Hülya Durur, Asif Yokuş, and Kashif Ali Abro*

Computational and traveling wave analysis of Tzitzéica and Dodd-Bullough-Mikhailov equations: An exact and analytical study

<https://doi.org/10.1515/nleng-2021-0021>

Received Apr 11, 2021; accepted Jul 22, 2021.

Abstract: Computational and travelling wave solutions provide significant improvements in accuracy and characterize novelty of imposed techniques. In this context, computational and travelling wave solutions have been traced out for Tzitzéica and Dodd-Bullough-Mikhailov equations by means of $(1/G')$ -expansion method. The different types of solutions have constructed for Tzitzéica and Dodd-Bullough-Mikhailov equations in hyperbolic form. Moreover, solution function of Tzitzéica and Dodd-Bullough-Mikhailov equations has been derived in the format of logarithmic nature. Since both equations contain exponential terms so the solutions produced are expected to be in logarithmic form. Traveling wave solutions are presented in different formats from the solutions introduced in the literature. The reliability, effectiveness and applicability of the $(1/G')$ -expansion method produced hyperbolic type solutions. For the sake of physical significance, contour graphs, two dimensional and three dimensional graphs have been depicted for stationary wave. Such graphical illustration has been contrasted for stationary wave verses traveling wave solutions. Our graphical comparative analysis suggests that imposed method is reliable and powerful method for obtaining exact solutions of nonlinear evolution equations.

Keywords: Tzitzéica equation, $(1/G')$ -expansion method, Dodd-Bullough-Mikhailov equation, traveling wave solutions, exact solutions

1 Introduction

Nonlinear evolution equations are studied in many areas such as physics, engineering, social sciences and chemistry. Particularly in the last years, many studies have been conducted to attain either exact or analytical solutions of nonlinear evolution equations. Analytic solutions of nonlinear evolution equations have been investigated and traced out by means of several mathematical methods. For the sake of confirmation and evaluations, Baskonus and Cattani [1, 2] invoked extended sinh-gordon equation expansion method. Improved Bernoulli sub-equation function method has been employed by Bulut and Dusunceli [3, 4] via fractional and non-fractional approaches. Maih et al. [5] suggested $(G'/G - 1/G)$ -expansion method for closed form traveling wave solutions of integro-differential equations. Yokus and Durur presented (1+1)-dimensional resonant nonlinear Schrödinger's equation by means of (G'/G) -expansion method in [6–8]. Same author worked on sub equation method in [9] and $(1/G')$ -expansion method in [10–12]. The partial solutions via decomposition method and residual power series method have been found for solitary and periodic solutions for the coupled higher-dimensional Burgers equations in [13–17]. Additionally Sumudu transform method is employed in [18]. The numerical study and three-dimensional elliptic partial differential equations have been haar wavelet collocation method [19, 20] and first integral method [21]. The most important advantage of the expansion methods is the generation of traveling wave solutions of partial differential equations, such solutions are difficult for investigation. The most important common feature of the expansion methods are easy to apply and useful for traditional wave transform. With the help of this transformation, partial differential equation can be converted into an ordinary differential equation. In this continuity, we consider the Tzitzéica equation of the form [22]:

$$u_{tt} - u_{xx} - e^u + e^{-2u} = 0. \quad (1)$$

Hülya Durur, Department of Computer Engineering, Faculty of Engineering, Ardahan University, Ardahan, 75000, Turkey

Asif Yokuş, Department of Mathematics, Faculty of Science, Firat University, Elazığ, 23200, Turkey

***Corresponding Author: Kashif Ali Abro**, Department of Basic Sciences and Related Studies, Mehran University of Engineering and Technology, Jamshoro, Pakistan, E-mail: kashif.abro@faculty.muet.edu.pk

We consider Dodd-Bullough-Mikhailov equation [22]

$$u_{xt} + e^u + e^{-2u} = 0. \quad (2)$$

Equations (1-2) play an important role in the field of solid-state physics and nonlinear optics. Various studies on equations (1-2) have been carried out by many researchers; for instance, Kumar et al. [23] have explored traveling wave solutions of the Tzitzéica type equations by invoking sine-Gordon expansion method. The new types of exact solutions and class of solitary-like solutions of the Tzitzéica equation have been explored by means of similarity reduction and classical Lie symmetry analysis by Huber [24]. Solitons and periodic solutions have been obtained for equations (1-2) using tan h method in [25] in which analytic solutions for equations (1-2) are obtained using improved $\tan(\phi(\xi)/2)$. Moreover, recent attempts have been explored on the basis of variety of the solutions as expansion method [26], exponential rational function method [27], $\exp_a$ function method [28], dressing factor method [29]. In brevity, we derived and obtained analytic solutions of equations (1-2) using $(1/G')$ -expansion method. Additionally, the recent attempts of analytical techniques and exact solutions can be traced out [30–32, 34–43] and [44–47] therein.

