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Magnetized couple stress fluid flow past a vertical cylinder under thermal radiation and viscous dissipation effects

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Abstract: Contemporary investigation studies the silent features of the dissipative free convection couple stress fluid flow over a cylinder under the action of magnetic field, thermal radiation and porous medium with chemical reaction effect. Present two-dimensional viscous incompressible physical model is designed based on the considered flow geometry. Present physical problem gives the highly complicated nonlinear coupled partial differential equations (PDE's) which are not amenable to any of the known techniques. Thus, unconditionally stable, most accurate and speed converging with flexible finite difference implicit technique is utilized to simplify the dimensionless flow field equations. It is apparent from the current results that; the velocity profiles are diminished with enhancing values of magnetic field. Temperature profile increases with enhancing values of thermal radiation parameter. Velocity contours deviates away from the wall with enhancing magnetic parameter. Also, the effects of magnetic field, porous medium, thermal radiation, chemical reaction, buoyancy ratio parameter and Eckert number on couple stress flow velocity, temperature, and concentration profiles are studied. However, the present study has good number of applications in the various fields of engineering such as; polymer processing, solidification of liquid crystals, colloidal solutions, synovial joints, geophysics, chemical engineering, astrophysics and nuclear reactors etc. Finally, the current solutions are validated with the available results in the literature review and found to be in good agreement.

Keywords: couple stress fluid, vertical cylinder, chemical reaction, radiation effect, viscous dissipation

1 Introduction

Incompressible couple stress fluid flow obeying Prandtl's theory with distinct effects about a vertical cylinder kept in a porous medium is a significant study area related to many of the manufacturing process. The non-Newtonian fluids have rich set of biomedical and industrial applications in the field of science and engineering. Numerous non-Newtonian fluids are available in nature which has significant number of applications in the various fields of science and engineering. Particularly, the time-dependent couple stress fluid have important applications in numerous processes that are recognized in the industry like the extrusion of polymer fluids, solidification of liquid crystals, and colloidal solutions. Application of couple stress fluid is fairly found in synovial joints (shoulder, hip, knee, and ankle), geophysics, chemical engineering, and astrophysics. Based on the literature review it is observed that, many researchers modelled the synovial fluids as couple stress fluid and they investigated the thermodynamic behaviour. In the last few decades, the flow of couple stress liquid around a plate and cylinder with different physical effects has acknowledged the consideration of various researchers and engineers.

Soundalgekar [1] investigated the natural convection flow over an infinite vertical plate by considering Stokes problem in accordance with the mass transfer applications. The action of variable temperature on fluid flow about a moving plate was investigated by Murty and Soundalgekar [2]. Soundalgekar and Takher [3] employed the series solution for magnetized free convection flow past an infinite vertical plate under the action of uniform transverse magnetic field with dissipation effect. Their investigation demonstrates that, the rising magnetic field parameter and the Prandtl number enhances the dissipation effects in the flow regime. Ganesan and Rani [4, 5] were disclosed the transient natural convection fluid flow with heat and mass transfer process along the vertical cylinder by considering MHD effects. Fathia *et al.* [6] studied the dissipative unsteady natural convective viscous fluid flow with concentration diffusion about an infinite

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vertical plate. Rani [7] has examined the time-dependent buoyancy motivated flow with thermal and species transfer effects over a vertical cylinder. Unsteady magnetized natural convective viscous dissipative fluid flow about a vertical plate was studied by Singh *et al.* [8] and they visualized that, the rising magnetic field raises the temperature field.

Palani [9] studied the viscous dissipative fluid flow along a vertical plate with variable temperature field. The research revealed that, the increasing dissipation effect results in the rise of temperature. Rani and Kim studied the time-dependent natural convection flow over a vertical cylinder with variable viscosity and thermal conductivity along with temperature dependent viscosity effect [10, 11]. Rani and Kim examined the unsteady natural convection flow about an isothermal cylinder with Dufour and Soret effects [12]. Magnetised fluid flow with diffusion of concentration field about a vertical plate was studied by Palani and Srikanth [13]. They found that, the rising buoyancy force significantly raises the velocity field. The effects of variable viscosity and heat conduction on velocity, temperature and Nusselt number on vertical geometry were studied by Palani and Kim [14]. Rani and Kim studied the time-dependent free convective micro-polar fluid flow about a cylinder with heat and mass transfer [15]. Rani *et al.* numerically examined the chemically reactive couple stress fluid flow about a vertical cylinder [16, 17]. In the same direction, the problem of fluid moment about a moving cylinder with magnetic and thermal radiation impacts was discussed by Loganathan *et al.* [18].

Neog and Das investigated the unsteady free convective magnetised fluid flow past a plate with varying temperature by employing Laplace transform method [19]. Rani *et al.* studied the buoyancy motivated unsteady couple stress fluid flow over a cylinder under the action of Prandtl number [20]. Bella and Naikoti inspected the impacts of dissipation and thermal radiative on time-dependent natural convective magnetized chemically reactive thermal and mass moment about a plumb plate using FEM method [21]. The application of viscosity on free convection flow of air around a vertical cylinder was studied by Hossain *et al.* [22]. Loganathan and Sivapoornapriya studied the free convective species concentration transfer past an infinite plate which is kept in porous medium [23]. Their analysis shows that, the rising permeability parameter enhances the velocity field and rising Prandtl number diminishes the thermal field.

