Dinkar Sharma*, Ramandeep Kaur, and Honey Sharma

Investigation of thermo-elastic characteristics in functionally graded rotating disk using finite element method

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Abstract: In this research paper, displacement, stresses and strains are presented for rotating FGM disk with variable thickness by using finite element method (FEM). Thermo-elastic material properties and thickness of FGM disk continuously vary as exponential and power law function in radial direction along radius of disk. The value of Poisson's ratio is taken as constant. The problem of thermo-elasticity is converted into second order governing differential equation in terms of radial coordinate. This conversion is based upon equilibrium equation for disk and stress-strain relationship. The influence of variable thickness, angular velocity and functionally graded materials is discussed on thermo-elastic characteristics of rotating disk for exponential variation of material properties. Further, these thermo-elastic characteristics of disk are plotted for various values of non-homogeneity parameter under power law distribution of material properties. Thus, the investigations done in this research paper may be useful for industrial area in construction an appropriate FGM disk by controlling above mentioned parameters.

Keywords: FGM, FEM, thickness, stress, strain and material properties

1 Introduction

Functionally graded materials are a new class of composites that are also known as multifunctional materials due to their spatial variation composition. The positive impact of compressive residual surface stresses on wear

resistance as well as on strength may be counted as a potential advantage of these multifunctional materials. So, it become a matter of concern to design an appropriate blueprint of the progressive compositional gradient for an optimal distribution of the residual stresses to correlate with mechanical properties [1]. Composite materials have found vast applications in distinguished fields of structural engineering such as aerospace, automobile, bio-medical, civil structures, etc. [2]. In the past few decades, many researchers have investigated the behavior of these multifunctional materials in the presence of different environmental conditions. Wang and Zu [3, 4] published and concluded that the damping, translational speed, and the excitation amplitude significantly affect the nonlinear dynamical responses of the translational FG plate using an analytical and numerical method approach. The nonlinear vibration analysis of rotating functionally graded cylindrical shell has been reported by Sheng and Wang [5]. Liew *et al.* [6] have studied the nonlinear vibration behaviour of a laminated FG cylindrical panel. Talha and Singh [7] have presented the nonlinear free flexural vibration analysis of an FG plate using an improved higher-order theory. Akbari and Ghanbari [8] studied influence of internal pressure, thermal load and rotation on functionally graded hollow discs from exact analytical solution under radially varying material properties. Kalali *et al.* [9] derived elasto-plastic stress solution numerically in axisymmetric functionally graded rotating disk, cylindrical and spherical vessel with help of Hencky's stress-strain relation. Salehi *et al.* [10] used linear plane elasticity theory to find analytical solution for axisymmetric thick-walled FGM cylinders. Habib *et al.* [11] made stress and strain analysis in functionally graded cylinder under exponentially varying material properties by finite element method and ANSYS software. Yadav and Jiwari [12, 13] used finite element to solve differential equations in various mathematical models. Afsar and Go [14] shown effect of radial thickness, angular speed and temperature profile on thermoelastic field in a thin circular FGM disk for exponentially varying material properties. Khorsand and Tang [15] used differential quadrature method and co-evolutionary particle

***Corresponding Author: Dinkar Sharma**, Department of Mathematics, Lyallpur Khalsa College, Jalandhar, India, E-mail: dinkar.nitj@gmail.com

Ramandeep Kaur, Department of Mathematics, I.K.G. Punjab Technical University, Kapurthala, India

Honey Sharma, Department of Mathematics, Gulzar group of Institute, Khanna, India

