Research Article

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Finite element method for stress and strain analysis of FGM hollow cylinder under effect of temperature profiles and inhomogeneity parameter

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Abstract: This study represents a numerical analysis of stress and strain in the functionally graded material (FGM) hollow cylinder subjected to two different temperature profiles and inhomogeneity parameter. The thermo-mechanical properties of a cylinder are assumed to vary continuously as power law function along the radial coordinate of a cylinder. Based on equilibrium equation, Hooke's law, stress-strain relationship in the cylinders, and other theories from mechanics second order differential equation is obtained that represents the thermoelastic field in hollow FGM cylinder. To find a numerical solution of governing differential equation, the finite element method (FEM) with standard discretization approach is used. The analysis of numerical results reveals that stress and strain in the FGM cylinder are significantly depend upon variation made in temperature profile and inhomogeneity parameter n . The results show good agreement with results available in the literature. It is shown that thermoelastic characteristics of the FGM cylinder are controlled by controlling the value of the above discussed parameters. Moreover, these results are very useful in various fields of engineering and science as FGM cylinders have a wide range of applications in these fields.

Keywords: Cylinder, stress, strain, temperature profile, FEM

Notations

r	Radial coordinate
r_a	Inner radius of a cylinder
r_b	Outer radius of a cylinder
T	Temperature profile
T_0	Constant temperature
α	Coefficient of thermal expansion
E	Coefficient of modulus of elasticity
n	Inhomogeneity parameter
α_0	Constant of thermal expansion
E_0	Constant of modulus of elasticity
σ_r	Radial stress
σ_θ	Circumferential stress
ε_r	Radial strain
ε_θ	Circumferential strain
ε_z	Axial strain
σ_z	Axial stress
ν	Poisson's ratio
φ	Stress function
u	Displacement

1 Introduction

Functionally graded materials were developed by a group of researchers in Japan by taking thermo-mechanical properties varying through thickness. FGM's are composite non-homogeneous materials obtained by mixing two or more materials [1]. Due to the toughness, strength and varying composition of these materials, these materials are used in the construction of various solid structures (e.g. disk, sphere, cylinder and beam etc.). FGM's structures work very effectively under higher environmental temperature, internal-external pressure and nonhomogeneity parameters. In the last few years, researchers are doing many experimental and theoretical studies to find the thermoelastic characteristics of solids that construct from functionally graded materials. Ngueyep *et al.* [2] developed an analytical

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and numerical solution in FGM drilling wicks or rotating cutter under axial load. To find stress field, deformation and displacement the material properties were assumed to vary in a radial direction according to power law distribution. Nkene *et al.* [3] shown the effect of the inhomogeneity parameter on stress, strain and displacement in rotating the FGM hollow cylinder under radially varying material properties. Dai *et al.* [4] used the infinitesimal theory of magneto-thermoelasticity to make stress analysis in the FGM hollow cylinder for different values of inhomogeneous constants. Habib *et al.* [5] represented radial stress, hoop stress and longitudinal stress theoretically under exponentially varying material properties in the FGM cylinder. Manthana *et al.* [6] studied heat conduction and thermal stress in a hollow cylinder in form of Bessel's and trigonometric function under the influence of nonhomogeneous material properties. Thakare and Warbhe [7] investigated temperature and stress distribution in a thick hollow cylinder for inhomogeneous material properties under heat exchange boundary conditions at curved surface. Abrinia *et al.* [8] obtained radial and circumferential stresses under an effect of internal pressure and temperature in FGM thick cylinder for power law distribution for material properties. Evci and Gulgec [9] developed an analytical solution for stress and displacement by using an airy stress function in FGM hollow cylinder. This solution was presented under influence of uniform heat generation and internal pressure. Rahimi and Nejad [10] found an exact solution to present thermal stresses in a hollow rotating thick-walled FGM cylinder for different values of material properties by using the theory of elasticity under the effect of internal and external pressure. Sharma *et al.* [11–13] made thermoelastic characteristics analysis with help of finite element method in FGM disk under the influence of Kibel number, thickness, angular speed and temperature distributions under variable material properties. Neghdabadi and Kordkheili [14] developed finite element analysis to represent thermoelastic characteristics in FGM plates and shells. Go [15–17] investigated temperature distribution, strain and stress in a circular disk under the effect of angular velocity, contact force and thickness by using finite element and finite volume method. Sharma *et al.* [18–21] studied thermoelastic characteristics in the FGM disk and a cylinder with the help of a finite element method under exponentially and linearly varying material properties. The analysis was carried out under different types of temperature profiles, grading index and boundary conditions. Sarkar and Rahman [22] employed the finite difference method to study thermo-elastic response in an FGM disk under steady state and transient state temperature distribution. Farhan *et al.* [23] obtained displacement, temperature and stress numerically under thermal

