Research Article

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Novel solitons solutions of two different nonlinear PDEs appear in engineering and physics

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Abstract: In this piece of research, our aim is to investigate the novel solitons solutions of nonlinear (4+1)-dimensional Fokas equation (FE) and (2+1)-dimensional Breaking soliton equation (BSE) via new extended direct algebraic method. New acquired solutions are bright, singular, dark, periodic singular, combined dark-bright and combined dark-singular solitons along with hyperbolic and trigonometric functions solutions. We achieved different kinds of solitons solutions contain key applications in engineering and physics. By taking the appropriate values of these parameters, numerous novel structures are also plotted. These solutions define the wave performance of the governing models, actually. Furthermore, the physical understanding of the acquired solutions is revealed in forms of 3-D, 2-D and contour graphs for different appropriate parameters. From results, we conclude that the applied computational method is straight, talented and can be applied in more complex phenomena for such models.

Keywords: (4+1)-dimensional Fokas equation; (2+1)-dimensional Breaking soliton equation; new extended direct algebraic method

1 Introduction

A large number of existing phenomena in science and engineering are connected to non-linear partial differential equations (NLPDEs). These are much significant in explaining everyday problems growing in science and nature such as waves, geology, population ecology, solid-

state physics, wave propagation, fluid dynamics, biology, computer science and birefringent fibers, etc. Many mathematical approaches have been efficiently applied to study the valuable results of NLPDEs such as the Kudryashov's method [1], homogenous balance method [2], the tanh method [3], the sine-cosine method [4], modified extended tanh-function method [5], the extended and improved F-expansion method [6], the inverse scattering method [7], the Backlund transformation method [8, 9], kernel Hilbert space method [10] and so on [11–41]. Using these approaches, various novel properties of wave behavior of these NLPDEs have been observed. The observed results have much impact in many scientific phenomena such optics, optical fibers, long distance high speed transmission lines and in optics as temporal or spatial optical solitons. Looking for the exact solutions of NLPDEs has the importance to measure the solidity of numerical solutions. Exact solutions to NLPDEs play a dynamic role in non-linear sciences, meanwhile they can deal much physical info and extra understanding of the physical constructions of the problem. Moreover, many typical researchers are giving more consideration to determine and rise the optical transmission situations using optical fibers. Also, the transmission of soliton through optical fibers directs next generation technology. So, the study of optical soliton is one of the most exciting and interesting regions of investigation in nonlinear optics. In this paper, we have constructed novel dark, singular, bright, periodic and mixed solitons solutions to (4+1)-dimensional Fokas equation (FE) and (2+1)-dimensional Breaking soliton equation (BSE) by applying new extended direct algebraic method (EDAM) [42, 43]. The (4+1)-dimensional FE is given by [44]

$$4\Phi_{tx} - \Phi_{xxxxy} + \Phi_{xyyy} + 12\Phi_x\Phi_y + 12\Phi\Phi_{xy} - 6\Phi_{zw} = 0. \quad (1)$$

Here, Φ is a function of x, y, z, w and t . This model has been discovered from the Kadomtsev-Petviashvili (KP) and Davey-Stewartson (DS) equations, which have been frequently used to indicate internal and shallow waves in the canals [45]. One another model, deliberated in this paper, (2+1)-dimensional Breaking soliton equation (BSE) stated as which is used to explain the collaboration of propagating

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Riemann waves, which was first introduced by Degasperis and Calogero in 1977. This model was explored by considering HDM and so on [46–49].

$$\Phi_{xxxxy} - 2\Phi_y\Phi_{xx} - 4\Phi_x\Phi_{xy} + \Phi_{xt} = 0. \quad (2)$$

This paper is organized as: In section 2 the description of the new extended direct algebraic method is described. Section 3 contains the application of the proposed method for the solutions of Fokas equation and Breaking soliton equation. Section 4 contains results and discussion while conclusion of this work is given in Section 5.

2 Description of the new EDAM

The steps of new EDAM [42, 43] are, as follows

Step 1. Let the PDE

$$G(\Phi, \Phi_x, \Phi_t, \Phi_{tx}, \Phi_{xx}, \Phi_{xt}, \Phi_{tt}, \dots) = 0, \quad (3)$$

where $\Phi = \Phi(x, t)$ is an unknown function. Put on the wave transformation

$$\Phi(x, t) = P(\xi), \quad \xi = \mu(x - ct), \quad (4)$$

(3) converted into ODE as follows

$$H(P, P', P'', \dots) = 0. \quad (5)$$

Step 2. Consider (5) has a solution as follows

$$P(\xi) = \sum_{j=0}^N F_j Q^j(\xi), \quad F_N \neq 0, \quad (6)$$

where F_j ($0 \leq j \leq N$) are constants and $Q(\xi)$ admits the ODE, as follows

$$Q'(\xi) = Ln(A)(\delta + \theta Q(\xi) + \varsigma Q^2(\xi)), \quad A \neq 1, 0. \quad (7)$$