swarm optimization approach to minimize stress and displacement in functionally graded hollow circular disk under variable thickness. Go [16] derived second order differential equation based upon two-dimensional thermal elastic theory to study stress, strain and temperature distribution in rotating circular disk. Thawati *et al.* [17] represented stress and deformation in functionally graded disk for three types of material properties named as: mori-tanaka scheme, power law distribution and exponential distribution under variable thickness. Zheng *et al.* [18] developed stress field numerically by using finite difference method in functionally graded rotating disk. To carried out stress field material properties are assumed to followed power law distribution. Nejad *et al.* [19] obtained elasto-plastic deformation and stress from exact form of analytical solution in FGM rotating disk for constant Poisson's ration. Calliogue *et al.* [20] made elastic-plastic stress analysis in functionally graded rotating disk from analytical and numerical solution for different values of angular velocities under radially varying material properties. Kordkheili and Livani [21] used material properties as function of temperature to study thermoelastic creep behaviour in functionally graded rotating disk with variable thickness. Arnab *et al.* [22] employed power and exponential function variation in radial direction to study thermoelastic field in thin circular FGM disk. Allam *et al.* [23] used infinitesimal theory to drive accurate and effective solutions for displacement and stress in rotating annular disk with variable thickness. Jalali and Shahriari [24] obtained stresses and deformation in rotating variable disk for three types of boundary conditions by finite difference method. Torabnia *et al.* [25] shown effect of young's modulus, density and yield stress on stress, strain and radial displacement in FGM hollow rotors from analytical solution of equilibrium equation. Garg [26] carried out elastic stress and strain in rotating FGM disk for different values of thickness gradation index under variable material properties in radial direction. Abdalla *et al.* [27] studied thermomechanical stress in functionally graded rotating disk with finite element method in two-dimensional model. Kursun *et al.* [28] made stress analysis in FGM discs under condition of uniform pressure on inner surface and linearly decreasing temperature distribution for radially varying material properties. Nkene *et al.* [29] considered second law of Newton, Hooke's law and stress-strain relationship to drive analytical and numerical solution in FGM rotating hollow cylinder for different values of inhomogeneity parameter. Saadt-far [30] analysed effect of angular velocity, hygrothermal loading and moisture concentration on piezomagnetic rotating thick walled cylinder for hygrothermal boundary

conditions. Abrinia *et al.* [31] assumed material properties varying along thickness as power law function to obtained radial and circumferential stress in FGM cylinder under internal pressure and temperature. Rahimi and Nejad [32] investigated thermal stresses from an exact solution in FGM rotating hollow thick-walled cylinder under internal and external pressure. Evci and Gulgec [33] derived analytical solution to represent stresses and displacement in FGM hollow cylinder by using airy stress function. Further, failure analysis done with coulomb-mohr theory and tresca yield criterion. Mantena *et al.* [34] obtained solution in form of bessels and trigonometric functions to study heat conduction and thermal stresses in hollow cylinder under non-homogeneous material properties. Yildirim [35] presented thermomechanical analysis analytically with Navier equations under assumption of spherically-symmetric plain strain for non-homogeneous isotropic spheres. Farhan *et al.* [36] proposed finite difference method to study thermoelasticity in isotropic and infinitely long circular cylinder under temperature dependent material properties. Hu *et al.* [37] investigated influence of various parameters on natural vibration in rotating functionally graded ring plate. Go [38] made thermoelastic characteristics analysis numerically under variable contact force and homogeneous thickness. Thakare and Warbhe [39] studied temperature distribution and stress distribution in thick hollow cylinder for homogeneous and inhomogeneous material properties. Saeedi *et al.* [40] used successive approximation method to study stresses and strains in thick walled FGM cylindrical shells under internal pressure and temperature gradient. In the present work, rotating disks of functionally graded materials are constructed due to its utilization in different fields of Engineering and Science. A stress field in FGM isotropic and anisotropic cylinder under internal pressure and uniform heat generation for different values of gradient index, is represented. These disks are modelled under variety of thermo-mechanical properties to extend our published work [41–45].

Basic Equations of the problem: For rotating disk, the strain-displacement relationship irrespective of thickness can be written as:

$$\varepsilon_r = \frac{du}{dr}, \varepsilon_\theta = \frac{u}{r} \quad (1)$$

Where u is displacement, r is radial coordinate and $(\varepsilon_r, \varepsilon_\theta)$ are strains in radial and circumferential direction.

For elastic deformation, by using Hooke's law relations between strains and stresses can be written as:

$$\sigma_r = \frac{E(r)}{1 - \nu^2} [\varepsilon_r + \nu \varepsilon_\theta] \quad (2)$$

$$\sigma_\theta = \frac{E(r)}{1-\nu^2} [\varepsilon_\theta + \nu \varepsilon_r] \quad (3)$$

where $(\sigma_r, \sigma_\theta)$ represents stresses in radial and circumferential direction, $E(r)$ is young's modulus and ν is Poisson's ratio of disk.