Reddy and Raju analyzed the radiation and chemical reaction effects on unsteady magnetized mixed convective flow around a vertical porous plate [24]. Palani *et al.* disclosed the impacts of MHD and viscous dissipation

on buoyancy motivated fluid moment about a plate with varying thermal field [25]. Loganathan and Divya numerically discussed the effects of chemical reaction and dissipation on Newtonian fluid past a cylinder [26]. Reddy studied the impact thermal radiation and chemical application on time-dependent magnetized free convective parabolic fluid flow over isothermal vertical plate with viscous dissipation effect [27]. Basha *et al.* studied the impacts of free convection on supercritical water flow past a vertical using Redlich-Kwong equation of state approach [28]. Basha *et al.* studied the chemically reactive unsteady buoyancy motivated couple stress fluid flow about a vertical plate [29]. Recently, Shekar and Kishan studied the impact of thermal radiation on natural convection thermal transfer in a square cavity filled with nanofluid under the influence of saturated porous medium with the different nanoparticles [30]. It is found from their investigation that, the heat transfer increases with increase in Rayleigh number. Haritha *et al.* investigated the impact of magnetic and viscous dissipation on free convection heat transfer process in a square cavity filled with nanofluid under the effect of saturated porous medium with the different nanoparticles [31]. It is observed from their investigation that, the enhancing magnetic number increases the flow field. Balla *et al.* numerically studied the effect of chemical reaction on bioconvective flow in a porous square cavity containing oxytactic microorganism [32]. Further, the bio-convection thermal and flow behavior is studied under the impact of Darcy model. It is recorded from their investigation that, chemical reaction increases the flow cell and causes the splitting of the cell in the fluid medium.

Acharya *et al.* studied the impact of thermal radiation on laminar free convective two-dimensional couple stress fluid flow over a stretching cylinder under magnetic field [33]. It is recorded from their study that, the increasing magnetic number increases the entropy generation process. Basha *et al.* investigated supercritical free convection in couple stress and Newtonian fluids under the influence EOS approach [34]. It is found from their investigation that, the supercritical fluid is sensible to thermal changes in the fluid medium. Das *et al.* [35] numerically investigated the importance of Joule heating, viscous dissipation, magnetic field and slip condition on the two-dimensional flow of an electrically conducting couple stress fluid induced by an exponentially elongating sheet kept in a porous medium. It is recorded from their study that, the increasing radiation number increases the thermal field in the flow regime. Ibrahim and Gadisa [36] studied the double stratified mixed convection flow of couple stress nanofluid about an inclined elongating cylinder under the impact of Cattaneo-Christov heat and mass flux model. It is found

from their study that, the increasing Reynolds number increases the concentration distribution in the flow field. The detailed literature on couple stress fluid can be found in [37–40].

By observing the above published results, the combined effects of chemical reaction with radiation, dissipation and porous medium on incompressible unsteady buoyancy driven couple stress fluid flow around a vertical cylinder has obtained rare consideration in the literature. In the current work, authors made an attempt to examine the behaviour of time-dependent couple stress fluid flow about a cylinder with magnetic force, porous medium, thermal radiation and chemical reaction. However, yet the exact solution is not available for the current physical problem with above mentioned physical effects. The governing highly nonlinear physical system of flow-field equations are simplified by employing the unconditionally stable finite difference scheme.

2 Mathematical formulation and governing equations

Transient natural convective magnetised incompressible thermally radiative viscous couple stress fluid about a vertical cylinder with viscous dissipation and chemical reaction impacts under the influence of porous medium is studied. To define the problem well, a rectangular region of geometry is considered in which x -axis is measured along the axial path and r -coordinate is taken normal to cylinder. The schematic representation of the physical problem shown in Figure 1. The free stream thermal field (T'_∞) and species diffusion field (C'_∞) are expected to be same as the thermal and mass diffusion of the geometry which is maintained at the time $t' = 0$. As time starts $t' > 0$, the thermal field and concentration diffusion raised to $T'_w (> T'_\infty)$ and $C'_w (> C'_\infty)$ and this is same for all time $t' > 0$. With this thermal and mass diffusion differences within the boundary layer in the vicinity of the flow configuration density changes occurs and that in turn interacts with the gravity field and produces the free convective couple stress fluid flow about a cylinder. Based on the fluid flow and geometry, the governing two-dimensional nonlinear coupled equations of the present problem are described below with Boussinesq's approximation [10–12, 15–17].

Continuity equation:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \quad (1)$$

Figure 1: Schematic representation of the problem.

Momentum equation:

$$\rho \left(\frac{\partial u}{\partial t'} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \right) = \rho g \beta_T (T' - T'_\infty) + \rho g \beta_C (C' - C'_\infty) + \frac{1}{r} \frac{\partial}{\partial r} \left(\mu r \frac{\partial u}{\partial r} \right) - \frac{\mu}{K_1} u - \eta \nabla^4 u - \sigma B_0^2 u \quad (2)$$

Here η is the constant related to couple stress liquid flow.