and mechanical boundary conditions for an isotropic circular cylinder. Nejad *et al.* [24] derived an exact solution for rotating functionally graded cylindrical pressure vessels for nonlinear material properties that vary along the radial direction. Furthermore, in this study temperature distribution was obtained from one dimensional heat conduction equation. Bidgoli *et al.* [25] investigated stress and strain distribution in FG rotating cylindrical shells under the effect of external-internal pressure, surface shear stress and constant temperature. Saadtfar [26] made an analysis in a piezomagnetic rotating thick-walled cylinder under the effect of angular velocity, hygrothermal loading and moisture concentration. Vaziri *et al.* [27] made stress analysis in the FGM cylinder using different methods (separation of variables, generalized Bessel's function and an eigen function method) under power law variation in mechanical and thermal properties. Bayat *et al.* [28] investigated stress and displacement in a functionally graded rotating disk subject to uniform, parabolic or hyperbolic thickness profiles that subjected to a steady state temperature profile. Nejad *et al.* [29] obtained an analytical solution in exact closed form for the FGM cylindrical pressure vessels under plane strain condition for exponential varying material properties. Peng and Lai [30] analyzed steady thermal stresses by converting the thermoelastic problem into Fredholm integral equation for the functionally graded hollow cylinder. Yeo *et al.* [31] obtained temperature distribution, stress and displacement by recursive method for the multilayered hollow cylinder. Atli and Lak [32] found stresses, strains and displacement in a functionally graded piezoelectric hollow cylinder for elastic-plastic material properties and internal and external pressure. Wang [33] divided transient solution into two parts to done transient thermal analysis in a functionally graded hollow cylinder. Tanvir *et al.* [34] developed analysis in the FGM cylinder to the show effect of internal pressure, temperature, wall thickness and material distribution on stress and strain. Garg [34] carried out elastic stress and strain in rotating FGM disk for different values of thickness gradation index under variable material properties in a radial direction. Saadatfar [35] analyzed time dependent stress distribution in a thick-walled rotating cylinder under the condition of plane strain. Saeedi *et al.* [36] used the successive approximation method to study thermo-elasto-plastic behavior of the thick-walled cylindrical shells subjected to internal pressure and temperature gradient. Abdalla *et al.* [37] presented thermo-mechanical stresses in rotating hollow disks under variable thickness by using the finite element method. Thawait *et al.* [38] reported deformation and stress field in the two types of FGM disks under three types of thickness profile namely power law, exponential law and mori-tanaka scheme. Hu *et al.*

[39] analyzed nonlinear vibration under the effect of rotating speed, surface temperature, volume fraction index and initial condition in FGM ring plate in which material properties vary with power law distribution along a radial direction. Bezzie and Woldmichael [40] carried out the numerical solution to the shown effect of graded index and Poisson's ratio on stress, strain and displacement in FGM thick-walled cylinder. Magar and Lal [41] used a complex variable method to present stress distribution in an infinite laminated composite plate under the influence of fibre angle, plane loading, the volume fraction of fibre, temperature and environmental conditions. Dhotre and Srinivasa [42] applied the finite element method to present analysis in the sandwich skew plates under the effect of variable parameters. Yadav and Jiwari [43] used the finite element method to check the existence and uniqueness of a weak solution of Burger's fisher's equation. Alimoradzadeh *et al.* [44] made nonlinear vibration analysis with the help of Galerkin's in the axially functionally graded beam under power law variation in material properties. In the present study, thermoelastic characteristics are obtained for the FGM cylinder due to its wide range of applications in the field of Engineering and Science. The material properties of a cylinder are assumed to vary in accordance with power law distribution along the radius of the cylinder. The effect of inhomogeneity parameter and temperature are disused on stress, strain and displacement. Thus, the results reveal that thermoelastic filed in an FGM cylinder is controlled by controlling the above-mentioned parameters.

