoresis parameter (N_t). An augmentation in thermophoresis variable results in temperature enhancement. Figure 9 presents Prandtl number (Pr) outcomes for temperature. Temperature boosts against higher Prandtl number. Decline in thermal diffusivity occurs for higher Prandtl number and temperature enhances against higher (Pr). Figure 10 depict Schmidt number (Sc) variations for temperature. Clearly temperature augmentation occurs for higher Schmidt number.

Figure 5: $\theta(\eta)$ versus Me .

Table 1: Convergence of $f'''(0), \theta'(0), \phi'(0), \chi'(0)$ when $\alpha = \beta = Pr = Pe = 0.5, N_t = N_b = Rd = 0.01, Me = Sc = 0.1, Lb = 0.2$ at various approximations.

Order of approximation	$-f'''(0)$	$\theta'(0)$	$\phi'(0)$	$\chi'(0)$
1	0.8125	0.6664	0.4833	0.667
5	0.8284	0.2883	0.2039	0.3045
10	0.8329	0.2063	0.1691	0.2161
15	0.8304	0.1839	0.1504	0.1909
20	0.8304	0.1839	0.1504	0.1803
22	0.8304	0.1839	0.1504	0.1803

Figure 6: $\theta(\eta)$ versus N_b .

Figure 8: $\theta(\eta)$ versus N_t .

Figure 7: $\theta(\eta)$ versus Rd .

Figure 9: $\theta(\eta)$ versus Pr .

6.3 Concentration

Variations of concentration for different variables are presented in Figures 11-14. Figure 11 display outcome of melting parameter for concentration ($\phi(\eta)$). An augmentation in melting parameter result in concentration decay. Figure 12 depict variations of Brownian motion parameter on concentration. Concentration boosts against higher (N_b). Physically molecular diffusivity decreases with augmentation in Brownian motion parameter which results in concentration enhancement. Figure 13 display thermophore-

sis outcome on concentration ($\phi(\eta)$). A decline in concentration is observed for larger values of thermophoresis (N_t). Figure 14 elucidate outcome of Schmidt number for concentration ($\phi(\eta)$). With an increment in Schmidt number concentration boosts. Basically mass transfer decays from hot fluid to cold surface which is responsible for concentration enhancement.

Figure 10: $\theta(\eta)$ versus Ec . Figure 13: $\phi(\eta)$ versus N_t . Figure 11: $\phi(\eta)$ versus Me . Figure 14: $\phi(\eta)$ versus Sc . Figure 12: $\phi(\eta)$ versus N_b .

6.4 Microorganism

Influences of bio convection lewis number (Lb), microorganism concentration difference (Ω), Peclet number (Pe) and melting parameter (Me) on microorganism field are displayed in Figures 15-18. Figure 15 displays impact of bio convection Lewis number on microorganism field. A de-

cline in microorganism field occurs for higher values bio convection Lewis number. Influence of Peclet number on microorganism field is plotted in Figure 16. Higher Peclet number results in microorganism field enhancement. Figure 17 illustrates the impact of (Ω) on microorganism field. An augmentation in microorganism field occurs against higher (Ω). Figure 18 depict outcomes of melting parameter for microorganism field. Microorganism field reduces against higher melting variable (Me).

6.5 Entropy rate

Variations of radiation (Rd) and Brinkman number (Br) on entropy rate are presented in Figures 19-20. Figure 19 depicts outcomes of Brinkman number for entropy rate ($S_G(\eta)$). An augmentation in entropy rate occurs against higher Brinkman number. Since viscous forces increases for higher Brinkman number which causes disturbance in the system and consequently entropy rate boosts against Brinkman number. Figure 20 shows outcome of radiation

Figure 15: $\chi(\eta)$ versus Lb . Figure 18: $\chi(\eta)$ versus Me . Figure 16: $\chi(\eta)$ versus Pe . Figure 19: $S_G(\eta)$ versus Br . Figure 17: $\chi(\eta)$ versus Ω . Figure 20: $S_G(\eta)$ versus Rd .

for entropy rate ($S_G(\eta)$). Clearly entropy rate boosts against higher radiation variable (Rd).

