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Dynamical behavior of fractionalized simply supported beam: An application of fractional operators to Bernoulli-Euler theory

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Abstract: The complex structures usually depend upon unconstrained and constrained simply supported beams because the passive damping is applied to control vibrations or dissipate acoustic energies involved in aerospace and automotive industries. This manuscript aims to present an analytic study of a simply supported beam based on the modern fractional approaches namely Caputo-Fabrizio and Atanagna-Baleanu fractional differential operators. The governing equation of motion is fractionalized for knowing the vivid effects of principal parametric resonances. The powerful techniques of Laplace and Fourier sine transforms are invoked for investigating the exact solutions with fractional and non-fractional approaches. The analytic solutions are presented in terms of elementary as well as special functions and depicted for graphical illustration based on embedded parameters. Finally, effects of the amplitude of vibrations and the natural frequency are discussed based on the sensitivities of dynamic characteristics of simply supported beam.

Keywords: simply supported beam, integral transform, analytic solutions, fractional approaches, rheological analysis

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1 Introduction

A beam is a load-bearing, standard and rigid element which is frequently utilized in structural engineering; this is because they have several applications, ranging from complex structure to simple structure for resisting vertical loads, bending moments and shear forces. Engineers use the different concepts of beams like simply supported beam, cantilever beam, uniformly distributed beam, continuously supported beam, fixed beam and many others in mechanical analyses for capturing the realistic simplicity of distribution of forces [1–7]. Zhu *et al.* [8] presented an analytical study of viscoelastic Timoshenko beam based on the fractional derivative in which three-dimensional fractional derivative constitutive relation has been traced on the basis of quasi-static behavior of the viscoelastic Timoshenko beam with step loading. Here they focused the dynamical response of deflection subjected to a periodic excitation. Hedrih [9] analyzed the vibrations of the beam based on nonhomogeneous continuously creeping material subject to the modulus of elasticity. Here the mathematical modeling has been performed on the basis of partial differential equation through constitutive relation of each layer. The fractional techniques were invoked for knowing the rotation of inertia for different boundary conditions. Freundlich [10] observed the simply supported beam for vibration analysis at steady-state position of beam. They modeled the fractional differential equations for knowing the amplitude-frequency characteristics based on the memory effects from Bernoulli-Euler beam model. They compared simply supported beam for vibration analysis for integer order derivative with fractional order derivative. Di-paola *et al.* [11] investigated the dynamic and quasi-static loads for the fractional viscoelastic Euler-Bernoulli beam in which axial strain and axial stress was focused. They traced out the fractional analytical solutions subject to mechanical boundary conditions for investigating gradient of curvature, shearing force, curvature and bending moment. Martin [12] observed an interested study on simply supported beam on the basis of uniformly distributed load with fractional and non-fractional approaches. The Laplace transform was invoked on classical and fractional Zener model for knowing the rheological influences and dynamic responses of the structure. Zhu and Chung [13] worked on simply supported beam with spinning and axially moving motion for checking the stability and vibration in which their main focus was to investigate the natural frequencies. They concluded on the basis of the rotary inertia that computed dynamic responses and natural frequencies are more reliable than the previous equations. Stepa *et al.* [14] examined the study of fractional viscoelastic beam by invoking the Galerkin method for finding approximate solutions of system of coupled fractional order differential equations. They emphasized the concentrated masses and base excitation by the comparison of classical and fractional order model. The numerical study of Euler-Bernoulli beams based on the fractional constitutive equations of viscoelastic beam has been analyzed by Yu *et al.* [15]. Here, Quasi-Legendre polynomial in the time domain has been invoked on the constitutive equation of the beam for transferring into matrix equation and then discretized and solved via numerical solutions for finding the displacements under different external loads. Qin *et al.* [16] examined the comparative analysis of numerical and analytical approaches for the simply supported horizontally composite curved I-beam. They found numerical solutions by employing finite element method and analytical solution by using trigonometric series for knowing the accuracy of deflection. In this continuity, debuting modern fractional differential operators have become a burning topic due the influence of memory effects during the deformation of certain types of beams. The modern fractional derivatives vary from singular to non-singular kernel and local to non-local kernel. The kernel depends upon the domain of definition involved in the modern fractional derivatives. The well-known modern fractional derivatives are (Atangana–Baleanu fractional derivative, Caputo–Fabrizio fractional derivative, Riemann–Liouville fractional derivative, Caputo fractional derivative, Hadamard fractional derivative, Riesz fractional derivative, Weyl fractional derivative, Hilfer fractional derivative and Erdérlyi–Kober fractional derivative) and few others [17–20, 36–43]. Additionally, the fractionalized mathematical models [44–58] with different mathematical techniques have been studied in different perspectives [59–68] and varying conditions. In brevity, our aim is to present an analytic study of a simply supported beam based on the modern fractional approaches namely Caputo-Fabrizio and Atanagna-Baleanu fractional differential operators. The governing equation of motion is fractionalized for knowing the vivid effects of principal parametric resonances. The powerful techniques of Laplace and Fourier sine transforms are invoked for investigating the exact solutions with fractional and non-fractional approaches. The analytic solutions are presented in terms of elementary as well as special functions and depicted for graphical illustration based on embedded parameters. Finally, effects of the amplitude of vibrations and the natural frequency are discussed based on the sensitivities of dynamic characteristics of simply supported beam.