2 Problem formulation

Let us assume, a FGM hollow cylinder with r_a (inner) and r_b (outer) radius of cylinder respectively. For a FGM hollow cylinder equilibrium equation in terms of cylindrical coordinates is written as:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (1)$$

By using generalized Hooke's Law, stresses and strains are related as:

$$\varepsilon_r = \frac{1}{E} [\sigma_r - \nu(\sigma_\theta + \sigma_z)] + \alpha T \quad (2)$$

$$\varepsilon_\theta = \frac{1}{E} [\sigma_\theta - \nu(\sigma_r + \sigma_z)] + \alpha T \quad (3)$$

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_\theta + \sigma_r)] + \alpha T \quad (4)$$

For the FGM cylinder with free ends, it is assumed that axial strain is independent of the radial coordinate. So, the

value of axial strain becomes constant. Therefore, Eq. (4) becomes:

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_\theta + \sigma_r)] + \alpha T = \varepsilon_0 \quad (5)$$

From Eq. (5), the Axial stress component can be written as:

$$\sigma_z = \nu(\sigma_r + \sigma_\theta) + (\varepsilon_0 - \alpha T)E \quad (6)$$

Eqs. (3) and (4) after putting a value of axial stress, will take the form as written below:

$$\varepsilon_r = \frac{1}{E} [(1 - \nu^2)\sigma_r - \nu(1 + \nu)\sigma_\theta + (1 + \nu)E\alpha T - \nu\varepsilon_0 E] \quad (7)$$

$$\varepsilon_\theta = \frac{1}{E} [(1 - \nu^2)\sigma_\theta - \nu(1 + \nu)\sigma_r + (1 + \nu)E\alpha T - \nu\varepsilon_0 E] \quad (8)$$

The equilibrium equation in terms of stress function can be written as:

$$\sigma_r = \frac{\varphi}{r}, \quad \sigma_\theta = \frac{d\varphi}{dr} \quad (9)$$

The radial and circumferential strain components are related to radial coordinates as written below:

$$\varepsilon_r = \frac{du}{dr}, \quad \varepsilon_\theta = \frac{u}{r} \quad (10)$$

By using the relation between strain components, we obtain a differential equation as given below:

$$\frac{\varepsilon_r - \varepsilon_\theta}{r} - \frac{d\varepsilon_\theta}{dr} = 0 \quad (11)$$

2.1 Material properties

Here we consider that thermo-mechanical material of cylinder properties follow the power law variation along a radial coordinate of a cylinder. The material properties modulus of elasticity E and coefficient of thermal expansion α are given by:

$$E(r) = E_0 \left(\frac{r}{r_a} \right)^n \quad (12)$$

$$\alpha(r) = \alpha_0 \left(\frac{r}{r_a} \right)^n \quad (13)$$

Where exponent n can be a positive or negative real number reflecting the degree of non-uniformity of the material.

2.2 Temperature profiles

The following two temperature profiles are assumed to find thermoelastic characteristics in a cylinder:

2.2.1 Uniform temperature profile

In this case, the temperature remains constant then the temperature profile is given as:

$$T(r) = T_0, \quad \frac{dT}{dr} = 0 \quad (14)$$

2.2.2 Steady state temperature profile

In this case, the temperature is independent of time then the temperature profile is given by:

$$T(r) = r_b T_0 \log\left(\frac{r}{r_a}\right), \quad \frac{dT}{dr} = r_b \frac{T_0}{r} \quad (15)$$

2.3 Differential equation of problem

From Eq. (11), with the help of Eqs. (7) and (8) and two temperate profiles we drive differential equations as written below:

2.3.1 In case of uniform temperature profile

$$\frac{d^2\varphi}{dr^2} - \frac{A}{r} \frac{d\varphi}{dr} + \frac{B}{r^2} \varphi = Pr^{2n-1} \quad (16)$$