(7) gives the solution as

Family 1: When $\theta^2 - 4\delta\varsigma < 0$ and $\varsigma \neq 0$,

$$Q_1(\xi) = -\frac{\theta}{2\varsigma} + \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \tan_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2} \xi \right),$$

$$Q_2(\xi) = -\frac{\theta}{2\varsigma} - \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \cot_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2} \xi \right),$$

$$Q_3(\xi) = -\frac{\theta}{2\varsigma} + \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \tan_A \left(\sqrt{-(\theta^2 - 4\delta\varsigma)} \xi \right)$$

$$\pm \frac{\sqrt{-pq(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \sec_A \left(\sqrt{-(\theta^2 - 4\delta\varsigma)} \xi \right),$$

$$Q_4(\xi) = -\frac{\theta}{2\varsigma} - \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \cot_A \left(\sqrt{-(\theta^2 - 4\delta\varsigma)} \xi \right) \\ \pm \frac{\sqrt{-pq(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \csc_A \left(\sqrt{-(\theta^2 - 4\delta\varsigma)} \xi \right),$$

$$Q_5(\xi) = -\frac{\theta}{2\varsigma} + \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{4\varsigma} \tan_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{4} \xi \right) \\ - \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{4\varsigma} \cot_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{4} \xi \right).$$

Family 2: When $\theta^2 - 4\delta\varsigma > 0$ and $\varsigma \neq 0$,

$$Q_6(\xi) = -\frac{\theta}{2\varsigma} - \frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \tanh_A \left(\frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{2} \xi \right),$$

$$Q_7(\xi) = -\frac{\theta}{2\varsigma} - \frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \coth_A \left(\frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{2} \xi \right),$$

$$Q_8(\xi) = -\frac{\theta}{2\varsigma} - \frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \tanh_A \left(\sqrt{(\theta^2 - 4\delta\varsigma)} \xi \right) \\ \pm \frac{\sqrt{pq(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \operatorname{sech}_A \left(\sqrt{(\theta^2 - 4\delta\varsigma)} \xi \right),$$

$$Q_9(\xi) = -\frac{\theta}{2\varsigma} - \frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \coth_A \left(\sqrt{(\theta^2 - 4\delta\varsigma)} \xi \right) \\ \pm \frac{\sqrt{pq(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \operatorname{csch}_A \left(\sqrt{(\theta^2 - 4\delta\varsigma)} \xi \right),$$

$$Q_{10}(\xi) = -\frac{\theta}{2\varsigma} - \frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{4\varsigma} \tanh_A \left(\frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{4} \xi \right) \\ - \frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{4\varsigma} \coth_A \left(\frac{\sqrt{(\theta^2 - 4\delta\varsigma)}}{4} \xi \right).$$

Family 3: When $\delta\varsigma > 0$ and $\theta = 0$,

$$Q_{11}(\xi) = \sqrt{\frac{\delta}{\varsigma}} \tan_A \left(\sqrt{\delta\varsigma} \xi \right),$$

$$Q_{12}(\xi) = -\sqrt{\frac{\delta}{\varsigma}} \cot_A \left(\sqrt{\delta\varsigma} \xi \right),$$

$$Q_{13}(\xi) = \sqrt{\frac{\delta}{\varsigma}} \tan_A \left(2\sqrt{\delta\varsigma} \xi \right) \pm \sqrt{pq\frac{\delta}{\varsigma}} \sec_A \left(2\sqrt{\delta\varsigma} \xi \right),$$

$$Q_{14}(\xi) = -\sqrt{\frac{\delta}{\varsigma}} \cot_A \left(2\sqrt{\delta\varsigma}\xi \right) \pm \sqrt{pq\frac{\delta}{\varsigma}} \csc_A \left(2\sqrt{\delta\varsigma}\xi \right),$$

$$Q_{15}(\xi) = \frac{1}{2} \left(\sqrt{\frac{\delta}{\varsigma}} \tan_A \left(\frac{\sqrt{\delta\varsigma}}{2} \xi \right) - \sqrt{\frac{\delta}{\varsigma}} \cot_A \left(\frac{\sqrt{\delta\varsigma}}{2} \right) \right).$$

Family 4: When $\delta\varsigma < 0$ and $\Theta = 0$,

$$Q_{16}(\xi) = -\sqrt{-\frac{\delta}{\varsigma}} \tanh_A \left(\sqrt{-\delta\varsigma}\xi \right),$$

$$Q_{17}(\xi) = -\sqrt{-\frac{\delta}{\varsigma}} \coth_A \left(\sqrt{-\delta\varsigma}\xi \right),$$

$$Q_{18}(\xi) = -\sqrt{-\frac{\delta}{\varsigma}} \tanh_A \left(2\sqrt{-\delta\varsigma}\xi \right) \\ \pm \iota \sqrt{-pq\frac{\delta}{\varsigma}} \operatorname{sech}_A \left(2\sqrt{-\delta\varsigma}\xi \right),$$

$$Q_{19}(\xi) = -\sqrt{-\frac{\delta}{\varsigma}} \coth_A \left(2\sqrt{-\delta\varsigma}\xi \right) \\ \pm \sqrt{-pq\frac{\delta}{\varsigma}} \operatorname{csch}_A \left(2\sqrt{-\delta\varsigma}\xi \right),$$

$$Q_{20}(\xi) = -\frac{1}{2} \left(\sqrt{-\frac{\delta}{\varsigma}} \tanh_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) \right. \\ \left. + \sqrt{-\frac{\delta}{\varsigma}} \coth_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) \right).$$

Family 5: When $\Theta = 0$ and $\varsigma = \delta$,

$$Q_{21}(\xi) = \tan_A(\delta\xi),$$

$$Q_{22}(\xi) = -\cot_A(\delta\xi),$$

$$Q_{23}(\xi) = \tan_A(2\delta\xi) \pm \sqrt{pq} \sec_A(2\delta\xi),$$

$$Q_{24}(\xi) = -\cot_A(2\delta\xi) \pm \sqrt{pq} \csc_A(2\delta\xi).$$

$$Q_{25}(\xi) = -\frac{1}{2} \left(\tan_A \left(\frac{\delta}{2} \xi \right) + \cot_A \left(\frac{\delta}{2} \xi \right) \right).$$