2 $(1/G')$ -Expansion method

We get the general form of non-linear partial differential equation

$$\sigma\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \dots\right) = 0, \quad (3)$$

in the general form. Here, let $u = u(x, t) = U(\xi)$, $\xi = x + vt$, $v \neq 0$, where v is a constant and the velocity of the wave. One can convert into following nonlinear ordinary differential equation (ODE) for $U(\xi)$:

$$\tau(U, U', U'', U''', \dots) = 0. \quad (4)$$

The solution of Eq. (4) is assumed to have the form

$$U(\xi) = a_0 + \sum_{i=1}^N a_i \left(\frac{1}{G'}\right)^i, \quad (5)$$

where a_i , $(i = 0, 1, \dots, N)$ are constants and $G = G(\xi)$ provides the following second-order integer order differential equation as

$$G'' + \lambda G' + \mu = 0, \quad (6)$$

where λ and μ are constants to be determined after,

$$\frac{1}{G'(\xi)} = \frac{1}{-\frac{\mu}{\lambda} + A \cosh[\xi\lambda] - A \sinh[\xi\lambda]}, \quad (7)$$

where, A is integral constant. Some constraints are needed for the solution of Eq. (7). These restrictions are the ignoring of values that makes the solution undefined. The asymptotic behavior of the solution to include a singular point is attractive for employees. In addition, the existence of a single point in the solution produced by the method offers a different perspective to the shock wave phenomenon. If the desired derivatives of the Eq. (5) are calculated and substituting in the Eq. (4), a polynomial with the argument $(1/G')$ is attained. An algebraic equation is created by equalizing the coefficients of this polynomial to zero. These equations are solved with the help of a package program and put into place in the default Eq. (4) solution function. Finally, the solutions of Eq. (1) and Eq. (2) are obtained.

3 Application of $(1/G')$ -expansion method

3.1 The solution of Tzitzéica equation

We consider Eq. (1) by employing transformation $u = \ln v$, we get

$$vv_{tt} - v_t^2 - vv_{xx} + v_x^2 - v^3 + 1 = 0. \quad (8)$$

We want to get the different type of exact solutions for the Eq. (8) by using $(1/G')$ -expansion method, with the help of transmutation $u = \ln v$, the general solution of Eq. (8) can be found. Suppose that the solution Eq. (8) is in the following

$$v = V(\xi), \quad \xi = kx + wt, \quad (9)$$

where k, w are real numbers. Replacing Eq. (9) into Eq. (8) for obtaining nonlinear ODE:

$$(w^2 - k^2)VV'' + (k^2 - w^2)(V')^2 - V^3 + 1 = 0. \quad (10)$$

In Eq. (10), we find balancing term say $N = 2$ and in Eq. (5), solving both the following situation is obtained

$$V = a_0 + a_1 \left(\frac{1}{G'}\right) + a_2 \left(\frac{1}{G'}\right)^2, \quad a_1 \neq 0 \quad \text{or} \quad a_2 \neq 0. \quad (11)$$

Replacing Eq. (11) into Eq. (10) and the coefficients of the algebraic Eq. (1) are equal to zero, we can establish the following algebraic equation systems:

$$\left(\frac{1}{G'[\xi]}\right)^0 : 1 - a_0^3 = 0,$$

$$\left(\frac{1}{G'[\xi]}\right)^1 : -k^2\lambda^2 a_0 a_1 + w^2\lambda^2 a_0 a_1 - 3a_0^2 a_1 = 0,$$