2 Fractional modeling of simply supported beam with theoretical background

Different levels of accuracy have lead several beam theories due to various assumptions in which Euler-Bernoulli beam theory one of the most useful and simplest. This is because, Euler-Bernoulli beam theory arises from a combination of four distinct terminologies namely the force resultant, kinematic, equilibrium and constitutive, equations. An idealized problem of a long beam subjected to two bending moments which are usually constant properties in structures. The cross-section of the beam is assumed to be symmetric and bending takes place in that plane of symmetry. This type of loading is often referred to as pure bending as shown in Fig. 1 which reflects before and after deformation:

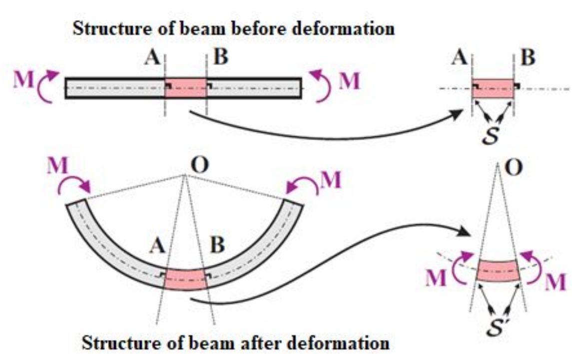

Figure 1: Structure of beam with before and after deformation

By neglecting shear deformation and rotary inertia, the equation of motion of the beam is derived under the assumptions of the Bernoulli-Euler theory, for which the governing equation of beam is described along with the initial and boundary conditions as [10]:

$$\frac{\partial^2 u(y,t)}{\partial y^2}\left(1+b\frac{d}{dt}\right)=-a^2\frac{\partial^2 u}{\partial t^2}(y,t) \tag{1}$$

$$u(y,0)=u'(y,0)=0,\ u(0,t)=t^p, \tag{2}$$

Here, the letting parameters for equation (1) are $b = \mu E^{-1}J^{-1}$ and $= A\rho E^{-1}J^{-1}$. The passive parameters for Eq. (1) are defined as μ is damping parameter, Young modulus of the beam material is represented by E, J refers moment of inertia of the beam cross-section, the cross-section area of the beam is measured by A, ρ denotes material mass density of the beam, t is a time, y presents longitudinal coordinate and $u(y,t)$ is transversal displacement of a beam. There is no denying fact that several deficiencies have been admitted by some fractional derivatives consisting kernels of singular nature. The main significance of kernels is to explain the entire memory effects. Different fractional derivatives have different limitations based on the nature of kernels. All of them, the fractional derivative of Caputo-Fabrizio is based on non-singular exponential kernel and Atangana-Baleanu fractional derivative is based on non-singular, non-local Mittag-Leffler kernel. In order to highlight the hidden phenomenon of both fractional operators, the governing equation of beam is developed in terms of fractional differentiations through which explanation of the full effect of the memory can be traced out. The fractionalized governing equation of beam is described as:

$$\frac{\partial^2 u(y,t)}{\partial y^2}\left(1+b\frac{d^\alpha}{dt^\alpha}\right)=-a^2\frac{\partial^{2\alpha} u(y,t)}{\partial t^{2\alpha}} \tag{3}$$

$$\frac{\partial^2 u(y,t)}{\partial y^2}\left(1+b\frac{d^\beta}{dt^\beta}\right)=-a^2\frac{\partial^{2\beta} u(y,t)}{\partial t^{2\beta}} \tag{4}$$

Where, $\frac{d^\alpha}{dt^\alpha}$ and $\frac{d^\beta}{dt^\beta}$ are the fractional operators namely Caputo-Fabrizio fractional operator [19, 21–25] and Atangana-Baleanu fractional operator [20, 26–30] respectively. The fractionalized governing equations of beam (3-4) are capable to model the intermediate material properties, for instance deflection and stress at any point, modulus of elasticity, moment of inertia of the cross-section, moment of resistance and few others. The modeling of fractionalized governing equations of beam (3-4) is based on Mittage-Leffler function as a kernel and exponential function as a kernel respectively and also proposed by [19, 20]:

$$D_t^\alpha u(y,t)=\int_0^\tau (1-\alpha)^{-1} exp\left(\frac{-\alpha(z-t)}{1-\alpha}\right)u'(y,t)dt.$$

$$0\le\alpha\le1, \tag{5}$$

$$D_t^\beta u(y,t)=\int_0^\tau (1-\beta)^{-1}E_\beta\left(\frac{-\beta(z-t)^\beta}{1-\beta}\right)u'(y,t)\,dt,$$

$$0\le\beta\le1. \tag{6}$$

While, the normalization functions for equations (5-6) are $M(\alpha)=M(\beta)=M(0)=M(1)=1$.

3 Methodology for analytic solutions

In this section, fractional order linear differential equations of beam (3-4) under imposed conditions (2) are investigated by employing Fourier sine and Laplace transforms. The basic principle of these methods is to analyze the problem for spatial and time domain variable. In brief, definitions of Laplace and Fourier sine transforms are as $F_s\{f(x)\}=\sqrt{\frac{2}{\pi}}\int_0^\infty sin(kx)\,f(x)\,dx=F_s(k)$, $L\{f(t)\}=\int_0^\infty e^{-qt}f(t)\,dt=F(q)$ respectively.

3.1 Caputo-Fabrizio fractional solution for simply supported beam

Applying the Fourier sine transform [31, 32] and appendix (A1-A2) on the governing fractional differential equation of simply supported beam (3), we obtain

$$-\xi^2 u_s(\xi,t)+\xi\sqrt{\frac{2}{\pi}}\,t^p-\xi^2\,b\frac{d^\alpha}{dt^\alpha}u_s(\xi,t)+\xi b\sqrt{\frac{2}{\pi}}t^p\ \frac{d^\alpha}{dt^\alpha}$$
$$=-a^2\frac{\partial^{2\alpha}u_s(\xi,t)}{\partial t^{2\alpha}}, \tag{7}$$

By imposing conditions ($2_{1,2,3}$) and Laplace transform [33, 34] on Eqs. (7). Here, $\overline{u}_s(\xi,\tau)$ is an image of Laplace transform of $u_s(\xi,t)$, we have,

$$\overline{u}_s(\xi,\tau)=\sqrt{\frac{2}{\pi}}\frac{\xi\,p!}{\tau^{p+1}}\left(\frac{M_1\tau^2+M_2\tau+M_3}{M_4\tau^2+M_5\tau+M_6}\right), \tag{8}$$