2.3.2 In case of steady state temperature profile

$$\frac{d^2\varphi}{dr^2} - \frac{A}{r} \frac{d\varphi}{dr} + \frac{B}{r^2} \varphi = P \left(r^{2n-1} + nr^{2n-1} \log\left(\frac{r}{r_a}\right) \right) \quad (17)$$

Where values A , B and P are given as:

$$A = (n-1), \quad B = \frac{\nu(n+1)-1}{1-\nu}, \quad P = \frac{-E_0 T_0 \alpha_0 n}{(r_a)^{2n}(1-\nu)}$$

3 Finite element formulation

In this part of the study, we find a solution to governing differential equations by using the finite element method. To achieve numerical results, a finite element with a standard discretization approach is used here. We divide the total domain into N elements of equal length. After the implementation of FEM, the problem is converted into N algebraic equations. The algebraic equations are of the form:

$$\sum_{j=1}^2 K_{ij}^e F_j^e = L_i^e; \quad i = 1, 2; \quad e = 1, 2, 3, \dots, n \quad (18)$$

Where the value of K_{ij}^e and L_i^e are obtained from:

$$K_{ij}^e = \left[\varphi_i \frac{d\varphi_j}{dr} \right]_{r_e}^{r_{e+1}} - \int_{r_e}^{r_{e+1}} \frac{d\varphi_i}{dr} \frac{d\varphi_j}{dr} dr \quad (19)$$

$$- A \int_{r_e}^{r_{e+1}} \frac{\varphi_i}{r} \frac{d\varphi_j}{dr} dr$$

$$+ B \int_{r_e}^{r_{e+1}} \frac{\varphi_i \varphi_j}{r^2} dr$$

$$L_i^e = P \int_{r_e}^{r_{e+1}} \varphi_i r^{2n-1} dr \quad (20)$$

(In case of uniform temperature profile)

$$L_i^e = P \int_{r_e}^{r_{e+1}} \varphi_i r^{2n-1} \left(1 + n \log\left(\frac{r}{r_a}\right) \right) dr \quad (21)$$

(In case of steady state temperate profile)

4 Numerical results

In this section, we study the influence of temperature profiles and inhomogeneity parameter n on stress and strain. To find numerical results, the value of the inhomogeneity parameter n is taken between -1 and 1 . The value of material constants is taken as:

$$\alpha_0 = 4.9 \times 10^{-6} (K^{-1}), \quad E_0 = 330 (GPa), \quad \nu = 0.3$$

4.1 Distribution of material properties for different values inhomogeneity parameter

Figure 1, presents a variation of modulus of elasticity against radial coordinate against different values of inhomogeneity parameter under uniform temperature distribution. From Figure 1(a), it is cleared that, there is a direct relation between r and E i.e. as the value of r increases E also increases. The highest and lowest value of modulus of elasticity is obtained corresponding to the highest and lowest values of n respectively. As Figure 1(b) shows that modulus of elasticity curves decreases as r increases. The profile of coefficient of thermal expansion shows the same behavior as the profile of modulus of elasticity as cleared from Figure 2. From Figure 2(a), it is observed that the value

Figure 1: Represents variation in modulus of elasticity versus radial coordinate corresponding to positive and negative values of inhomogeneity parameter

Figure 2: Represents variation in Coefficient of thermal expansion versus radial coordinate corresponding to positive and negative values of inhomogeneity parameter

of coefficient of thermal expansion increases as the value of n and r increases. From Figure 2(b), we check that the maximum value of α is obtained corresponding to $r = 0.2$ and $n = -0.3$. Also, both material properties are more elastic at the most outer surface of a cylinder.