7 Conclusion

Presented work deals with flow of Prandtl fluid in the presence of microorganisms. Surface is subjected to melting heat. Boungiorno model for nanofluid describes nanofluid characteristics while heat transfer character-

istics are explored via thermal radiation and viscous dissipation. It is concluded that velocity increases with an increase in fluid and melting parameters. Decline in temperature of the fluid is noticed for both radiation and melting parameters while it intensifies for larger Brownian motion and thermophoresis parameters. Higher melting parameter causes reduction in concentration while it increases with higher Brownian motion parameter and Schmidt number. Higher Peclet number leads to intensification in microorganism field while it decays against bio-concentration Lewis number. Entropy production rate is higher for Brinkman number and radiation parameter. In future this work can be extended for cavity problems and can be handled by finite element method.

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Table 2: Nomenclature for the considered problem.

Expression	Name	Unit
Pe	Peclet number	Dimensionless
Sc	Schmidt number	Dimensionless
N_t	Thermophoresis parameter	Dimensionless
W_c	Swimming cell speed	$m.s^{-1}$
T	Temperature	K
Br	Brinkman number	Dimensionless
U_w	Stretching velocity	$m.s^{-1}$
$\frac{A}{c^*}$	Fluid material coefficient	$kg.m^{-1}.s^{-1}$
k	Thermal conductivity	$s^{-3}.kg.m.K^{-1}$
α_1, α_2	Temperature and concentration difference parameters	Dimensionless
Me	Melting parameter	Dimensionless
Pr	Prandtl number	Dimensionless
C_{fx}	Skin friction coefficient	Dimensionless
c_s	Heat capacity	$m^2.s^{-1}.K^{-1}$
b	Chemotaxis constant	M
μ_f	Dynamic viscosity	$kg.m^{-1}.s^{-1}$
Re_x	Local Reynold number	Dimensionless
T_m	Reference temperature	K
c_p	Specific heat	$m^2.s^{-1}.K^{-1}$
Sh_x	Sherwood number	Dimensionless
Nu_x	Nusselt number	Dimensionless
$(\rho c_p)_f$	Fluid heat capacity	$kg.m^{-1}.s^{-2}.K^{-1}$
ρ_f	Density of fluid	$kg.m^{-3}$
Ec	Eckert number	Dimensionless
f	Dimensionless velocity profile	Dimensionless
D_T	Thermophoresis diffusion coefficient	$m^2.s^{-1}$
C, N	(Concentration, Microorganisms) fluid	$mol.m^{-3}$
ϕ	Dimensionless concentration profile	Dimensionless
$\phi'(0)$	(Concentration, Microorganisms) surface	$mol.m^{-3}$
a	Dimensional constant (positive)	s^{-1}
θ	Dimensionless temperature profile	Dimensionless
(ν_f)	Kinematic viscosity	$m^2.s^{-1}$
u, v	Components of velocity along x, y direction	$m.s^{-1}$
$(\rho c_p)_{nf}$	Nanofluid heat capacity	$kg.m^{-1}.s^{-2}.K^{-1}$
D_m	Microorganism diffusion coefficient	$m^2.s^{-1}$
N_b	Brownian motion parameter	Dimensionless
Nn_x	Microorganism density number	Dimensionless
k^*	Mean absorption coefficient	m^{-1}
h_f	Coefficient of heat transfer	$W.m^{-2}.K^{-1}$
S_G	Entropy rate	Dimensionless
Ω	Microorganism concentration difference parameter	Dimensionless
Rd	Radiation parameter	Dimensionless
L_1	Concentration diffusion parameter	Dimensionless
L_2	Microorganism diffusion parameter	Dimensionless
D_B	Brownian motion coefficient	$m^2.s^{-1}$
α, β	Material parameters	Dimensionless
C_∞, N_∞	(Concentration, Microorganisms) ambient	$mol.m^{-3}$
τ	Ratio of heat capacities	Dimensionless
T_w, T_∞	Surface and ambient temperature	K
χ	Dimensionless microorganism profile	Dimensionless
$\theta'(0)$	Stefan-Boltzmann constant	$kg.s^{-3}.K^{-4}$
λ_1	Latent heat	$kJ.kg^{-1}$
Lb	Bioconvection Lewis number	Dimensionless
α_3	Microorganism difference parameter	Dimensionless