$$\begin{aligned}
\left(\frac{1}{G'[\xi]}\right)^2 &: -3k^2\lambda\mu a_0 a_1 + 3w^2\lambda\mu a_0 a_1 - 3a_0 a_1^2 - 4k^2\lambda^2 a_0 a_2 + 4w^2\lambda^2 a_0 a_2 - 3a_0^2 a_2 = 0, \\
\left(\frac{1}{G'[\xi]}\right)^3 &: -2k^2\mu^2 a_0 a_1 + 2w^2\mu^2 a_0 a_1 - k^2\lambda\mu a_1^2 + w^2\lambda\mu a_1^2 - a_1^3 - 10k^2\lambda\mu a_0 a_2 \\
&\quad + 10w^2\lambda\mu a_0 a_2 - k^2\lambda^2 a_1 a_2 + w^2\lambda^2 a_1 a_2 - 6a_0 a_1 a_2 = 0, \\
\left(\frac{1}{G'[\xi]}\right)^4 &: -k^2\mu^2 a_1^2 + w^2\mu^2 a_1^2 - 6k^2\mu^2 a_0 a_2 + 6w^2\mu^2 a_0 a_2 - 5k^2\lambda\mu a_1 a_2 + 5w^2\lambda\mu a_1 a_2 - 3a_1^2 a_2 - 3a_0 a_2^2 = 0, \\
\left(\frac{1}{G'[\xi]}\right)^5 &: -4k^2\mu^2 a_1 a_2 + 4w^2\mu^2 a_1 a_2 - 2k^2\lambda\mu a_2^2 + 2w^2\lambda\mu a_2^2 - 3a_1 a_2^2 = 0, \\
\left(\frac{1}{G'[\xi]}\right)^6 &: -2k^2\mu^2 a_2^2 + 2w^2\mu^2 a_2^2 - a_2^3 = 0.
\end{aligned} \tag{12}$$

Case1.

$$a_0 = 1, \quad a_1 = \frac{6\mu}{\lambda}, \quad a_2 = \frac{6\mu^2}{\lambda^2}, \quad k = -\frac{\sqrt{-3+w^2\lambda^2}}{\lambda}, \quad \xi = kx + wt. \tag{13}$$

Considering Eq. (7) and substituting in Eq. (13) and Eq. (11), the following solution is obtained.

$$\begin{aligned}
V = 1 + & \frac{6\mu^2}{\lambda^2 \left(-\frac{\mu}{\lambda} + A \cosh \left[\lambda \left(tw - \frac{x\sqrt{-3+w^2\lambda^2}}{\lambda} \right) \right] - A \sinh \left[\lambda \left(tw - \frac{x\sqrt{-3+w^2\lambda^2}}{\lambda} \right) \right] \right)^2} \\
& + \frac{6\mu}{\lambda \left(-\frac{\mu}{\lambda} + A \cosh \left[\lambda \left(tw - \frac{x\sqrt{-3+w^2\lambda^2}}{\lambda} \right) \right] - A \sinh \left[\lambda \left(tw - \frac{x\sqrt{-3+w^2\lambda^2}}{\lambda} \right) \right] \right)},
\end{aligned} \tag{14}$$

In addition, if Eq. (14) is written instead of $u = \ln v$ transformation, the analytical solution of Eq. (1) is as follows.

$$\begin{aligned}
u_1(x, t) = \ln & \left[1 + \frac{6\mu^2}{\lambda^2 \left(-\frac{\mu}{\lambda} + A \cosh \left[\lambda \left(tw - \frac{x\sqrt{-3+w^2\lambda^2}}{\lambda} \right) \right] - A \sinh \left[\lambda \left(tw - \frac{x\sqrt{-3+w^2\lambda^2}}{\lambda} \right) \right] \right)^2} \right. \\
& \left. + \frac{6\mu}{\lambda \left(-\frac{\mu}{\lambda} + A \cosh \left[\lambda \left(tw - \frac{x\sqrt{-3+w^2\lambda^2}}{\lambda} \right) \right] - A \sinh \left[\lambda \left(tw - \frac{x\sqrt{-3+w^2\lambda^2}}{\lambda} \right) \right] \right)} \right].
\end{aligned} \tag{15}$$

Case 2.

$$a_0 = \frac{1}{2}(-1 + i\sqrt{3}), \quad a_1 = \frac{3i(i\mu + \sqrt{3}\mu)}{\lambda}, \quad a_2 = \frac{3i(i\mu^2 + \sqrt{3}\mu^2)}{\lambda^2}, \quad k = -\sqrt{w^2 + \frac{3}{2\lambda^2} - \frac{3i\sqrt{3}}{2\lambda^2}}, \quad \xi = kx + wt, \tag{16}$$

where $i = \sqrt{-1}$. Considering Eq. (7), substituting Eq. (16) into Eq. (11), the following solution is obtained.