Energy equation:

$$\left(\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial r} \right) = \frac{\alpha}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T'}{\partial r} \right) + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial r} \right)^2 - \frac{1}{\rho C_p} \frac{1}{r} \frac{\partial}{\partial r} (r q_r) \quad (3)$$

Concentration equation:

$$\left(\frac{\partial C'}{\partial t'} + u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial r} \right) = \frac{D}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C'}{\partial r} \right) - K_r (C' - C'_\infty) \quad (4)$$

Based on the Rosseland theory of thermal radiation [41], the radiative heat flux (q_r) can be approximated as:

$$q_r = -\frac{4\sigma_s}{3k^*} \frac{\partial T'^4}{\partial r} \quad (5)$$

Stefan-Boltzmann constant is σ_s and k^* is the mean absorption coefficient. If difference of the temperature within the flow is too small, the term T'^4 can be written as a linear temperature function, then T'^4 can be written by utilizing Taylor's expansion about T'_∞ with ignoring terms of greater order is given by:

$$T'^4 \cong 4T_\infty'^3 - 3T_\infty'^4 \quad (6)$$

Using equations (5) and (6), equation (3) reduces to

$$\left(\frac{\partial T'}{\partial t'} + u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial r} \right) = \frac{\alpha}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T'}{\partial r} \right) + \frac{\mu}{\rho C_p} \left(\frac{\partial u}{\partial r} \right)^2 + \frac{16\sigma_s T_\infty'^3}{3\rho C_p k^*} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T'}{\partial r} \right) \quad (7)$$

Stokes [16, 17] given the two conditions for couple stress liquid moment in the boundary layer regime. However, second condition is employed for simplifying the current problem. Thus, the required boundary conditions of the current physical problem are [10–12, 15–17, 20, 34]:

$$t' \leq 0 : \quad u = 0, \quad v = 0, \quad T' = T'_\infty, \quad C' = C'_\infty$$

for all x & r

$$t' > 0 : \quad u = 0, \quad v = 0, \quad T' = T'_w, \quad C' = C'_w \quad \text{at } r = r_0 \quad (8)$$

$$u = 0, \quad v = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{at } x = 0$$

$$T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty, \quad u \rightarrow 0, \quad v \rightarrow 0 \quad \text{as } r \rightarrow \infty$$

and

$$\frac{\partial u}{\partial r} = \frac{\partial v}{\partial x} \quad \text{at } r = r_0 \quad \text{and } r \rightarrow \infty \quad (9)$$

Utilized non-dimensional variables and parameters are as follows:

$$\begin{aligned} X &= Gr_T^{-1} \frac{x}{r_0}, \quad R = \frac{r}{r_0}, \quad U = Gr_T^{-1} \frac{ur_0}{v}, \quad V = \frac{vr_0}{v}, \quad t = \frac{vt'}{r_0^2}, \\ T &= \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\ Gr_T &= \frac{g\beta_T r_0^3 (T'_w - T'_\infty)}{v^2}, \quad Gr_C = \frac{g\beta_C r_0^3 (C'_w - C'_\infty)}{v^2}, \\ Pr &= \frac{v}{a}, \quad r_0 = \left(\frac{\eta}{\mu} \right)^{1/2}, \quad Sc = \frac{v}{D}, \quad Bu = \frac{Gr_C}{Gr_T}, \\ M &= \frac{\sigma B_0^2 r_0^2}{\rho v}, \quad K1 = \frac{k'_1}{r_0^2}, \quad Kr = \frac{K'_r r_0^2}{v}, \quad Nr = \frac{k^* k}{4\sigma_s T_\infty'^3}, \\ Ec &= \frac{Gr_T^2 v^2}{C_p r_0^2 (T'_w - T'_\infty)}. \end{aligned} \quad (10)$$

By introducing the above dimensionless quantities in the Eqs. (1)–(4) and (7), the reduced non-dimensional governing equations are obtained:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial R} + \frac{V}{R} = 0 \quad (11)$$

$$\left(\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial R} \right) = T + BuC - \frac{U}{K1} - MU + \left(1 + \frac{1}{R^2} \right) \frac{\partial^2 U}{\partial R^2} + \left(\frac{1}{R} - \frac{1}{R^3} \right) \frac{\partial U}{\partial R} - \frac{2}{R} \frac{\partial^3 U}{\partial R^3} - \frac{\partial^4 U}{\partial R^4} \quad (12)$$

$$\left(\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial R} \right) = \frac{1}{Pr} \left(1 + \frac{4}{3Nr} \right) \left(\frac{\partial^2 T}{\partial R^2} + \frac{1}{R} \frac{\partial T}{\partial R} \right) + Ec \left(\frac{\partial U}{\partial R} \right)^2 \quad (13)$$

$$\left(\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial R} \right) = \frac{1}{Sc} \left(\frac{\partial^2 C}{\partial R^2} + \frac{1}{R} \frac{\partial C}{\partial R} \right) - KrC \quad (14)$$

The corresponding initial and boundary conditions in dimensionless form are given as follows:

$$\left. \begin{aligned} &\text{For all } X \text{ and } R, \quad C = 0, \quad T = 0, \quad U = 0, \quad V = 0, \quad \text{when } t \leq 0 \\ &\text{at } R = 1, \quad C = 1, \quad T = 1, \quad U = 0, \quad V = 0 \\ &\text{at } X = 0 \quad C = 0, \quad T = 0, \quad U = 0, \quad V = 0 \\ &\text{as } R \rightarrow \infty \quad C \rightarrow 0, \quad T \rightarrow 0, \quad U \rightarrow 0, \quad V \rightarrow 0 \end{aligned} \right\} \quad \text{when } t > 0 \quad (15)$$

Similarly, the Eq. (9) also changed to the following dimensionless form.

$$\frac{\partial U}{\partial R} = \frac{\partial V}{\partial X} \frac{1}{Gr_T^2} \quad \text{at } R = 1 \quad \text{and as } R \rightarrow \infty \quad (16)$$