Where, the letting notations from (M_0-M_6) are defined in the equation (9) as

$$\begin{gathered}M_0=(1-\alpha)^{-1},\ M_1=1+bM_0,\ M_2=2\alpha M_0+b\alpha M_0^2,\\ M_3=M_0^2\alpha^2,\ M_4=\xi^2+M_0\xi^2 b-M_0^2a^2,\\ M_5=2M_0\xi^2\alpha+b\xi^2M_0^2\alpha,\ M_6=M_0^2\xi^2\alpha^2\end{gathered} \tag{9}$$

In order to justify the imposed conditions, we write Eq.(8) into suitable format, we arrive at

$$\bar{u}_s(\xi,\tau)=\sqrt{\frac{2}{\pi}}\frac{\xi\,p!}{\tau^{p+1}}+\frac{M_7}{M_4}\sqrt{\frac{2}{\pi}}\frac{\xi\,p!}{\tau^{p}}\left(\frac{\tau^2+M_{10}\tau+M_{11}}{\tau(\tau-M_{14})(\tau-M_{15})}\right),\tag{10}$$

The simplified form of Eq. (10) is obtained with following expressions defined in Eq. (11),

$$\begin{array}{c}M_7=M_1-M_4,\,M_8=M_2-M_5,\,M_9=M_3-M_6,\\ M_{10}=\frac{M_8}{M_7},\,M_{11}=\frac{M_9}{M_7},\,M_{12}=\frac{M_5}{M_4},\,M_{13}=\frac{M_6}{M_4},\\ \left(\tau^2+M_{12}\tau+M_{13}\right)=(\tau-M_{14})(\tau-M_{15})\end{array}.\tag{11}$$

Now, inverting Eq.(10) by means of Fourier sine transform and using appendix (A3), we get Eq.(10) in spatial variable format as

$$\bar{u}(y,\tau)=\frac{p!}{\tau^{p+1}}+\frac{2}{\pi}\frac{M_7}{M_4}p!\int_0^\infty \xi^2\sin(y\xi)\frac{1}{\tau^p}\left(\frac{\tau^2+M_{10}\tau+M_{11}}{\tau(\tau-M_{14})(\tau-M_{15})}\right)d\xi,\tag{12}$$

For converting Eq.(12) from frequency domain to time domain, we invert Eq. (12) via Laplace transform and invoking appendix (A4-A6), we have final expression in terms of product of convolution as

$$\begin{aligned}u(y,t)&=t^p+\frac{2}{\pi}\frac{M_7}{M_4}p!\int_0^\infty \xi^2\sin(y\xi)\int_0^t (t-z)^p\\&\times\left\{\frac{M_{11}}{M_{14}M_{15}}+\left(\frac{M_{14}^2+M_{10}M_{14}+M_{11}}{M_{14}(M_{14}+M_{15})}\right)\times\exp(M_{14}t)\right.\\&\left.+\left(\frac{M_{15}^2+M_{10}M_{15}+M_{11}}{M_{15}(M_{15}+M_{14})}\right)\exp(M_{15}t)\right\}d\xi\,dz\end{aligned}\tag{13}$$

3.2 Atangana-Baleanu fractional solution for simply supported beam

Applying the Fourier sine transform and appendix (A1-A2) on the governing fractional differential equation of simply supported beam (4), we obtain

$$-\xi^2u_s(\xi,t)+\xi\sqrt{\frac{2}{\pi}}t^p-\xi^2b\frac{d^\beta}{dt^\beta}u_s(\xi,t)+\xi b\sqrt{\frac{2}{\pi}}t^p\frac{d^\beta}{dt^\beta}=-a^2\frac{\partial^{2\beta}u_s(\xi,t)}{\partial t^{2\beta}},\tag{14}$$