Equilibrium equation for disk of variable thickness is given as:

$$\frac{d}{dr} [h(r)r\sigma_r] - h(r)\sigma_\theta + h(r)\rho(r)\omega^2 r^2 = 0 \quad (4)$$

where ω is angular velocity, $\rho(r)$ is density of rotating disk and $h(r)$ is thickness of disk.

Case I: Exponential variation of material properties

In this section, we assumed that material properties of rotating disk are varying exponentially along radial direction. Therefore, $E(r)$, $\rho(r)$ and $h(r)$ vary as:

$$E(r) = E_0 e^{-m(r-a)/b} \quad (5)$$

$$\rho(r) = \rho_0 e^{-n(r-a)/b} \quad (6)$$

$$h(r) = h_0 e^{-k(r-a)/b} \quad (7)$$

Where r is radial coordinate, E_0 , ρ_0 and h_0 are constants of young's modulus, mass density and thickness respectively at inner surface of FGM disk. The index m and n are material parameters, the k is the geometric parameter.

After substituting values from (5-7) into equation (4), governing differential equation of problem in terms of radial displacement become:

$$r^2 \frac{d^2 u}{dr^2} + r(1-rP) \frac{du}{dr} - (1+r\nu P)u = Qr^3 e^{-R(r-a)} \quad (8)$$

Where $P = \frac{k+m}{b}$, $Q = \frac{\rho_0}{E_0} (1-\nu^2)\omega^2$ and $R = \frac{-(n-m)}{b}$

Case II: Power law variation of material properties

In this case, to study thermoelastic characteristics in FGM disk material properties of disk are consider to follow power law variation along radius as follows:

$$E(r) = E_0 r^{n_1} \quad (9)$$

$$\rho(r) = \rho_0 r^{n_2} \quad (10)$$

$$h(r) = h_0 r^{n_3} \quad (11)$$

By substituting values of various material parameters from Eqs. (9-11) into Eq. (4), the differential equation which represents problem of thermoelasticity is obtained, as:

$$\frac{d^2 u}{dr^2} + \frac{n_1 + n_3 + 1}{r} \frac{du}{dr} + \frac{1}{r^2} (\nu(n_1 + n_3) - 1)u + \frac{\rho_0}{E_0} (1-\nu^2)\omega^2 r^{n_2-n+1} = 0 \quad (12)$$

After simplification equation (15), we get:

$$\frac{d^2 u}{dr^2} + \frac{P}{r} \frac{du}{dr} + \frac{Q}{r^2} u + Rr^{n_2-n+1} = 0 \quad (13)$$

Where $P = n_1 + n_3 + 1$, $Q = \nu(n_1 + n_3) - 1$ and $R = \frac{\rho_0}{E_0} (1-\nu^2)\omega^2$

2 Finite element formulation

In this section, FEM is applied to Eq. (8) and (13). To solve these equations standard discretization approach of finite element method is used. After applying FEM differential equation is converted into simultaneous linear equation as:

$$\sum_{j=1}^2 K_{ij}^e F_j^e = L_i^e; i = 1, 2; e = 1, 2, 3, \dots, n \quad (14)$$

Case I:

$$K_{ij}^e = \left[\phi_i \frac{d\phi_j}{dr} \right]_{r_e}^{r_{e+1}} - \int_{r_e}^{r_{e+1}} \frac{d\phi_i}{dr} \frac{d\phi_j}{dr} dr + \int_{r_e}^{r_{e+1}} \phi_i \left(\frac{1}{r} - P \right) \frac{d\phi_j}{dr} dr + \int_{r_e}^{r_{e+1}} \phi_i \phi_j \left(\frac{\nu P}{r} + \frac{1}{r^2} \right) dr \quad (15)$$

$$L_i^e = Q \int_{r_e}^{r_{e+1}} \phi_i r e^{-R(r-a)} dr \quad (16)$$

Case II:

$$K_{ij}^e = \left[\phi_i \frac{d\phi_j}{dr} \right]_{r_e}^{r_{e+1}} - \int_{r_e}^{r_{e+1}} \frac{d\phi_i}{dr} \frac{d\phi_j}{dr} dr + \int_{r_e}^{r_{e+1}} \frac{\phi_i}{r} \frac{d\phi_j}{dr} dr + \int_{r_e}^{r_{e+1}} \frac{\phi_i \phi_j}{r^2} dr \quad (17)$$

$$L_i^e = R \int_{r_e}^{r_{e+1}} \phi_i r^{n_2-n_1+1} dr \quad (18)$$

where $\varphi_1^e = \frac{r_{e+1}-r}{r_{e+1}-r_e}$, $\varphi_2^e = \frac{r-r_e}{r_{e+1}-r_e}$

3 Numerical results and discussions

In this section, numerical results for thermo-elastic characteristics have been presented graphically for rotating FGM disk under exponential and power-law distribution of material properties. The value of Poisson's ratio is taken as 0.3 constant throughout material.