2.1 Average momentum, heat transfer and mass transfer rates

In view of real-life engineering applications, the researchers required the good knowledge on the values average momentum, heat and mass transfer rates. When compared to Nusselt and Sherwood numbers, skin friction coefficient plays the key role in the flow phenomena. In many of the technical applications, the raising skin-friction coefficient is unwanted but even though the raised skin-friction can be utilised in some applications such as thermal and mass exchanger's systems. In the current investigation, the momentum, thermal and concentration flow rates are obtained from the literature Rani *et al.* [16]. Thus, the dimensionless average momentum transport coefficient ($\bar{C}_f$), heat transfer ($\bar{Nu}$) and mass transfer ($\bar{Sh}$) rates are defined as follows:

$$\bar{C}_f = \int_0^1 \left(\frac{\partial U}{\partial R} \right)_{R=1} dX \quad (17)$$

$$\bar{Nu} = - \int_0^1 \left(\frac{\partial T}{\partial R} \right)_{R=1} dX \quad (18)$$

$$\bar{Sh} = - \int_0^1 \left(\frac{\partial C}{\partial R} \right)_{R=1} dX \quad (19)$$

The differentials involved in Eqs. (17)-(19) are evaluated through a five-point formula, later integrals are computed through a Newton-Cotes scheme. Simulated dimensionless momentum, thermal and species diffusion rates for couple stress liquid are shown graphically and discussed below.

3 Numerical solution procedure

Governing non-dimensional unsteady fluid flow equations Eqs. (11)-(14) are simplified by employing implicit finite difference method. The associated finite difference equations are listed as follows:

$$\frac{U_{l,m}^{n+1} - U_{l-1,m}^{n+1} + U_{l,m}^n - U_{l-1,m}^n}{2\Delta X} + \frac{V_{l,m}^{n+1} - V_{l,m-1}^{n+1} + V_{l,m}^n - V_{l,m-1}^n}{2\Delta R} + \frac{V_{l,m}^{n+1}}{(1 + (m-1)\Delta R)} = 0 \quad (20)$$

$$\begin{aligned} & \frac{U_{l,m}^{n+1} - U_{l,m}^n}{\Delta t} + \frac{U_{l,m}^n}{2\Delta X} \left(U_{l,m}^{n+1} - U_{l-1,m}^{n+1} + U_{l,m}^n - U_{l-1,m}^n \right) + \frac{V_{l,m}^n}{4\Delta R} \left(U_{l,m+1}^{n+1} - U_{l,m-1}^{n+1} + U_{l,m+1}^n - U_{l,m-1}^n \right) \\ &= \left(\frac{T_{l,m}^{n+1} + T_{l,m}^n}{2} \right) + Bu \left(\frac{C_{l,m}^{n+1} + C_{l,m}^n}{2} \right) - \frac{1}{K1} \left(\frac{U_{l,m}^{n+1} + U_{l,m}^n}{2} \right) - M \left(\frac{U_{l,m}^{n+1} + U_{l,m}^n}{2} \right) \\ &+ \left(\frac{U_{l,m+1}^{n+1} - 2U_{l,m}^{n+1} + U_{l,m-1}^{n+1} + U_{l,m+1}^n - 2U_{l,m}^n + U_{l,m-1}^n}{2\Delta R^2} \right) + \left(\frac{U_{l,m+1}^{n+1} - 2U_{l,m}^{n+1} + U_{l,m-1}^{n+1} + U_{l,m+1}^n - 2U_{l,m}^n + U_{l,m-1}^n}{2(1 + (m-1)\Delta R)^2 \Delta R^2} \right) \\ &+ \left(\frac{U_{l,m+1}^{n+1} - U_{l,m-1}^{n+1} + U_{l,m+1}^n - U_{l,m-1}^n}{4(1 + (m-1)\Delta R) \Delta R} \right) - \left(\frac{U_{l,m+1}^{n+1} - U_{l,m-1}^{n+1} + U_{l,m+1}^n - U_{l,m-1}^n}{4(1 + (m-1)\Delta R)^3 \Delta R} \right) \\ &- \left(\frac{U_{l,m+2}^{n+1} - 2U_{l,m+1}^{n+1} + 2U_{l,m-1}^{n+1} - U_{l,m-2}^{n+1} + U_{l,m+2}^n - 2U_{l,m+1}^n + 2U_{l,m-1}^n - U_{l,m-2}^n}{2(1 + (m-1)\Delta R) \Delta R^3} \right) \\ &- \left(\frac{U_{l,m+2}^{n+1} - 4U_{l,m+1}^{n+1} + 6U_{l,m-1}^{n+1} - 4U_{l,m-2}^{n+1} + U_{l,m+2}^n - 4U_{l,m+1}^n + 6U_{l,m-1}^n - 4U_{l,m-2}^n + U_{l,m-2}^n}{2\Delta R^4} \right) \end{aligned} \quad (21)$$

$$\begin{aligned}
& \frac{T_{l,m}^{n+1} - T_{l,m}^n}{\Delta t} + \frac{U_{l,m}^n}{2\Delta X} \left(T_{l,m}^{n+1} - T_{l-1,m}^{n+1} + T_{l,m}^n - T_{l-1,m}^n \right) + \frac{V_{l,m}^n}{4\Delta R} \left(T_{l,m+1}^{n+1} - T_{l,m-1}^{n+1} + T_{l,m+1}^n - T_{l,m-1}^n \right) \\
& = \frac{1}{Pr} \left(1 + \frac{4}{3Nr} \right) \left(\frac{T_{l,m+1}^{n+1} - 2T_{l,m}^{n+1} + T_{l,m-1}^{n+1} + T_{l,m+1}^n - 2T_{l,m}^n + T_{l,m-1}^n}{2\Delta R^2} \right) \\
& + \frac{1}{Pr} \left(1 + \frac{4}{3Nr} \right) \left(\frac{T_{l,m+1}^{n+1} - T_{l,m-1}^{n+1} + T_{l,m+1}^n - T_{l,m-1}^n}{4(1+(m-1)\Delta R)\Delta R} \right) + Ec \left(\frac{U_{l,m+1}^{n+1} - U_{l,m-1}^{n+1} + U_{l,m+1}^n - U_{l,m-1}^n}{4\Delta R} \right)^2 \quad (22)
\end{aligned}$$