By imposing conditions $(2_{1,2,3})$ and Laplace transform on Eqs. (14). Here, $\bar{u}_s(\xi,\tau)$ is an image of Laplace transform of $u_s(\xi,t)$, we have,

$$\bar{u}_s(\xi,\tau)=\sqrt{\frac{2}{\pi}}\frac{\xi\,p!}{\tau^{p+1}}\left(\frac{N_1\tau^{2\beta}+N_2\tau^\beta+N_3}{N_4\tau^{2\beta}+N_5\tau^\beta+N_6}\right),\tag{15}$$

Where, the letting notations from (N_0-N_6) are defined in the equation (9) as

$$\begin{array}{c}N_0=(1-\beta)^{-1},\,N_1=1+bN_0,\,N_2=2\beta N_0+b\beta N_0^2,\\ N_3=N_0^2\beta^2,\,N_4=\xi^2+N_0\xi^2b-N_0^2a^2,\\ N_5=2N_0\xi^2\beta+b\xi^2N_0^2\beta,\,N_6=N_0^2\xi^2\beta^2\end{array},\tag{16}$$

In order to justify the imposed conditions, we write Eq.(15) into suitable format, we arrive at

$$\bar{u}_s(\xi,\tau)=\sqrt{\frac{2}{\pi}}\frac{\xi\,p!}{\tau^{p+1}}+\frac{N_7}{N_4}\sqrt{\frac{2}{\pi}}\frac{\xi\,p!}{\tau^{p+1}}\left(\frac{\tau^{2\beta}+N_{10}\tau^\beta+N_{11}}{\tau^{2\beta}+N_{12}\tau^\beta+N_{13}}\right),\tag{17}$$

The simplified form of Eq. (10) is obtained with following expressions defined in Eq. (17),

$$\begin{array}{c}N_7=N_1-N_4,\,N_8=N_2-N_5,\,N_9=N_3-N_6,\,N_{10}=\frac{N_8}{N_7},\\ N_{11}=\frac{N_9}{N_7},\,N_{12}=\frac{N_5}{N_4},\,N_{13}=\frac{N_6}{N_4},\end{array}.\tag{18}$$

The concepts of infinite series [35] are utilized on Eq.(17) for separating the transformed variables as

$$\begin{aligned}\bar{u}_s(\xi,\tau)&=\sqrt{\frac{2}{\pi}}\frac{\xi\,p!}{\tau^{p+1}}+\sqrt{\frac{2}{\pi}}\frac{\xi\,p!}{\tau^{p+1}}\frac{N_7}{N_4}\sum_{q_0=0}^\infty(-N_{13})^{-q_0}\\&\sum_{q_1=0}^\infty\frac{(-N_{12})^{q_1}\Gamma(q_0+1)}{q_1!\Gamma(q_0-q_1+1)}\times\left(\tau^{2\beta}+N_{10}\tau^\beta+N_{11}\right)\tau^{2q_0\beta-\beta q_1},\end{aligned}\tag{19}$$

Inverting Eq.(17) by means of Fourier sine transform for converting the transformed variable into spatial variable, we have