$$\begin{aligned}
V = \frac{1}{2}(-1 + i\sqrt{3}) + & \frac{3i(i\mu^2 + \sqrt{3}\mu^2)}{\lambda^2 \left(-\frac{\mu}{\lambda} + A \cosh \left[\left(tw + x\sqrt{w^2 + \frac{3}{2\lambda^2} - \frac{3i\sqrt{3}}{2\lambda^2}} \right) \lambda \right] - A \sinh \left[\left(tw + x\sqrt{w^2 + \frac{3}{2\lambda^2} - \frac{3i\sqrt{3}}{2\lambda^2}} \right) \lambda \right] \right)^2} \\
& + \frac{3i(i\mu + \sqrt{3}\mu)}{\lambda \left(-\frac{\mu}{\lambda} + A \cosh \left[\left(tw + x\sqrt{w^2 + \frac{3}{2\lambda^2} - \frac{3i\sqrt{3}}{2\lambda^2}} \right) \lambda \right] - A \sinh \left[\left(tw + x\sqrt{w^2 + \frac{3}{2\lambda^2} - \frac{3i\sqrt{3}}{2\lambda^2}} \right) \lambda \right] \right)}.
\end{aligned} \tag{17}$$

In addition, if Eq. (17) is written instead of $u = \ln v$ transformation, the analytical solution of Eq. (1) is as follows.

$$\begin{aligned}
u_2(x, t) = \ln & \left[\frac{1}{2}(-1 + i\sqrt{3}) + \frac{3i(i\mu^2 + \sqrt{3}\mu^2)}{\lambda^2 \left(-\frac{\mu}{\lambda} + A \cosh \left[\left(tw + x\sqrt{w^2 + \frac{3}{2\lambda^2} - \frac{3i\sqrt{3}}{2\lambda^2}} \right) \lambda \right] - A \sinh \left[\left(tw + x\sqrt{w^2 + \frac{3}{2\lambda^2} - \frac{3i\sqrt{3}}{2\lambda^2}} \right) \lambda \right] \right)^2} \right. \\
& \left. + \frac{3i(i\mu + \sqrt{3}\mu)}{\lambda \left(-\frac{\mu}{\lambda} + A \cosh \left[\left(tw + x\sqrt{w^2 + \frac{3}{2\lambda^2} - \frac{3i\sqrt{3}}{2\lambda^2}} \right) \lambda \right] - A \sinh \left[\left(tw + x\sqrt{w^2 + \frac{3}{2\lambda^2} - \frac{3i\sqrt{3}}{2\lambda^2}} \right) \lambda \right] \right)} \right].
\end{aligned} \tag{18}$$

Case 3.

$$\begin{aligned} a_0 &= 1, \quad a_1 = 2\sqrt{3}\sqrt{-k^2\mu^2 + w^2\mu^2}, \quad a_2 = 2(-k^2\mu^2 + w^2\mu^2), \\ \lambda &= \left(\frac{\sqrt{3}\sqrt{(-k^2 + w^2)\mu^2}}{(-k^2 + w^2)\mu} \right), \quad \xi = kx + wt. \end{aligned} \quad (19)$$

Considering Eq. (7), substituting Eq. (19) into Eq. (11), the following solution is obtained.

$$V = \frac{9A^2 + 3e^{\frac{2\sqrt{3}(tw+kx)\mu}{\sqrt{(-k^2+w^2)\mu^2}}}(-k+w)(k+w)\mu^2 + 12\sqrt{3}Ae^{\frac{\sqrt{3}(tw+kx)\mu}{\sqrt{(-k^2+w^2)\mu^2}}}\sqrt{(-k^2+w^2)\mu^2}}{\left(-3A + \sqrt{3}e^{\frac{\sqrt{3}(tw+kx)\mu}{\sqrt{(-k^2+w^2)\mu^2}}}\sqrt{(-k^2+w^2)\mu^2}\right)^2}. \quad (20)$$

In addition, if Eq. (20) is written instead of $u = \ln v$ transformation, the analytical solution of Eq. (1) is as follows.