$$\begin{aligned}
& \frac{C_{l,m}^{n+1} - C_{l,m}^n}{\Delta t} + \frac{U_{l,m}^n}{2\Delta X} \left(C_{l,m}^{n+1} - C_{l-1,m}^{n+1} + C_{l,m}^n - C_{l-1,m}^n \right) + \frac{V_{l,m}^n}{4\Delta R} \left(C_{l,m+1}^{n+1} - C_{l,m-1}^{n+1} + C_{l,m+1}^n - C_{l,m-1}^n \right) \\
& = \frac{1}{Sc} \left(\frac{C_{l,m+1}^{n+1} - C_{l,m-1}^{n+1} + C_{l,m+1}^n - C_{l,m-1}^n}{4(1+(m-1)\Delta R)\Delta R} \right) + \frac{1}{Sc} \left(\frac{C_{l,m+1}^{n+1} - 2C_{l,m}^{n+1} + C_{l,m-1}^{n+1} + C_{l,m+1}^n - 2C_{l,m}^n + C_{l,m-1}^n}{2\Delta R^2} \right) \\
& - Kr \left(\frac{C_{l,m}^{n+1} + C_{l,m}^n}{2} \right) \quad (23)
\end{aligned}$$

The solutions of the Eqs. (20)-(23) are obtained in the rectangular region of the geometry limiting from $X_{min} = 0$, $X_{max} = 1$, $R_{min} = 1$ and $R_{max} = 20$, here R_{max} assuming as $R = \infty$ be located far from the momentum, heat and mass diffusion boundary layers. Unsteady velocity, heat and mass diffusion values are obtained using the mesh space of 100×500 varies 2^{nd} decimal with 50×250 and 50×250 differing in 5^{th} decimal with 200×100 . The mesh space of 100×500 has been finalised for all numerical computations with 0.03 and 0.01 as mesh sizes along R and X directions. Also, the step size of time dependence has verified with $\Delta t = 0.01$ and provides good result. Further, for unsteady solutions of velocity, temperature and concentration, the absolute difference between the flow field at two successive times is smaller than 10^{-5} at all mesh nodes is maintained.

By solving the energy and concentration Eqs. (22)-(23), the temperature and mass diffusion fields are obtained, respectively. Further, the result of momentum and continuity equations, gives the velocity field. Finite difference equations of Eqs. (21)-(23) at $(n+1)^{th}$ computation can be written in pentadiagonal and tridiagonal formulae, respectively as listed below.

$$A_1 \Psi_{l,m-2}^{n+1} + B_1 \Psi_{l,m-1}^{n+1} + C_1 \Psi_{l,m}^{n+1} + D_1 \Psi_{l,m+1}^{n+1} + E_1 \Psi_{l,m+2}^{n+1} = F_1 \quad (24)$$

$$A_2 \varphi_{l,m-1}^{n+1} + B_2 \varphi_{l,m}^{n+1} + C_2 \varphi_{l,m+1}^{n+1} = D_2 \quad (25)$$

where Ψ represents the dependent variable velocity U and φ represents the dependent variables T and C . Therefore, the Eqs. (24) and (25) at each interior node on a specific l -level creates a pentadiagonal and tridiagonal equations. These equations are simplified through pentadiagonal and Thomas schemes. Explicitly the velocity V is found from the finite difference equation Eq. (20). This mechanism is continual for all successive l -levels with more time steps till the convergence criteria is attained 10^{-5} .

4 Results and discussions

Based on the present numerical analysis following results are demonstrated. To study the time-dependent behaviour of simulated patterns such as flow field, thermal, concentration and their numerical values at various places are shown and which are neighbouring to heated surface of vertical cylinder. At $X = 1.0$ along R -coordinate, the steady mass diffusion, thermal and flow fields are described.

4.1 Validation of currents with existing solutions

The steady-state simulated mass diffusion fields of couple stress liquid obtained in the current investigation is compared with the existing results of Rani *et al.* [17]. It is clear from Figure 2 that, the current findings are showing accurate matching with the existing solutions.

4.2 Analysis of numerical results

Current numerical solutions are expressed in terms of magnetic parameter (M), porous medium parameter ($K1$), radiation number (Nr), chemical reaction number (Kr) Buoyancy ratio parameter (Bu) and Eckert number and rest of the variables kept constant.

Figure 2: Validation of current results with the existed results of Rani *et al.* [17].

(i) Effect of magnetic field (M) on flow profiles

Velocity profile: The simulated velocity field U at position $(1, 1.38)$ versus t is illustrated in Figure 3(a). It is determined that, the unsteady-state flow field decayed with rising M . It is because of fact that, the increasing magnetic number amplifies the Lorenz forces in the flow region and hence velocity profile is diminished. Also, the boundary layer thickness diminished with rising M . It is witnessed that, for the enhancing M , the maximum velocity peaks diminished. Also, the time required to reach the time-independent state increases with amplifying M . Initially (*i.e.*, $t \ll 1$) velocity profiles are coinciding with each other due to convective thermal transfer process is dominated by the conduction heat transfer.