$$\begin{aligned}\bar{u}(y,\tau)&=\frac{p!}{\tau^{p+1}}+\frac{2N_7}{\pi N_4}\int_0^\infty \xi\,sin(y\xi)\sum_{q_0=0}^\infty(-N_{13})^{-q_0}\\&\sum_{q_1=0}^\infty\frac{(-N_{12})^{q_1}\Gamma(q_0+1)}{q_1!\Gamma(q_0-q_1+1)}\tau^{2q_0\beta-\beta q_1-2\beta-p-1}d\xi\\&+\frac{N_7N_{10}}{2^{-1}\pi N_4}\int_0^\infty \xi\,sin(y\xi)\sum_{q_0=0}^\infty(-N_{13})^{-q_0}\\&\sum_{q_1=0}^\infty\frac{(-N_{12})^{q_1}\Gamma(q_0+1)}{q_1!\Gamma(q_0-q_1+1)}\tau^{2q_0\beta-\beta q_1-\beta-p-1}d\xi\\&+\frac{2}{\pi}\frac{N_7N_{11}}{N_4}\int_0^\infty \xi\,sin(y\xi)\sum_{q_0=0}^\infty(-N_{13})^{-q_0}\\&\sum_{q_1=0}^\infty\frac{(-N_{12})^{q_1}\Gamma(q_0+1)}{q_1!\Gamma(q_0-q_1+1)}\tau^{2q_0\beta-\beta q_1-p-1}d\xi,\end{aligned}\tag{20}$$

Applying inverse Laplace transform on Eq. (18), we find final solution as

$$u(y,t)=t^p+\frac{2N_7}{\pi N_4}\int_0^\infty \xi\,sin(y\xi)\sum_{q_0=0}^\infty(-N_{13})^{-q_0}$$

$$\sum_{q_1=0}^{\infty} \frac{(-N_{12})^{q_1} \Gamma(q_0+1)\, t^{2q_0\beta-\beta q_1-2\beta-p}}{q_1! \Gamma(q_0-q_1+1)\Gamma(2q_0\beta-\beta q_1-2\beta-p-1)} d\xi$$

$$+ \frac{N_7 N_{10}}{2^{-1}\pi N_4} \int_0^{\infty} \xi \, sin(y\xi) \sum_{q_0=0}^{\infty} (-N_{13})^{-q_0}$$

$$\sum_{q_1=0}^{\infty} \frac{(-N_{12})^{q_1} \Gamma(q_0+1)\, t^{2q_0\beta-\beta q_1-\beta-p}}{q_1! \Gamma(q_0-q_1+1)\Gamma(2q_0\beta-\beta q_1-\beta-p-1)} d\xi$$

$$+ \frac{2}{\pi}\frac{N_7 N_{11}}{N_4} \int_0^{\infty} \xi \, sin?y\xi) \sum_{q_0=0}^{\infty} (-N_{13})^{-q_0}$$

$$\sum_{q_1=0}^{\infty} \frac{(-N_{12})^{q_1} \Gamma(q_0+1)\, t^{2q_0\beta-\beta q_1-p}}{q_1! \Gamma(q_0-q_1+1)\Gamma(2q_0\beta-\beta q_1-p-1)} d\xi. \quad (21)$$

is the analytical solution of transversal displacement of a beam via Atangana-Baleanu fractional operator. Furthermore, the analytical solutions (13) and (18) obtained through both fractional operators can be retrieved for classical solution by letting $\alpha = \beta = 1$ in Eqs. (13) and (18).

4 Results with parametric conclusion

In this manuscript, an analytic study of a simply supported beam based on the modern fractional approaches namely Caputo-Fabrizio and Atanagna-Baleanu fractional differential operators is investigated for knowing the effective role of non-singular and non-local kernels involved in fractional differentiations. Due to this fact, governing equation of simply supported beam is fractionalized and then their solutions have been investigated by means of Laplace and Fourier sine transforms. Such solutions are presented in terms of elementary as well as special functions and depicted for graphical illustration based on embedded parameters. The main findings regarding amplitude of vibrations and the natural frequency are discussed as per following results:

4.1 Role of damping parameter in fractional solutions

In order to characterize the energy dissipation through damping, the dynamic response of simply supported beam is analyzed in Figure 2. It is also well known fact that the damping ratio of the beam changes with moving load location, hence we depicted Figure 2 for effective role of damping on displacement by choosing physical values as $\mu = 0.05, 0.07, 0.09$. It is clear from Figure 2 that displacement obtained through Caputo-Fabrizio fractional operator has rapid cycling in comparison with displacement obtained through Caputo-Fabrizio fractional operator. Physically, both fractional operators decay in amplitude of motion of simply supported beam. One can observe that energy dissipation through fractional mechanisms cannot be ceased for the vibratory analysis of simply supported beam. On a view of criticism, classical models of simply supported beam are not capable of critical damping which provides the quickest approach to zero amplitude.