Family 6: When $\Theta = 0$ and $\varsigma = -\delta$,

$$Q_{26}(\xi) = -\tanh_A(\delta\xi),$$

$$Q_{27}(\xi) = -\coth_A(\delta\xi),$$

$$Q_{28}(\xi) = -\tanh_A(2\delta\xi) \pm \iota \sqrt{pq} \operatorname{sech}_A(2\delta\xi),$$

$$Q_{29}(\xi) = -\coth_A(2\delta\xi) \pm \sqrt{pq} \operatorname{csch}_A(2\delta\xi),$$

$$Q_{30}(\xi) = -\frac{1}{2} \left(\tanh_A \left(\frac{\delta}{2} \xi \right) + \coth_A \left(\frac{\delta}{2} \xi \right) \right).$$

Family 7: When $\Theta^2 = 4\delta\varsigma$,

$$Q_{31}(\xi) = \frac{-2\delta(\Theta\xi \operatorname{Ln}(A) + 2)}{\Theta^2 \xi \operatorname{Ln}(A)}.$$

Family 8: When $\Theta = k$, $\delta = mk$ ($m \neq 0$) and $\varsigma = 0$,

$$Q_{32}(\xi) = A^{k\xi} - m.$$

Family 9: When $\Theta = \varsigma = 0$,

$$Q_{33}(\xi) = \delta\xi \operatorname{Ln}(A).$$

Family 10: When $\Theta = \delta = 0$,

$$Q_{34}(\xi) = \frac{-1}{\varsigma \xi \operatorname{Ln}(A)}.$$

Family 11: When $\delta = 0$ and $\Theta \neq 0$,

$$Q_{35}(\xi) = -\frac{p\Theta}{\varsigma(\cosh_A(\Theta\xi) - \sinh_A(\Theta\xi + p))},$$

$$Q_{36}(\xi) = -\frac{\Theta(\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi))}{\varsigma(\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi + q))}.$$

Family 12: When $\Theta = k$, $\varsigma = mk$ ($m \neq 0$) and $\delta = 0$,

$$Q_{37}(\xi) = \frac{pA^{k\xi}}{q - \mp A^{k\xi}}.$$

Note: The generalized hyperbolic and triangular functions are given as [50]

$$\sinh_A(\xi) = \frac{pA^\xi - qA^{-\xi}}{2}, \quad \cosh_A(\xi) = \frac{pA^\xi + qA^{-\xi}}{2},$$

$$\tanh_A(\xi) = \frac{pA^\xi - qA^{-\xi}}{pA^\xi + qA^{-\xi}}, \quad \coth_A(\xi) = \frac{pA^\xi + qA^{-\xi}}{pA^\xi - qA^{-\xi}},$$

$$\operatorname{sech}_A(\xi) = \frac{2}{pA^\xi + qA^{-\xi}}, \quad \operatorname{csch}_A(\xi) = \frac{2}{pA^\xi - qA^{-\xi}},$$

$$\sin_A(\xi) = \frac{pA^{i\xi} - qA^{-i\xi}}{2i}, \quad \cos_A(\xi) = \frac{pA^{i\xi} + qA^{-i\xi}}{2},$$

$$\tan_A(\xi) = -i \frac{pA^{i\xi} - qA^{-i\xi}}{pA^{i\xi} + qA^{-i\xi}}, \quad \cot_A(\xi) = i \frac{pA^{i\xi} + qA^{-i\xi}}{pA^{i\xi} - qA^{-i\xi}},$$

$$\sec_A(\xi) = \frac{2}{pA^{i\xi} + qA^{-i\xi}}, \quad \csc_A(\xi) = \frac{2i}{pA^{i\xi} - qA^{-i\xi}}.$$

where $p, q > 0$.

Step 3. The integer $N > 0$ can be determined by balancing rule in (3). Inserting (4) in (3), gets algebraic equations having powers of $Q^j(\xi)$ ($j = 0, 1, 2, \dots$) and equating the coefficients of powers of $Q(\xi)$ to zero, provides system of equations.

Step 4. Solve system of equations and putting results in (4) to retrieve the exact solutions of (1).

3 Applications of the new EDAM

3.1 New EDAM for (4+1)-dimensional Fokas equation

Here, new EDAM is applied to discover some new solutions of (1).

$$\Phi(x, y, z, t, w) = P(\xi) \quad \xi = \alpha x + \gamma y + \chi z + \tau w + \epsilon t, \quad (8)$$

Using (8) into (1), then the following ODE is obtained

$$(4\alpha\epsilon - 6\chi\tau)P'' + (\alpha\gamma^3 - \alpha^3\gamma)P^{(4)} + 12\alpha\gamma(PP')' = 0. \quad (9)$$

Integration (9) and gets the following equation

$$(4\alpha\epsilon - 6\chi\tau)P + (\alpha\gamma^3 - \alpha^3\gamma)P'' + 6\alpha\gamma P^2 = 0. \quad (10)$$

Using balancing rule on (10), it gives $N = 2$, which converts (6) as

$$P(\xi) = F_0 + F_1 Q(\xi) + F_2 (Q(\xi))^2, \quad (11)$$

where F_0, F_1 and F_2 are constants. Substituting (11) into (10) and equating the coefficients of polynomials of $Q(\xi)$ to zero, we yield a set of equations in F_0, F_1, F_2 and ϵ .