4.2 Stress and strain in case of uniform temperature distribution

From Figure 3(a), it is cleared that positive values of radial stress increase gradually with r . As shown in Figure 3(b), when $0.2 \leq r \leq 0.7$ maximum radial stress is obtained for $n = -0.3$ and for remaining values of r maximum radial stress is corresponding to $n = -0.9$. Also, curves are more tensile when $0.2 \leq r \leq 0.7$ and compressive for resting values r . From Figure 4(a), it is observed that the behavior

of the circumferential stress curve changes from linear to nonlinear as the value of n increases. The circumferential stress profiles show decreasing behaviour with respect to radial coordinate. Figure 4(b) displays that Maximum variation in behaviour of circumferential stress curve occurs for $n = -0.9$ and maximum value of circumferential stress is obtained at the most inner surface of a cylinder. Circumferential stress curves are opposite in nature for $n = -0.9$ and other negative values of n . From Figures 5(a) and 5(b), it is cleared that axial stress profiles are more tensile at a most outer surface of a cylinder. Also, axial stress curves are increasing for positive values of n but curves show decreasing behaviour corresponding to different negative values of inhomogeneity parameter. The maximum magnitude of radial strain is obtained for the least value of n as indicates in Figure 6(a). The maximum and minimum value of radial strain exists for $n = -0.3$ and $n = -0.9$ when $0.1 \leq r \leq 0.4$

Figure 3: Variation in radial stress against radial coordinate for different values of n

Figure 4: Variation in circumferential stress against radial coordinate for different values of n

Figure 5: Variation in axial stress against radial coordinate for different values of n

Figure 6: Variation in radial strain against radial coordinate for different values of n

Figure 7: Variation in circumferential strain against radial coordinate for different values of n

Figure 8: Radial stress versus radial coordinate for various values of n under steady state temperate distribution

Figure 9: Circumferential stress versus radial coordinate for various values of n under steady state temperate distribution

Figure 10: Axial stress versus radial coordinate for various values of n under steady state temperate distribution

Figure 11: Radial strain versus radial coordinate for various values of n under steady state temperate distribution

Figure 12: Circumferential stress versus radial coordinate for various values of n under steady state temperature distribution

but this behaviour is changed at most outer surface of a cylinder. The Circumferential strain is more tensile at the outer surface of the cylinder for positive values of n but for negative values of n it is more tensile at the inner surface of a cylinder. The results obtained in this section show good agreement with the results given by Rahimi and Nejad [10].

4.3 Thermoelastic characteristics in case of steady state temperature distribution

From Figure 8(a) it is clear that there is an inverse relation between radial stress and n i.e. as n increases radial stress decreases. From Figure 8(b) it is clear that behaviour of radial stress curve is different at $n = -0.5$ because at $n = -0.3$, $n = -0.7$, $n = -0.9$ radial stress decreases as r increases but for $n = -0.5$ curve shows increasing behaviour w.r.t. r . Figure 9 and 10 presents a variation in circumferential and axial stress against radial coordinate for steady state temperature profile. The maximum value of circumferential stress is obtained when $n = |0.3|$. From Figure 10(b) it is clear that the maximum value of axial stress occurs at $n = -0.5$ for outer surface of a cylinder. Radial strain curve behaviour is increasing at $n = -0.5$ but at $n = -0.3$, $n = -0.7$, $n = -0.9$ behaviour is decreasing w.r.t. r . From Figure 12, it is cleared that, circumferential strain curves are more tensile for the outer surface of a hollow cylinder.

5 Conclusion

In this study, strains and stresses are analyzed in an inhomogeneous hollow cylinder under a uniform and steady state temperature profile. The significant effect of the in-

homogeneity parameter is studied on the thermoelastic characteristics of a cylinder. The following conclusions are drawn from the study:

- Modulus of elasticity curve shows increasing behaviour for positive values of n and the decreasing behaviour for negative values of n against radial coordinate.
- The coefficient of thermal expansion profile is obtained a maximum value at the outer surface for the positive value of n and for negative values of n the maximum value is obtained at the inner surface of a cylinder.
- In uniform and steady state temperature profiles radial stress, circumferential stress and radial strain are the maximum value when $n = 0.3$ for all positive values of n .
- The Behaviour of radial stress, circumferential stress and radial strain curves at $n = -0.5$ under a steady state temperature profile are different for rest of the negative values of n .
- Circumferential strain curves are interesting at one common value of radial coordinate for positive values of inhomogeneity parameter in case of steady state temperature distribution.

The results obtained in this work are useful as these results give contribution for designing and modeling of rotating cylinders and cutters.

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