$$u_3(x, t) = \ln \left[\frac{9A^2 + 3e^{\frac{2\sqrt{3}(tw+kx)\mu}{\sqrt{(-k^2+w^2)\mu^2}}}(-k+w)(k+w)\mu^2 + 12\sqrt{3}Ae^{\frac{\sqrt{3}(tw+kx)\mu}{\sqrt{(-k^2+w^2)\mu^2}}}\sqrt{(-k^2+w^2)\mu^2}}{\left(-3A + \sqrt{3}e^{\frac{\sqrt{3}(tw+kx)\mu}{\sqrt{(-k^2+w^2)\mu^2}}}\sqrt{(-k^2+w^2)\mu^2}\right)^2} \right]. \quad (21)$$

3.2 The solution of the Dodd-Bullough-Mikhailov equation

We consider Eq. (2). Using transmutation $u = \ln v$, we get

$$v v_{xt} - v_x v_t + v^3 + 1 = 0. \quad (22)$$

Suppose that solution of Eq. (22) may be write as follows

$$v = V(\xi), \quad \xi = kx + wt, \quad (23)$$

where k, w are real numbers. Replacing Eq. (23) into Eq. (22) to obtain nonlinear ODE:

$$kwVV'' - kw(V')^2 + V^3 + 1 = 0. \quad (24)$$

In Eq. (24), we obtain balancing term $N = 2$, then the following relationship is achieved:

$$V = a_0 + a_1 \left(\frac{1}{G'} \right) + a_2 \left(\frac{1}{G'} \right)^2, \quad a_1 \neq 0 \quad \text{or} \quad a_2 \neq 0. \quad (25)$$

Replacing Eq. (25) into Eq. (24), and the coefficients of the algebraic of Eq. (2) are equal to zero, we may find the following algebraic equation systems,

$$\begin{aligned} \left(\frac{1}{G'[\xi]} \right)^0 : 1 + a_0^3 &= 0, \\ \left(\frac{1}{G'[\xi]} \right)^1 : kw\lambda^2 a_0 a_1 + 3a_0^2 a_1 &= 0, \\ \left(\frac{1}{G'[\xi]} \right)^2 : 3kw\lambda\mu a_0 a_1 + 3a_0 a_1^2 + 4kw\lambda^2 a_0 a_2 + 3a_0^2 a_2 &= 0, \\ \left(\frac{1}{G'[\xi]} \right)^3 : 2kw\mu^2 a_0 a_1 + kw\lambda\mu a_1^2 + a_1^3 + 10kw\lambda\mu a_0 a_2 + kw\lambda^2 a_1 a_2 + 6a_0 a_1 a_2 &= 0, \\ \left(\frac{1}{G'[\xi]} \right)^4 : kw\mu^2 a_1^2 + 6kw\mu^2 a_0 a_2 + 5kw\lambda\mu a_1 a_2 + 3a_1^2 a_2 + 3a_0 a_2^2 &= 0, \\ \left(\frac{1}{G'[\xi]} \right)^5 : 4kw\mu^2 a_1 a_2 + 2kw\lambda\mu a_2^2 + 3a_1 a_2^2 &= 0, \end{aligned} \quad (26)$$

$$\left(\frac{1}{G'[\xi]}\right)^6 : 2kw\mu^2 a_2^2 + a_2^3 = 0.$$

Case 1.

$$a_0 = -1, \quad a_1 = 2\sqrt{3}\sqrt{k}\sqrt{w}\mu, \quad a_2 = -2kw\mu^2, \quad \lambda = -\frac{\sqrt{3}}{\sqrt{k}\sqrt{w}}, \quad \xi = kx + wt. \quad (27)$$

Considering Eq. (7), substituting Eq. (27) into Eq. (25), the following solution is obtained.

$$V = -1 - \frac{2kw\mu^2}{\left(\frac{\sqrt{k}\sqrt{w}\mu}{\sqrt{3}} + A \cosh\left[\frac{\sqrt{3}(tw+kx)}{\sqrt{k}\sqrt{w}}\right] + A \sinh\left[\frac{\sqrt{3}(tw+kx)}{\sqrt{k}\sqrt{w}}\right]\right)^2} + \frac{2\sqrt{3}\sqrt{k}\sqrt{w}\mu}{\frac{\sqrt{k}\sqrt{w}\mu}{\sqrt{3}} + A \cosh\left[\frac{\sqrt{3}(tw+kx)}{\sqrt{k}\sqrt{w}}\right] + A \sinh\left[\frac{\sqrt{3}(tw+kx)}{\sqrt{k}\sqrt{w}}\right]}. \quad (28)$$