Case I:

In this section, an effect of angular velocity, thickness and functionally graded materials study on stresses, strains and displacement under exponentially varying material properties.

Figure 1: Young's modulus versus radial coordinate

Figure 2: Density versus radial coordinate

Figure 3: Thickness versus radial coordinate

Figure 4: Displacement against radial coordinate for different values of angular velocity

Figure 5: Circumferential strain against radial coordinate for different values of angular velocity

Figure 6: Radial strain against radial coordinate for different values of angular

Figure 9: Displacement versus radial coordinate for variable thickness

Figure 7: Circumferential stress against radial coordinate for different values of angular velocity

Figure 10: Circumferential strain versus radial coordinate for variable thickness

Figure 8: Radial stress against radial coordinate for different values of angular

Figures 1 – 3 present variation in thermomechanical material properties along radial direction in rotating FGM disk. From Figure 1, we can see that as value of m increases, modulus of elasticity decreases. Maximum variation in $E(r)$ exists when $m = -2$. From Figure 2 and 3, one can observe that behaviour of density and thickness curves are same in nature. The maximum value of density and thickness is obtained for minimum value of parameter. For +ve values of parameter behaviour of all material properties are decreasing in nature but for -ve values of parameter behaviour is increasing in nature.

Figures 4 – 8 summarize variation in thermoelastic characteristics for various values of angular velocity against radial coordinate r . Figure 4 is evident displacement for different values of angular velocity when $0.2 \leq r \leq 1$. It is clear that as value of angular velocity increases,

Figure 11: Radial strain versus radial coordinate for variable thickness

Figure 14: Displacement for radial coordinate for different FGM's

Figure 12: Circumferential stress versus radial coordinate for variable thickness

Figure 15: Circumferential strain radial coordinate for different FGM's

Figure 13: Radial stress versus radial coordinate for variable thickness

Figure 16: Radial strain radial coordinate for different FGM's

Figure 17: Circumferential stress radial coordinate for different FGM's

Figure 18: Radial stress radial coordinate for different FGM's

the displacement also increases. For various values of ω maximum displacement exists for minimum value of r . Figure 5 reveals that the behaviour of circumferential strain and displacement curves are same for different values of ω . The maximum and minimum numerical value of circumferential strain exists corresponding maximum and minimum value of angular velocity respectively. The displacement and circumferential strain curves are tensile for inner surface of disk. Figure 6 exhibits that, radial strain and angular velocity are opposite in nature for different values of r i.e. ϵ_r decreases as value of r increases. Figure 7, illustrate circumferential stress against radial coordinate for different values of ω . The behaviour of circumferential stress curves are changes at $r = 0.8$. At inner surface of disk maximum value of circumferential stress is obtained for maximum value of angular velocity but at outer surface of disk maximum value is achieved for minimum value for angular velocity. Figure 8 represents, effect of angular ve-

locity on radial stress. For $\omega = 100$ radial stress curve is very fluctuating in nature but for $\omega = 10$ curve shows steady behaviour.

Figures 9 – 13 show the effect of thickness on thermo-elastic characteristics by taking different values of thickness parameter k . As per Figure 9, for inner surface of disk maximum value of displacement is obtained corresponding to $k = 0.1$ and for outer surface maximum value it exists for $k = 0.7$. Figure 10 shows that, circumferential strain curves are more variable for inner surface of disk. All circumferential curves are decreasing in nature i.e. as r increases ϵ_θ decreases. But from Figure 11, one can see that behaviour of radial strain curve is increasing and maximum value of radial strain is achieved corresponding to maximum value of thickness parameter. From Figures 12 and 13 it is noted that behaviour of circumferential and radial stress curves are different in nature. Maximum value of circumferential stress and minimum value of radial stress are obtained for $k = 0.1$. Also, circumferential stress is maximum at inner surface of disk but radial stress is minimum at this surface.