The computer generated flow field U versus R at $X = 1$ is demonstrated in the Figure 3(b) under the action of magnetic field. Initially flow field begun with zero near the cylindrical hot surface, touch its greatest values and monotonically decays to value zero along R -direction for all t . It is recorded from Figure 3(b) that, the steady-state velocity acts as a decaying function of magnetic parameter.

(a)

(b)

Figure 3: (a) Transient and (b) Steady state velocity profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $Nr = 2.0$, $K1 = 1.0$, $Ec = 0.1$.

(a)

(a)

(b)

(b)

Figure 4: (a) Transient and (b) Steady state temperature profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $Nr = 2.0$, $K1 = 1.0$, $Ec = 0.1$.

Figure 5: (a) Transient and (b) Steady state concentration profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $Nr = 2.0$, $K1 = 1.0$, $Ec = 0.1$.

(a)

(a)

(b)

(b)

Figure 6: (a) Transient and (b) Steady state velocity profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $Nr = 1.0$, $M = 0.2$, $Ec = 0.1$.

Figure 7: (a) Transient and (b) Steady state temperature profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $Nr = 1.0$, $M = 0.2$, $Ec = 0.1$.

(a)

(a)

(b)

(b)

Figure 8: (a) Transient and (b) Steady state velocity profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $K1 = 1.0$, $M = 0.2$, $Ec = 0.1$.

Figure 9: (a) Transient and (b) Steady state temperature profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $K1 = 1.0$, $M = 0.2$, $Ec = 0.1$.

(a)

(a)

(b)

(b)

Figure 10: (a) Transient and (b) Steady state concentration profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $K1 = 1.0$, $M = 0.2$, $Ec = 0.1$.

Figure 11: (a) Transient and (b) Steady state velocity profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Nr = 1.0$, $K1 = 1.0$, $M = 0.2$, $Ec = 0.1$.

(a)

(a)

(b)

(b)

Figure 12: (a) Transient and (b) Steady state concentration profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Nr = 1.0$, $K1 = 1.0$, $M = 0.2$, $Ec = 0.1$.

Figure 13: (a) Transient and (b) Steady state velocity profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Kr = -0.2$, $Nr = 1.0$, $K1 = 1.0$, $M = 0.2$, $Ec = 0.1$.

(a)

(b)

Figure 14: (a) Transient and (b) Steady state temperature profiles with fixed $Sc = 0.6$, $Pr = 0.7$, $Kr = -0.2$, $Bu = 1.0$, $Nr = 1.0$, $K1 = 1.0$, $M = 0.2$.

Temperature profile: The simulated transient temperature profile with enhancing values of magnetic field M at $(1, 1.05)$ is demonstrated in Figure 4(a). This plot shows that, the merged thermal curves indicates that, initially conduction process dominates the convection phenomena. Transient temperature profile rises with time, reaches maximum values, shortly later, diminished and again gradually increases, at last asymptotically reached steady state. Unsteady temperature profile rises with amplifying values of magnetic parameter.

At $X = 1.0$ along radial axis, the time-independent temperature profile is shown in the Figure 4(b) for the giving different values of M . The temperature curves originate from the hot cylinder wall at $T = 1$ and diminished monotonically to zero irrespective of time along the radial coordinate. However, steady state velocity profile enhanced with increasing M .

Concentration profile: The computer-generated unsteady concentration diffusion field for the various magnetic parameter M values at location $(1, 1.06)$ is displayed in the Figure 5(a). Initially all the concentration diffusion arcs are fully merged with one another and diverged after some time. Also, it is noticed that, the unsteady concentration field increases, attains maximum as well as minimum and again increases, there after decreases, later asymptotically attains the stable state condition. Further, for the rising M , the mass diffusion field increases.

The time-independent simulated concentration profile at $X = 1.0$ versus R for various values of M is shown in the Figure 5(b). The concentration diffusion curves are originated from the heated cylinder wall at $C = 1$ and diminished monotonically to zero along R -axis irrespective of t . Steady-state concentration profile rises with rising M values.

(ii) Effect of porous parameter ($K1$) on velocity and temperature profile

Velocity Profile: The simulated time-dependent velocity field for different values of porous parameter $K1$ is shown in the Figure 6(a) at the location $(1, 1.3)$. It is found from this figure that, the enhancing porous parameter amplifies the velocity profiles. According to Darcy's law, the flow between the two points is directly related to the distance between points, pressure difference between the points and connectivity of the flow between the points within the medium. Thus, an increment in porous number results the free flow of couple stress fluid in the medium, hence velocity boundary layer thickness increases. It is shown that velocity profile slowly rises with time, attained maximum around $t = 4.5$ and then decreases attain the temporal minima and again increases, decreases slightly, later attain the steady-state.

Figure 15: Profiles of average (a) skin friction coefficient, (b) Nusselt number and (c) Sherwood number for fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $Nr = 2.0$, $K1 = 1.0$, $Ec = 0.1$.

The time-independent simulated velocity profile at $X = 1$ versus R is demonstrated in the Figure 6(b) for various $K1$ values. This graph shows that, the flow field starts initially zero at heated wall of cylinder, reaches the highest value and shortly later diminished to zero monotonically in R -direction. Also, for the increasing values of $K1$, the steady state velocity profile are increases.