In addition, if Eq. (28) is written instead of $u = \ln v$ transformation, the analytical solution of Eq. (2) is as follows.

$$u_1(x, t) = \ln \left[-1 - \frac{2kw\mu^2}{\left(\frac{\sqrt{k}\sqrt{w}\mu}{\sqrt{3}} + A \cosh\left[\frac{\sqrt{3}(tw+kx)}{\sqrt{k}\sqrt{w}}\right] + A \sinh\left[\frac{\sqrt{3}(tw+kx)}{\sqrt{k}\sqrt{w}}\right]\right)^2} + \frac{2\sqrt{3}\sqrt{k}\sqrt{w}\mu}{\frac{\sqrt{k}\sqrt{w}\mu}{\sqrt{3}} + A \cosh\left[\frac{\sqrt{3}(tw+kx)}{\sqrt{k}\sqrt{w}}\right] + A \sinh\left[\frac{\sqrt{3}(tw+kx)}{\sqrt{k}\sqrt{w}}\right]} \right]. \quad (29)$$

Case 2.

$$a_0 = \frac{1}{2}(1 + i\sqrt{3}), \quad a_1 = -\sqrt{6}\sqrt{-kw\mu^2 - i\sqrt{3}kw\mu^2}, \quad a_2 = -2kw\mu^2, \quad \lambda = \frac{\sqrt{\frac{3}{2}}\sqrt{-i(-i + \sqrt{3})}kw\mu^2}{kw\mu}, \quad \xi = kx + wt. \quad (30)$$

Considering Eq. (7), substituting Eq. (30) into Eq. (25), the following solution is obtained

$$V = \frac{1}{2}(1 + i\sqrt{3}) - \frac{2kw\mu^2}{\left(-\frac{\sqrt{\frac{3}{2}}kw\mu^2}{\sqrt{-i(-i + \sqrt{3})}kw\mu^2} + A \cosh\left[\frac{\sqrt{\frac{3}{2}}(tw+kx)\sqrt{-i(-i + \sqrt{3})}kw\mu^2}{kw\mu}\right] - A \sinh\left[\frac{\sqrt{\frac{3}{2}}(tw+kx)\sqrt{-i(-i + \sqrt{3})}kw\mu^2}{kw\mu}\right]\right)^2} - \frac{\sqrt{6}\sqrt{-kw\mu^2 - i\sqrt{3}kw\mu^2}}{-\frac{\sqrt{\frac{3}{2}}kw\mu^2}{\sqrt{-i(-i + \sqrt{3})}kw\mu^2} + A \cosh\left[\frac{\sqrt{\frac{3}{2}}(tw+kx)\sqrt{-i(-i + \sqrt{3})}kw\mu^2}{kw\mu}\right] - A \sinh\left[\frac{\sqrt{\frac{3}{2}}(tw+kx)\sqrt{-i(-i + \sqrt{3})}kw\mu^2}{kw\mu}\right]}. \quad (31)$$

In addition, if Eq. (31) is written instead of $u = \ln v$ transformation, the analytical solution of Eq. (2) is as follows

$$u_2(x, t) = \ln \left[\frac{1}{2}(1 + i\sqrt{3}) - \frac{2kw\mu^2}{\left(-\frac{\sqrt{\frac{3}{2}}kw\mu^2}{\sqrt{-i(-i + \sqrt{3})}kw\mu^2} + A \cosh\left[\frac{\sqrt{\frac{3}{2}}(tw+kx)\sqrt{-i(-i + \sqrt{3})}kw\mu^2}{kw\mu}\right] - A \sinh\left[\frac{\sqrt{\frac{3}{2}}(tw+kx)\sqrt{-i(-i + \sqrt{3})}kw\mu^2}{kw\mu}\right]\right)^2} - \frac{\sqrt{6}\sqrt{-kw\mu^2 - i\sqrt{3}kw\mu^2}}{-\frac{\sqrt{\frac{3}{2}}kw\mu^2}{\sqrt{-i(-i + \sqrt{3})}kw\mu^2} + A \cosh\left[\frac{\sqrt{\frac{3}{2}}(tw+kx)\sqrt{-i(-i + \sqrt{3})}kw\mu^2}{kw\mu}\right] - A \sinh\left[\frac{\sqrt{\frac{3}{2}}(tw+kx)\sqrt{-i(-i + \sqrt{3})}kw\mu^2}{kw\mu}\right]} \right]. \quad (32)$$

Figure 1: 3D, 2D and contour graphics for $A = 1$, $\mu = -4$, $\lambda = 1$, $w = 1.8$ values of Eq. (1).