On solving the set of equations, we get

$$\begin{aligned} F_0 &= \frac{1}{6}(\alpha^2 - \gamma^2)(2\theta\lambda + \mu^2)Ln(A)^2, \\ F_1 &= (\alpha^2 - \gamma^2)\theta\mu Ln(A)^2, \\ F_2 &= (\alpha^2 - \gamma^2)\theta^2 Ln(A)^2, \\ \epsilon &= \frac{6\tau\chi + \alpha\gamma(\alpha^2 - \gamma^2)(4\theta\lambda - \mu^2)Ln(A)^2}{4\alpha}. \end{aligned} \quad (12)$$

From Eqs. (4), (11) and (12), we acquired new solutions of the model as follows.

Family 1: When $\theta^2 - 4\delta\varsigma < 0$ and $\varsigma \neq 0$, then

$$\begin{aligned} \Phi_1(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ &\quad \left. + 6\theta\mu \left(-\frac{\theta}{2\varsigma} - \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \right) \right. \end{aligned} \quad (13)$$

$$\begin{aligned} &\cdot \tan_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2} \xi \right) \\ &+ 6\theta^2 \left(-\frac{\theta}{2\varsigma} - \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \right. \\ &\cdot \tan_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2} \xi \right) \left. \right)^2, \end{aligned}$$

$$\begin{aligned} \Phi_2(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ &+ 6\theta\mu \left(-\frac{\theta}{2\varsigma} - \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \right. \\ &\cdot \cot_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2} \xi \right) \\ &+ 6\theta^2 \left(-\frac{\theta}{2\varsigma} - \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \right. \\ &\cdot \cot_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2} \xi \right) \left. \right)^2, \end{aligned} \quad (14)$$

$$\begin{aligned} \Phi_3(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ &+ 6\theta\mu \left(-\frac{\theta}{2\varsigma} + \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \right. \\ &\times \tan_A \left(\sqrt{-(\theta^2 - 4\delta\varsigma)}\xi \right) \\ &\pm \frac{\sqrt{-pq(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \\ &\sec_A \left(\sqrt{-(\theta^2 - 4\delta\varsigma)}\xi \right) \\ &+ 6\theta^2 \left(-\frac{\theta}{2\varsigma} + \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \right. \\ &\tan_A \left(\sqrt{-(\theta^2 - 4\delta\varsigma)}\xi \right) \\ &\pm \frac{\sqrt{-pq(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \\ &\sec_A \left(\sqrt{-(\theta^2 - 4\delta\varsigma)}\xi \right) \left. \right)^2, \end{aligned} \quad (15)$$

$$\begin{aligned} \Phi_4(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ &+ 6\theta\mu \left(-\frac{\theta}{2\varsigma} - \frac{\sqrt{-(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \right. \\ &\times \cot_A \left(\sqrt{-(\theta^2 - 4\delta\varsigma)}\xi \right) \\ &\pm \frac{\sqrt{-pq(\theta^2 - 4\delta\varsigma)}}{2\varsigma} \end{aligned} \quad (16)$$

$$\begin{aligned}
& \csc_A \left(\sqrt{-(\Theta^2 - 4\delta\zeta)} \xi \right) \\
& + 6\theta^2 \left(-\frac{\Theta}{2\zeta} - \frac{\sqrt{-(\Theta^2 - 4\delta\zeta)}}{2\zeta} \right. \\
& \cot_A \left(\sqrt{-(\Theta^2 - 4\delta\zeta)} \xi \right) \\
& \pm \frac{\sqrt{-pq(\Theta^2 - 4\delta\zeta)}}{2\zeta} \\
& \left. \csc_A \left(\sqrt{-(\Theta^2 - 4\delta\zeta)} \xi \right) \right)^2, \\
\Phi_5(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2) \ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(-\frac{\Theta}{2\zeta} + \frac{\sqrt{-(\Theta^2 - 4\delta\zeta)}}{4\zeta} \right) \\
& \times \tan_A \left(\frac{\sqrt{-(\Theta^2 - 4\delta\zeta)}}{4} \xi \right) \\
& - \frac{\sqrt{-(\Theta^2 - 4\delta\zeta)}}{4\zeta} \\
& \cdot \cot_A \left(\frac{\sqrt{-(\Theta^2 - 4\delta\zeta)}}{4} \xi \right) \\
& + 6\theta^2 \left(-\frac{\Theta}{2\zeta} + \frac{\sqrt{-(\Theta^2 - 4\delta\zeta)}}{4\zeta} \right) \\
& \cdot \tan_A \left(\frac{\sqrt{-(\Theta^2 - 4\delta\zeta)}}{4} \xi \right) \\
& - \frac{\sqrt{-(\Theta^2 - 4\delta\zeta)}}{4\zeta} \\
& \cdot \cot_A \left(\frac{\sqrt{-(\Theta^2 - 4\delta\zeta)}}{4} \xi \right) \left. \right)^2. \quad (17)
\end{aligned}$$