Temperature profile: The Figure 7(a) at $(1, 1.05)$ shows the simulated unsteady temperature profile for var-

ious $K1$ values. Temperature profile diminished with rising $K1$. It is due to fact that, the rising $K1$ raises the flow between the points and pressure difference between the points decreases, then this decrease in the heat generation. Also, noticed that, initially all the thermal curves merged with each other, shortly later reaches temporal maxima as well as minima and again increases, later it attains the steady state. The Figure 7(b) shows the simulated steady state temperature profile at $X = 1.0$ against R

Figure 16: Steady state velocity contours (a) $M = 0.0$ (b) $M = 0.2$ for fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $Nr = 2.0$, $K1 = 1.0$, $Ec = 0.1$.

Figure 17: Steady state temperature contours (a) $M = 0.0$ (b) $M = 0.2$ for fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $Nr = 2.0$, $K1 = 1.0$, $Ec = 0.1$.

(a)

(b)

Figure 18: Steady state concentration contours (a) $M = 0.0$ (b) $M = 0.2$ for fixed $Sc = 0.6$, $Pr = 0.7$, $Bu = 1.0$, $Kr = -0.2$, $Nr = 2.0$, $K1 = 1.0$, $Ec = 0.1$.

for different values of $K1$. The temperature profile starts at the hot cylindrical wall with $T = 1$ and then for all time

diminished to zero in R -direction monotonically. Steady state temperature profile decayed with enhancing $K1$.

(iii) Influence of radiation parameter (Nr) on flow profiles

Velocity profile: Figure 8(a) disclose the simulated transient velocity profile for distinct Nr values at position (1, 1.3). It is observed that, rising Nr values raises the flow field. It is because of the reason that, the bond holding tendency of components of the fluid particles are easily broken when the intensity of heat produced through convected heat is raised, hence by increasing the thermal radiation parameter, the translational velocity increases. From this figure it is noticed that, the transient velocity increases gradually, reached maxima as well as minima and slightly increases then shortly later attains the steady state.

Figure 8(b) describes the computer generated flow field at $X = 1$ against R for distinct Nr . Initially all velocity curves are originated from the zero value and are merged with each other, after some distance they separate with each other and attains their maximum and gradually decreases to zero. Steady state flow field magnifies with rising Nr .

Temperature profile: The Figure 9(a) provides the simulated time-dependent temperature profile for distinct Nr values at position (1, 1.05). The parameter Nr defines the comparative contribution of conduction heat transfer to convection heat transfer. It is seen from Figure 9(a) that, the rising value of Nr makes in increase in the temperature profile. It is because of fact that, the increasing Nr enhances the heat generation process and hence increases the thermal boundary layer thickness. Also, it is perceived that, initially all the thermal curves are coinciding with each other, later gradually increases and scatter with respect to time, attain slightly temporal maxima as well as minima, and then increases slowly there after reaches the steady state. The time required to attain the steady state decays with rising values of Nr .

Steady state temperature profile for varying values of Nr is shown in the Figure 9(b) at $X = 1$ against R . Also, the temperature profile generated from the heated cylinder wall at $T = 1$ and diminished to zero in radial path irrespective of t . Further, the steady state thermal field amplifies with enhancing Nr .

Concentration profile: Figure 10(a) shows the time-dependent mass diffusion profile at the location (1, 1.06) for various values of Nr . It is inspected that, the concentration field decayed for rising values of Nr . Since, the thermal radiation process produces the heat by increasing Nr consequently the concentration field decreases by rising the Nr . Further, the Figure 10(b) disclose the time-independent mass diffusion field for various values of Nr

at $X = 1$ verses R . The concentration profile begins from the hot wall of the cylinder at $C = 1$ and diminished to zero in radial path. Also, the steady state concentration profile decreases with increasing Nr values.

(iv) Influence of chemical reaction parameter (Kr) on velocity and concentration profiles

Velocity profile: Figure 11(a) indicates the impact of Kr on velocity profile at location $(1, 1.3)$. Initially all velocity curves rising with respect to time, attains its maximum, then declines, again enhances and there after accomplishes the stable state. Initially when $t \ll 1$, the thermal flow is because of conduction effect. Immediately after sometimes, there exists a convective heat transfer process and which dominates the conduction phenomena. It is observed that for rising the values of Kr , yields the diminished mass diffusion process, which in turns decays the values of BuC in Eq.(12), due to this fact, the acceleration reduces. Hence, velocity profile diminished for the rising values of Kr .

Figure 11(b) discloses the time-independent behaviour of simulated velocity field verses R at $X = 1$, at which flow curves begins with zero at cylinder, attains their highest value, then monotonically decayed to zero irrespective of time in R -path. Magnitude of the velocity field decreases with rising Kr , since rising Kr results the reduced mass diffusion process near the cylinder surface (ref. Eq. 14), and this fact decelerate the flow field (ref. Eq. (12)). Hence, steady-state velocity diminished with rising Kr .

Concentration profile: Figure 12(a) shows the effect of Kr on transient simulated concentration field at the location $(1, 1.06)$. Where the concentration profiles initially near the wall dominated by conduction process, later attains temporal maxima as well as minima, then marginally enhances and shortly later touches to time-independent state. Temporal maxima attains early for smaller Kr values. Species diffusion field declined with amplifying Kr , since higher values of Kr decreases the mass diffusion process in the flow domain (ref. Eq. (14)).

Simulated time-independent mass diffusion profile for various values of Kr at $X = 1$ along R path is described in Figure 12(b). The mass diffusion profile begins from the heated cylinder surface at $C = 1$ and monotonically diminished to zero. Further, as Kr rises, the species concentration profile decreases.

(v) Effect of buoyancy ratio parameter (Bu) on velocity profiles

Figures 13(a) and 13(b) describes the influence of buoyancy ratio parameter on velocity field. It is recorded from these figures that, the both transient and steady velocity profiles increases with increasing values of Bu , this is due to the fact that, the increasing Bu enhances buoy-

ancy forces and which in turn accelerates the flow field in the flow regime.