4.2 Role of moment of inertia in fractional solutions

The ability to resist bending of simply supported beam is usually measured by the geometric property so called moment of inertia which depends up on a reference axis. We depicted the measurement of resistance of simply supported beam in Figure 3 by choosing larger specific values of moment of inertia because more is the moment of inertia then more is the resistance offered by the simply supported beam to rotation. Here, the displacement obtained through both CF and AB fractional differentiations have opposite trends when the specific values of moment of inertia increase. Physically, displacement obtained through both CF and AB fractional differentiations have measured the tendency to move back to a low energy state or rest reciprocally. Due to this obvious fact, CF and AB fractional differentiations give an idea of how much force is needed to maintain or alter the current state.

4.3 Role of young modulus of elasticity in fractional solutions

For measuring the fractional stiffness whether tensile or compressive deformation in simply supported beam, we elucidate the structural implant of simply supported beam for deformation by the fractionally obtained displacement. Figure 4 is prepared for measuring the resistance of a simply supported beam to its elasticity in which the specific values are taken from 0-40 as a concrete. It is noted that displacement obtained via CF fractional operator has larger number of oscillations in comparison AB fractional operator. As the temperature increases the elasticity of a simply supported beam decreases. This may be due to the fact memory effects of exponential and Mittage-Leffler kernels involved in both types of fractional operators.

4.4 Role of comparative analysis of fractional solutions at smaller and larger time

Figures 5-6 present the comparison of displacement investigated via CF fractional derivative and AB fractional derivative at two different times i-e (larger time) $t = 10\ s$ and (smaller time) $t = 0.4\ s$ with three variants of fractional parameters. Figure 5 is prepared for smaller time $t = 0.4\ s$ to compare the two different displacements in which solution obtained via CF fractional derivative could not collected full memory effects in comparison with AB fractional derivative. On the contrary, Figure 6 is depicted for larger time $t = 10\ s$ for the comparison of displacements. It is obvious from the oscillations depicted in Figure 6 that AB fractional derivative is more dominant in collections of entire memory effects. Physically, such results for displacements reflect the non-local spatial description and prevention of bending rigidities within the qualitative and quantitative behavior of simply supported beam for fractional calculus.

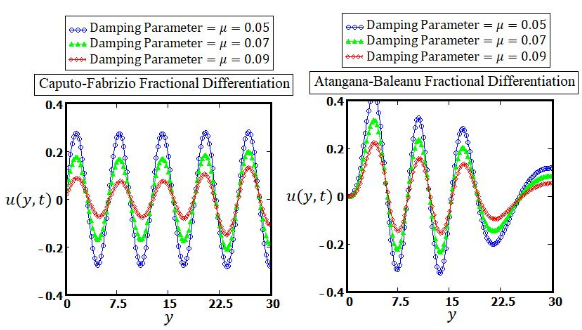

Figure 2: Graph of displacement via CF and AB fractional operator with three different variants of damping parameter μ

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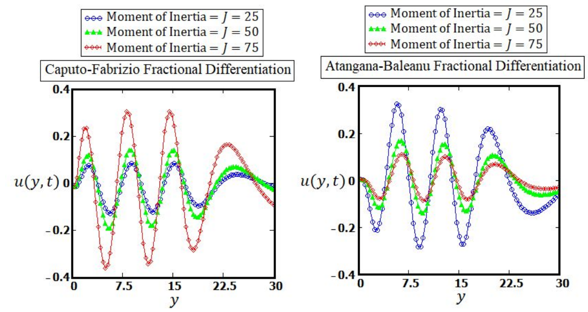

Figure 3: Graph of displacement via CF and AB fractional operator with three different variants of moment of inertia J.