Family 2: When $\Theta^2 - 4\delta\zeta > 0$. and $\zeta \neq 0$, then

$$\begin{aligned}
\Phi_6(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2) \ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(-\frac{\Theta}{2\zeta} - \frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2\zeta} \right) \\
& \cdot \tanh_A \left(\frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2} \xi \right) \\
& + 6\theta^2 \left(-\frac{\Theta}{2\zeta} - \frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2\zeta} \right) \\
& \cdot \tanh_A \left(\frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2} \xi \right) \left. \right)^2, \quad (18)
\end{aligned}$$

$$\Phi_7(x, y, z, w, t) = \frac{1}{6}(\alpha^2 - \gamma^2) \ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \quad (19)$$

$$\begin{aligned}
& + 6\theta\mu \left(-\frac{\Theta}{2\zeta} - \frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2\zeta} \right) \\
& \cdot \coth_A \left(\frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2} \xi \right) \\
& + 6\theta^2 \left(-\frac{\Theta}{2\zeta} - \frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2\zeta} \right) \\
& \cdot \coth_A \left(\frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2} \xi \right) \left. \right)^2, \\
\Phi_8(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2) \ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \quad (20) \\
& + 6\theta\mu \left(-\frac{\Theta}{2\zeta} - \frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2\zeta} \right) \\
& \times \tanh_A \left(\sqrt{(\Theta^2 - 4\delta\zeta)} \xi \right) \\
& \pm \frac{\sqrt{pq(\Theta^2 - 4\delta\zeta)}}{2\zeta} \\
& \cdot \operatorname{sech}_A \left(\sqrt{(\Theta^2 - 4\delta\zeta)} \xi \right) \\
& + 6\theta^2 \left(-\frac{\Theta}{2\zeta} - \frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2\zeta} \right) \\
& \cdot \tanh_A \left(\sqrt{-(\Theta^2 - 4\delta\zeta)} \xi \right) \\
& \pm \frac{\sqrt{pq(\Theta^2 - 4\delta\zeta)}}{2\zeta} \\
& \cdot \operatorname{sech}_A \left(\sqrt{(\Theta^2 - 4\delta\zeta)} \xi \right) \left. \right)^2,
\end{aligned}$$

$$\begin{aligned}
\Phi_9(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2) \ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \quad (21) \\
& + 6\theta\mu \left(-\frac{\Theta}{2\zeta} - \frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2\zeta} \right) \\
& \times \coth_A \left(\sqrt{(\Theta^2 - 4\delta\zeta)} \xi \right) \\
& \pm \frac{\sqrt{pq(\Theta^2 - 4\delta\zeta)}}{2\zeta} \\
& \cdot \operatorname{csch}_A \left(\sqrt{(\Theta^2 - 4\delta\zeta)} \xi \right) \\
& + 6\theta^2 \left(-\frac{\Theta}{2\zeta} - \frac{\sqrt{(\Theta^2 - 4\delta\zeta)}}{2\zeta} \right) \\
& \cdot \coth_A \left(\sqrt{(\Theta^2 - 4\delta\zeta)} \xi \right) \\
& \pm \frac{\sqrt{pq(\Theta^2 - 4\delta\zeta)}}{2\zeta} \\
& \cdot \operatorname{csch}_A \left(\sqrt{(\Theta^2 - 4\delta\zeta)} \xi \right) \left. \right)^2,
\end{aligned}$$

$$\begin{aligned} \Phi_{10}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ & + 6\theta\mu \left(-\frac{\theta}{2\zeta} - \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4\zeta} \right) \\ & \times \tanh_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \\ & - \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4\zeta} \\ & \cdot \coth_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \\ & + 6\theta^2 \left(-\frac{\theta}{2\zeta} - \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4\zeta} \right) \\ & \cdot \tanh_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \\ & - \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4\zeta} \\ & \cdot \coth_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \left. \right)^2. \end{aligned} \quad (22)$$

Family 3: When $\delta\zeta > 0$ and $\theta = 0$, then

$$\begin{aligned} \Phi_{11}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ & + 6\theta\mu \left(\sqrt{\frac{\delta}{\zeta}} \tan_A(\sqrt{\delta\zeta}\xi) \right) \\ & + 6\theta^2 \left(\sqrt{\frac{\delta}{\zeta}} \tan_A(\sqrt{\delta\zeta}\xi) \right)^2 \left. \right), \end{aligned} \quad (23)$$

$$\begin{aligned} \Phi_{12}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ & + 6\theta\mu \left(-\sqrt{\frac{\delta}{\zeta}} \cot_A(\sqrt{\delta\zeta}\xi) \right) \\ & + 6\theta^2 \left(-\sqrt{\frac{\delta}{\zeta}} \cot_A(\sqrt{\delta\zeta}\xi) \right)^2 \left. \right), \end{aligned} \quad (24)$$

$$\begin{aligned} \Phi_{13}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ & + 6\theta\mu \left(\sqrt{\frac{\delta}{\zeta}} \tan_A(2\sqrt{\delta\zeta}\xi) \right) \\ & \pm \sqrt{pq\frac{\delta}{\zeta}} \sec_A(2\sqrt{\delta\zeta}\xi) \\ & + 6\theta^2 \left(\sqrt{\frac{\delta}{\zeta}} \tan_A(2\sqrt{\delta\zeta}\xi) \pm \sqrt{pq\frac{\delta}{\zeta}} \right. \end{aligned} \quad (25)$$