(vi) Effect of Eckert number (Ec) on temperature profiles

Figures 14(a) and 14(b) describes the influence of Eckert number on thermal field. It is recorded from these figures that, the both transient and steady temperature fields enhanced with rising values of Ec , this is due to the fact that, the increasing Ec enhances frictional forces and which in turn accelerates the flow field in the flow regime. The physical justification behind this fact is that the presence of frictional forces in the fluid causes the release of heat energy into the fluid which gives the intensified temperature field in the flow region.

4.3 Influence of magnetic parameter on physical quantities of interest

Figure 15(a) reveals impact of magnetic parameter on transient average $\bar{C}_f$. This figure discloses that, enhancing magnetic variable diminished the skin-friction coefficient in the flow domain. Figure 15(b) describes the action of M on average Nusselt number ($\bar{Nu}$) and it is known that, $\bar{Nu}$ is the ratio of heat transfer due to convection to the heat transfer due to conduction, if its value is one, it reflects the thermal transfer by pure conduction. Figure 15(b) discloses that, amplifying M decays the $\bar{Nu}$ in the flow domain. Further, the Figure 15(c), displays the effect of M on average Sherwood number ($\bar{Sh}$). However, $\bar{Sh}$ is the ratio of convective mass transfer to the diffusive mass transport process. It is recorded from Figure 15(c) that, the rising M diminishes the $\bar{Sh}$ in the flow domain.

4.4 Contours of the flow fields

Figures 16(a) and 16(b) disclose the impact of M on the steady-state velocity contours in the two-dimensional domain. Figures 16(a) and 16(b) demonstrate that, velocity field of couple stress fluid for $M = 0.0$ is higher when compared to $M = 0.2$, it is due to Lorentz's effect. Further, it is observed that, as M increases, the velocity contours of couple stress fluid moves away from the heated cylindrical surface. Figures 17(a) and 17(b) describe the stable state thermal contours for different M values. Thermal field of couple stress fluid for $M = 0.0$ is less when equated to $M = 0.2$. Further, for the enhancing M , the thermal contours moves near to heated cylinder. Figure 18(a) and 18(b) shows the steady-state concentration contours for differ-

ent values of M . It is noticed that, as M increases, mass diffusion contours of couple stress fluid moves far from the heated cylindrical wall.

5 Concluding remarks

In the current investigation, authors obtained the simulated results on the boundary layer flow of viscous incompressible non-Newtonian unsteady magnetized thermally radiative couple stress fluid flow about a cylinder with viscous dissipation effect under the influence of porous medium and chemical reaction. The system of governing nonlinear non-dimensional equations with required conditions are simplified by employing the implicit FDM. Solutions are obtained for different values of magnetic field, porous medium parameter, thermal radiation parameter and chemical reaction parameter. From the present study following observations are made in limiting sense:

- Velocity profile decreases, whereas temperature and concentration profiles increase for amplifying M .
- Velocity field increases and temperature profile decreases with rising K_1 values.
- Velocity and temperature profiles increase whereas concentration profile decreased with enhancing values of Nr .
- Velocity and concentration profiles decreased with rising values of Kr .
- Momentum, thermal and species transport rates decreased for amplifying values of M .
- Velocity profile increases with increase buoyancy ratio parameter.
- Temperature enhanced with rising Eckert number.
- Time-independent state flow and mass diffusion contours deviates away from the cylinder surface, whereas, the temperature contours lies near to heated cylinder for the rising M .

However, it is anticipated that, the present numerical investigation provides the motivation for numerical modelling of two and three dimensional heat and mass transfer problems occurring in the field of science and engineering. Further, the present paper gives a motivation for the modeling of couple stress fluid flow problems past a horizontal cylinder, sphere, cone and stretching cylinder as well as sheets with various physical aspects. Also, the present problem can be extended to study the problems occurring in the field of bioengineering and medicine, solar collectors, nonmaterial synthesis, thermal

and chemical industries, etc.

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List of symbols:

u – velocity along the vertical axis
 v – velocity along the radial direction
 t' – time
 t – non-dimensional time
 T' – temperature
 T – dimensionless temperature
 C' – concentration
 C – dimensionless concentration
 K'_1 – permeability parameter of the medium
 K_1 – dimensionless Porous parameter
 Ec – Eckert number
 q_r – radiative heat flux
 K'_r – Coefficient of chemical reaction
 Kr – Dimensionless chemical reaction parameter
 B_0 – Magnetic field
 g – Gravitational field
 Gr_T, Gr_C – thermal and solute Grashof numbers
 M – magnetic number
 Bu – combined buoyancy ratio parameter
 r_0 – radius of the cylinder
 Sc – Schmidt number
 Pr – Prandtl number
 Nr – thermal radiation parameter
 k – thermal conductivity
 r – radial coordinate
 x – axial coordinate
 U, V – dimensionless velocities

X – non-dimensional axial coordinate
 R – non-dimensional radial coordinate
 ∇^4 – Biharmonic or Bi-Laplacian operator

Greek symbols:

α – thermal diffusive coefficient
 β_T – thermal expansion coefficient
 β_C – mass diffusion expansion coefficient
 μ – dynamic viscosity
 ν – kinematic viscosity
 ρ – density
 σ – electrical conductivity of the fluid
 η – material constant

Subscripts:

w – wall condition
 ∞ – ambient condition

Superscripts:

n – time step level

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