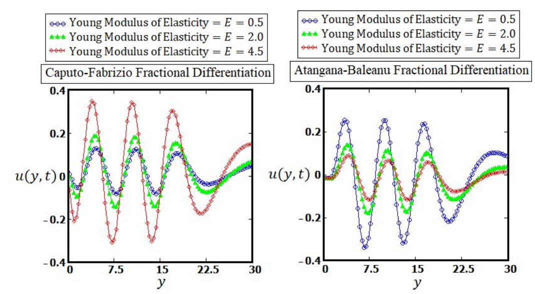

Figure 4: Graph of displacement via CF and AB fractional operator with three different variants of young modulus of elasticity E.

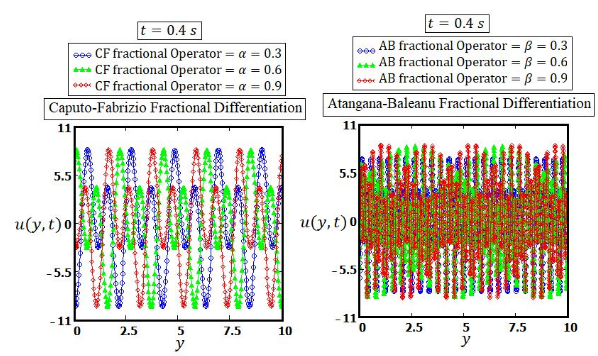

Figure 5: Graph of comparison of displacement via CF and AB fractional operator at smaller time $t = 0.04\ s$.

Conflict of interest: The authors state no conflict of interest.

Data Availability Statement: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

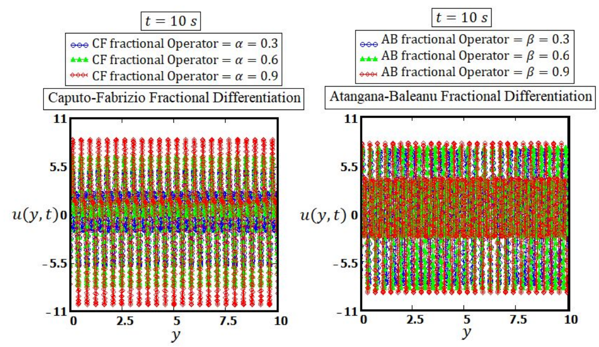

Figure 6: Graph of comparison of displacement via CF and AB fractional operator at larger time $t = 10\ s$.

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Appendix

$$F_s\left\{\frac{\partial^2 \bar{u}(y,q)}{\partial t^2}\right\} = -\xi^2 \bar{u}_s(\xi,q) + \xi\sqrt{\frac{2}{\pi}}\bar{u}_s(0,q), \tag{A1}$$

$$F_s\left\{\frac{\partial \bar{u}(y,q)}{\partial t}\right\} = \bar{u}_s(\xi,q), \tag{A2}$$

$$\int_0^\infty \frac{\sin(y\xi)}{\xi}d\xi = \frac{\pi}{2},\ y>0, \tag{A3}$$

$$L^{-1}\left\{\frac{\left(s^2+M_{12}s+M_{11}\right)}{s(s-M_{14})(s-M_{15})}\right\} = \frac{M_{11}}{M_{14}M_{15}} + \left(\frac{M_{14}^2+M_{12}M_{14}+M_{11}}{M_{14}(M_{14}+M_{15})}\right)\exp(M_{14}t) + \left(\frac{M_{15}^2+M_{12}M_{15}+M_{11}}{M_{15}(M_{15}+M_{14})}\right)\exp(M_{15}t), \tag{A4}$$

$$L^{-1}\left\{\frac{1}{s^p}\right\} = \frac{t^{p-1}}{\Gamma(p)}, \tag{A5}$$

$$(f*g)(t) = \int_0^t f(t)\,g(t-u)\,du, \tag{A6}$$