$$\left. \cdot \sec_A(2\sqrt{\delta\zeta}\xi) \right)^2 \left. \right),$$

$$\begin{aligned} \Phi_{14}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ & + 6\theta\mu \left(-\sqrt{\frac{\delta}{\zeta}} \cot_A(2\sqrt{\delta\zeta}\xi) \pm \sqrt{pq\frac{\delta}{\zeta}} \right. \\ & \cdot \csc_A(2\sqrt{\delta\zeta}\xi) \\ & + 6\theta^2 \left(-\sqrt{\frac{\delta}{\zeta}} \cot_A(2\sqrt{\delta\zeta}\xi) \pm \sqrt{pq\frac{\delta}{\zeta}} \right. \\ & \cdot \csc_A(2\sqrt{\delta\zeta}\xi) \left. \right)^2 \left. \right), \end{aligned} \quad (26)$$

$$\begin{aligned} \Phi_{15}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ & + 6\theta\mu \left(\frac{1}{2} \sqrt{\frac{\delta}{\zeta}} \tan_A \left(\frac{\sqrt{\delta\zeta}}{2} \xi \right) \right. \\ & - \sqrt{\frac{\delta}{\zeta}} \cot_A \left(\frac{\sqrt{\delta\zeta}}{2} \xi \right) \\ & + 6\theta^2 \left(\frac{1}{2} \sqrt{\frac{\delta}{\zeta}} \tan_A \left(\frac{\sqrt{\delta\zeta}}{2} \xi \right) \right. \\ & \left. \left. - \sqrt{\frac{\delta}{\zeta}} \cot_A \left(\frac{\sqrt{\delta\zeta}}{2} \xi \right) \right)^2 \right. \end{aligned} \quad (27)$$

Family 4: When $\delta\zeta < 0$ and $\theta = 0$, then

$$\begin{aligned} \Phi_{16}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ & + 6\theta\mu \left(-\sqrt{-\frac{\delta}{\zeta}} \tanh_A(\sqrt{-\delta\zeta}\xi) \right) \\ & + 6\theta^2 \left(-\sqrt{-\frac{\delta}{\zeta}} \tanh_A(\sqrt{-\delta\zeta}\xi) \right)^2 \left. \right), \end{aligned} \quad (28)$$

$$\begin{aligned} \Phi_{17}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\ & + 6\theta\mu \left(-\sqrt{-\frac{\delta}{\zeta}} \coth_A(\sqrt{-\delta\zeta}\xi) \right) \\ & + 6\theta^2 \left(-\sqrt{-\frac{\delta}{\zeta}} \coth_A(\sqrt{-\delta\zeta}\xi) \right)^2 \left. \right), \end{aligned} \quad (29)$$

$$\Phi_{18}(x, y, z, w, t) = \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \quad (30)$$

$$\begin{aligned}
& + 6\theta\mu \left(-\sqrt{\frac{\delta}{\varsigma}} \cot_A \left(2\sqrt{\delta\varsigma}\xi \right) \right. \\
& \pm \iota \sqrt{-pq\frac{\delta}{\varsigma}} \csc_A \left(2\sqrt{\delta\varsigma}\xi \right) \\
& + 6\theta^2 \left(-\sqrt{\frac{\delta}{\varsigma}} \cot_A \left(2\sqrt{\delta\varsigma}\xi \right) \right. \\
& \left. \left. \pm \iota \sqrt{-pq\frac{\delta}{\varsigma}} \csc_A \left(2\sqrt{\delta\varsigma}\xi \right) \right)^2 \right), \\
\Phi_{19}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(-\sqrt{\frac{\delta}{\varsigma}} \cot_A \left(2\sqrt{\delta\varsigma}\xi \right) \right. \\
& \pm \sqrt{-pq\frac{\delta}{\varsigma}} \csc_A \left(2\sqrt{\delta\varsigma}\xi \right) \\
& + 6\theta^2 \left(-\sqrt{\frac{\delta}{\varsigma}} \cot_A \left(2\sqrt{\delta\varsigma}\xi \right) \right. \\
& \left. \left. \pm \sqrt{-pq\frac{\delta}{\varsigma}} \csc_A \left(2\sqrt{\delta\varsigma}\xi \right) \right)^2 \right), \\
\Phi_{20}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(-\frac{1}{2} \sqrt{-\frac{\delta}{\varsigma}} \tanh_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) \right. \\
& + \sqrt{-\frac{\delta}{\varsigma}} \coth_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) \\
& + 6\theta^2 \left(-\frac{1}{2} \sqrt{-\frac{\delta}{\varsigma}} \tanh_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) \right. \\
& \left. \left. + \sqrt{-\frac{\delta}{\varsigma}} \coth_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) \right)^2 \right).
\end{aligned}$$