Figure 14 – 18 show distribution of thermo-elastic characteristics of rotating disk that disk is constructed from different FGM's. The thermo-mechanical properties of FGM's are presented in Table 1.

From Figure 14, it is observed that displacement decreases as value of radial coordinate increases. The maximum value of displacement is obtained for stainless steel and minimum value is attained for silicon nitride. Figure 15 displays that circumferential strain converges to zero as radial coordinate moves toward outer surface of disk. Radial strain curves are increasing in nature, according to Figure 16. The behaviour of all radial strain curves are linear when $0.2 \leq r \leq 0.3$ and for remaining region behaviour is non-linear in nature. The circumferential stress is maximum for silicon nitride and least for aluminium. The behaviour of displacement and circumferential strain curves are same in nature. As we can see from Figure 18, radial stress curve is firstly linearly decreasing in nature for $0.2 \leq r \leq 0.3$ and it is decreasing for remaining values of radial coordinate.

Case II:

In this section, graphical representation of thermoelastic characteristics of rotating disk is done under exponential material properties. The effect of non-homogeneity index ($n_1 = n_2 = n_3 = n$) is shown on stress, strain and displacement by taking its various values.

Figures 19 – 26 show thermo-mechanical properties and thermo-elastic characteristics for power law distribution of material properties. The effect of non-homogeneity

Table 1: Thermo-mechanical properties of various functionally graded materials

Material abbreviation	Material	$E_0(\text{GPa})$	$\rho_0(\text{g/cm}^3)$
FGM-I	Aluminium	71	2.70
FGM-II	Zirconium oxide	180	5.68
FGM-III	Stainless steel	200	7.9
FGM-IV	Silicon nitride	300	3.17

Figure 19: Distribution of Young's modulus with radial coordinate **Figure 21:** Distribution of thickness with radial coordinate **Figure 20:** Distribution of density with radial coordinate **Figure 22:** Distribution of displacement with radial coordinate

parameter is shown by taking different values of n . From Figures 19 – 21, one can see that maximum value of material properties obtained for $n = 2$ and minimum values are obtained for $n = 6$. Modulus of elasticity, density and thickness are constant when non homogeneity parameter is absent from material properties. As Figure 22 shows, displacement curve is decreasing for $n = 4$ and $n = 6$ but for $n = 2$ curve is strictly increasing. Maximum displacement is obtained when material properties follow quadratic variation along radius of disk. For $n = 2$ circum-

ferential strain Curve shows increasing behaviour in nature for inner surface of rotating disk but decreasing for outer surface of disk. For $n = 0$, $n = 4$ and $n = 6$ behaviour of circumferential strain linear and stable. Radial strain is fluctuating for $n = 0$ and $n = 2$ but this behaviour is linear for $n = 6$ and very less elastic for $n = 4$. From Figures 25 – 26, it is noted that both stress curves are increasing in nature for $n = 0$ but shows decreasing behaviour for remaining values of n . Maximum variation in stresses exits for outer surface of disk.

Figure 23: Distribution of circumferential strain with radial coordinate

Figure 26: Distribution of radial stress with radial coordinate

Figure 24: Distribution of radial strain with radial coordinate

Figure 25: Distribution of circumferential stress with radial coordinate

4 Conclusion

In presented research, thermo-elastic characteristics analysis is made for different material properties profiles in rotating disk which is constructed from FGM material. Finite element method is employed to find displacement, strains and stresses numerically from equilibrium equation. The following conclusion can be made:

- 1) In exponential distribution of material properties, the value of thermo-mechanical properties increases as material parameter increases but for power law distribution this behaviour of material properties is opposite in nature.
- 2) Maximum variation in displacement, strains and stresses exist for inner surface of rotating disk when thickness is taken as variable parameter in case-I.
- 3) The behaviour of thermo-elastic characteristics is same in nature but differ numerically for different types of functionally graded materials.
- 4) In power law distribution of material properties maximum value of thermo-elastic characteristics is obtained for $n = 2$.
- 5) For $n = 4$ and $n = 6$ thermo-elastic curves are more stable in nature in power law distribution of material properties.

Finally, these findings may be helpful in construction/designing of FGM rotating disks, which have a numerous application in different fields of engineering.

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