4 Results and discussion

One of the important instruments used in obtaining the exact solution of nonlinear evolution equations is the expansion method. In the literature, traveling wave solutions have been obtained by using many expansion methods. The different types of traveling wave solutions were constructed from (G'/G) -expansion method, novel exponential rational function method, sub equation method, extended auxiliary equation method, the tanh method, Kudryashov method, Sine-Gordon expansion method, the extended tanh method, the improved $\tan(\phi(\xi)/2)$ expansion method, auxiliary equation method and the extended trial equation method and several others. The analytic solution function is obtained by employed method for Eq. (7). In addition, the solution presented with the Eq. (21) contains the exponential solution for the basic equation. Such solution is similar to the format of Eq. (7) but the coefficients are different when the solution in Eq. (21) is written in trigonometric form. The Tzitzéica and the Dodd-Bullough-Mikhailov equations are same family as presented by using two transformations with $(1/G')$ -expansion method. Different arguments can be created by giving special values to constants in the solutions. The Tzitzéica and the Dodd-Bullough-Mikhailov equations contain an exponential function that indicates the solution function in logarithmic form. On the basis of $u = \ln v$ transformation, it verifies that the solutions are logarithmic in which v is the hyperbolic traveling wave solution. In addition, mobility of the wave can be achieved by giving appropriate values to the constants among the solutions. Let us specifically analyze the traveling wave solution presented by Eq. (15).

$$\lim_{x \rightarrow \pm\infty} u_1 = 0 \quad (33)$$

Considering the Eq. (33), it can be said that Eq. (15) exhibits a damped wave behavior independent of the variable t . Because for $x \rightarrow \pm\infty$ is $u_1 \rightarrow 0$. In addition, we can observe the progress of the following solitary wave for different values of t time parameter, $x \in [0, 25]$ of the traveling wave solution presented by Eq. (15). Figure 6 presents that a wave moves to the right. One of the most important reasons for this is that $w > 0$ represents the velocity of the wave. $w = 1.8$ that is chosen for Figure 6.

5 Conclusions

In this article, we have obtained traveling wave solutions for the Tzitzéica and the Dodd-Bullough-Mikhailov

equations by invoking of $(1/G')$ -expansion method. For both equations, the solutions are obtained in hyperbolic form. The validity of the $(1/G')$ -expansion method has been tested by checking the hyperbolic type solutions. Consequently, nonlinear part of the method can be recommended for generating differential equations in traveling wave solutions. The 3D, 2D and contour graphics representing the constant waves are presented by evaluating the constants in the traveling wave solutions. This method is an important instrument in obtaining the traveling wave solutions of nonlinear evolution equations. The reliability of this method can be used to obtain the analytical solution of differential equations in different classes. Due to this fact, this method increases the importance of its methodology. To conclude that complex processes and difficulties have been overcome with the help of computer programs.

Acknowledgements: The author Dr. Hülya Durur is highly thankful and grateful to Ardahan University, Ardahan, Turkey for generous support and facilities of this research work. The author Dr. Asif Yokuş is highly thankful and grateful to Firat University, Elazığ, Turkey for generous support and facilities of this research work. The author Dr. Kashif Ali Abro is highly thankful and grateful to Mehran university of Engineering and Technology, Jamshoro, Pakistan for generous support and facilities of this research work.

Funding information: The authors state no funding involved.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Data Availability Statement: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Figure 2: 3D, 2D and contour graphics for $A = -2$, $\mu = -2$, $\lambda = 0.5$, $w = 1.8$ values of Eq. (1).

Figure 3: 3D, 2D and contour graphics for $A = -2$, $\mu = -4$, $\lambda = 2$, $w = 2$, $k = 1$ values of Eq. (1).

Figure 4: 3D, 2D and contour graphics for $A = 1$, $\mu = -1$, $\lambda = -1$, $w = 1$, $k = 2$ values of Eq. (2).

Figure 5: 3D,2D and contour graphics for $A = 1, \mu = 2, \lambda = 1, w = 1, k = 1$ values of Eq. (2).

Figure 6: Simulation graphics for $A = 1, \mu = -4, \lambda = 1, w = 1.8$ values of Eq. (15).

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