Family 5: When $\Theta = 0$ and $\varsigma = \delta$, then

$$\begin{aligned}
\Phi_{21}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& \left. + 6\theta\mu (\tan_A(\delta\xi)) + 6\theta^2 (\tan_A(\delta\xi))^2 \right), \\
\Phi_{22}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& \left. + 6\theta\mu (-\cot_A(\delta\xi)) + 6\theta^2 (-\cot_A(\delta\xi))^2 \right), \\
\Phi_{23}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu (\tan_A(2\delta\xi) \pm \sqrt{pq} \sec_A(2\delta\xi)) \\
& + 6\theta^2 (\tan_A(2\delta\xi) \pm \sqrt{pq} \sec_A(2\delta\xi))^2 \Big), \\
\Phi_{24}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu (-\cot_A(2\delta\xi) \pm \sqrt{pq} \csc_A(2\delta\xi)) \\
& + 6\theta^2 (-\cot_A(2\delta\xi) \pm \sqrt{pq} \csc_A(2\delta\xi))^2 \Big), \\
\Phi_{25}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(\frac{1}{2} \tan_A \left(\frac{\delta}{2} \xi \right) - \cot_A \left(\frac{\delta}{2} \xi \right) \right) \\
& + 6\theta^2 \left(\frac{1}{2} \tan_A \left(\frac{\delta}{2} \xi \right) - \cot_A \left(\frac{\delta}{2} \xi \right) \right)^2 \Big), \\
\Phi_{26}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu (-\tanh_A(\delta\xi)) \\
& + 6\theta^2 (-\tanh_A(\delta\xi))^2 \Big), \\
\Phi_{27}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu (-\coth_A(\delta\xi)) \\
& + 6\theta^2 (-\coth_A(\delta\xi))^2 \Big), \\
\Phi_{28}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu (-\tanh_A(2\delta\xi) \pm \iota\sqrt{pq} \operatorname{sech}_A(2\delta\xi)) \\
& + 6\theta^2 (-\tanh_A(2\delta\xi) \\
& \left. \pm \iota\sqrt{pq} \operatorname{sech}_A(2\delta\xi))^2 \right), \\
\Phi_{29}(x, y, z, w, t) &= \frac{1}{6}(\alpha^2 - \gamma^2) \operatorname{Ln}(A)^2 \left(2\theta\lambda + \mu^2 \right.
\end{aligned}$$

Family 6: When $\Theta = 0$ and $\varsigma = -\delta$, then

$$\begin{aligned}
& + 6\theta\mu (-\coth_A(2\delta\xi) \pm \sqrt{pq}csch_A(2\delta\xi)) \\
& + 6\theta^2 (-\coth_A(2\delta\xi) \\
& \pm \sqrt{pq}csch_A(2\delta\xi))^2), \\
\Phi_{30}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(-\frac{1}{2} \tanh_A \left(\frac{\delta}{2}\xi \right) \right. \\
& + \coth_A \left(\frac{\delta}{2}\xi \right) \left. \right) \\
& + 6\theta^2 \left(-\frac{1}{2} \tanh_A \left(\frac{\delta}{2}\xi \right) \right. \\
& + \coth_A \left(\frac{\delta}{2}\xi \right) \left. \right)^2 \left. \right). \quad (42)
\end{aligned}$$

Family 7: When $\Theta^2 = 4\delta\varsigma$, then

$$\begin{aligned}
\Phi_{31}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(\frac{-2\delta(\Theta\xi Ln(A) + 2)}{\Theta^2\xi Ln(A)} \right) \\
& + 6\theta^2 \left(\frac{-2\delta(\Theta\xi Ln(A) + 2)}{\Theta^2\xi Ln(A)} \right)^2 \left. \right). \quad (43)
\end{aligned}$$

Family 8: When $\Theta = k$, $\delta = mk(m \neq 0)$ and $\varsigma = 0$, then

$$\begin{aligned}
\Phi_{32}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu (A^{k\xi} - m) \\
& + 6\theta^2 (A^{k\xi} - m)^2 \left. \right). \quad (44)
\end{aligned}$$

Family 9: When $\Theta = \varsigma = 0$, then

$$\begin{aligned}
\Phi_{33}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu (\delta\xi Ln(A)) \\
& + 6\theta^2 (\delta\xi Ln(A))^2 \left. \right). \quad (45)
\end{aligned}$$

Family 10: When $\Theta = \delta = 0$, then

$$\begin{aligned}
\Phi_{34}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(\frac{-1}{\varsigma\xi Ln(A)} \right) \\
& + 6\theta^2 \left(\frac{-1}{\varsigma\xi Ln(A)} \right)^2 \left. \right). \quad (46)
\end{aligned}$$

Family 11: When $\delta = 0$ and $\Theta \neq 0$, then

$$\begin{aligned}
\Phi_{35}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(-\frac{p\Theta}{\varsigma(\cosh_A(\Theta\xi) - \sinh_A(\Theta\xi + p))} \right) \\
& + 6\theta^2 \left(-\frac{p\Theta}{\varsigma(\cosh_A(\Theta\xi) - \sinh_A(\Theta\xi + p))} \right)^2 \left. \right), \quad (47)
\end{aligned}$$

$$\begin{aligned}
\Phi_{36}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(-\frac{\Theta(\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi))}{\varsigma(\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi + q))} \right) \\
& + 6\theta^2 \left(-\frac{\Theta(\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi))}{\varsigma(\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi + q))} \right)^2 \left. \right). \quad (48)
\end{aligned}$$

Family 12: When $\Theta = k$, $\varsigma = mk$, ($m \neq 0$) and $\delta = 0$, then

$$\begin{aligned}
\Phi_{37}(x, y, z, w, t) = & \frac{1}{6}(\alpha^2 - \gamma^2)Ln(A)^2 \left(2\theta\lambda + \mu^2 \right. \\
& + 6\theta\mu \left(\frac{pA^{k\xi}}{q - \mp A^{k\xi}} \right) \\
& + 6\theta^2 \left(\frac{pA^{k\xi}}{q - \mp A^{k\xi}} \right)^2 \left. \right). \quad (49)
\end{aligned}$$

where

$$\xi = \alpha x + \gamma y + \chi z + \tau w + \epsilon t$$

3.2 EDAM for (2+1)-dimensional Breaking soliton equation

Here, (2+1)-dimensional BSE (2) is used to construct novel solitons solutions

$$\Phi(x, y, t) = P(\xi), \quad \xi = kx + ly - ct, \quad (50)$$

Using (50) into (2), then the following ODE is obtained as

$$k^3 l P^{(4)} - 3k^2 l ((P')^2)' - kc P'' = 0. \quad (51)$$

Integrating (51), we get

$$k^3 l P''' - 3k^2 l (P'^2) - kc P' = 0. \quad (52)$$

for simplicity, when we can consider

$$V = P', \quad V' = P'', \quad V'' = P''', \quad (53)$$

$$k^3 l V'' - 3k^2 l V^2 - kc V = 0. \quad (54)$$

Using balancing rule in (54), it yields $N = 2$, (6) converts to

$$P(\xi) = F_0 + F_1 Q(\xi) + F_2 (Q(\xi))^2, \quad (55)$$

Where F_0 , F_1 and F_2 are constants. Substituting (55) into (54) and equating coefficients of polynomials of $Q(\xi)$ to zero, get a set of equations in F_0 , F_1 , F_2 , and c .

On solving the set of equations, we get

$$\begin{aligned} F_0 &= 2k\theta\lambda\ln(A)^2, \\ F_1 &= 2k\theta\mu\ln(A)^2, \\ F_2 &= 2k\theta^2\ln(A)^2, \\ c &= -4k^2l\theta\lambda\ln(A)^2 + kl\mu^2\ln(A)^2. \end{aligned} \quad (56)$$

Using Eqs. (50), (55) and (56) the new solutions of (2) are obtained as follows

Family 1: When $\theta^2 - 4\delta\zeta < 0$ and $\zeta \neq 0$, then

$$\begin{aligned} \Phi_1(x, y, t) &= 2k\theta\ln(A)^2 \left(\lambda \right. \\ &\quad + \mu \left(-\frac{\theta}{2\zeta} + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2\zeta} \tan_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2} \xi \right) \right) \\ &\quad \left. + \theta \left(-\frac{\theta}{2\zeta} + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2\zeta} \tan_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2} \xi \right) \right)^2 \right), \end{aligned} \quad (57)$$

$$\begin{aligned} \Phi_2(x, y, t) &= 2k\theta\ln(A)^2 \left(\lambda \right. \\ &\quad + \mu \left(-\frac{\theta}{2\zeta} + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2\zeta} \cot_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2} \xi \right) \right) \\ &\quad \left. + \theta \left(-\frac{\theta}{2\zeta} + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2\zeta} \cot_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2} \xi \right) \right)^2 \right), \end{aligned} \quad (58)$$

$$\begin{aligned} \Phi_3(x, y, t) &= 2k\theta\ln(A)^2 \left(\lambda + \mu \left(-\frac{\theta}{2\zeta} \right. \right. \\ &\quad + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2\zeta} \tan_A \left(\sqrt{-(\theta^2 - 4\delta\zeta)} \xi \right) \\ &\quad \left. \pm \frac{\sqrt{-pq(\theta^2 - 4\delta\zeta)}}{2\zeta} \sec_A \left(\sqrt{-(\theta^2 - 4\delta\zeta)} \xi \right) \right) \\ &\quad + \theta \left(-\frac{\theta}{2\zeta} \right. \\ &\quad + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2\zeta} \times \tan_A \left(\sqrt{-(\theta^2 - 4\delta\zeta)} \xi \right) \\ &\quad \left. \pm \frac{\sqrt{-pq(\theta^2 - 4\delta\zeta)}}{2\zeta} \sec_A \left(\sqrt{-(\theta^2 - 4\delta\zeta)} \xi \right) \right)^2 \right), \end{aligned} \quad (59)$$

$$\Phi_4(x, y, t) = 2k\theta\ln(A)^2 \left(\lambda + \mu \left(-\frac{\theta}{2\zeta} \right. \right. \quad (60)$$

$$\begin{aligned} &\quad + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2\zeta} \cot_A \left(\sqrt{-(\theta^2 - 4\delta\zeta)} \xi \right) \\ &\quad \pm \frac{\sqrt{-pq(\theta^2 - 4\delta\zeta)}}{2\zeta} \csc_A \left(\sqrt{-(\theta^2 - 4\delta\zeta)} \xi \right) \Bigg) \\ &\quad + \theta \left(-\frac{\theta}{2\zeta} \right. \\ &\quad + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{2\zeta} \times \cot_A \left(\sqrt{-(\theta^2 - 4\delta\zeta)} \xi \right) \\ &\quad \left. \pm \frac{\sqrt{-pq(\theta^2 - 4\delta\zeta)}}{2\zeta} \csc_A \left(\sqrt{-(\theta^2 - 4\delta\zeta)} \xi \right) \right)^2 \Bigg), \end{aligned}$$

$$\begin{aligned} \Phi_5(x, y, t) &= 2k\theta\ln(A)^2 \left(\lambda + \mu \left(-\frac{\theta}{2\zeta} \right. \right. \\ &\quad + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{4\zeta} \tan_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \\ &\quad - \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{4\zeta} \cot_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \Bigg) \\ &\quad + \theta \left(-\frac{\theta}{2\zeta} + \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{4\zeta} \right. \\ &\quad \times \tan_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \\ &\quad \left. - \frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{4\zeta} \cot_A \left(\frac{\sqrt{-(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \right)^2 \Bigg). \end{aligned} \quad (61)$$

Family 2: When $\theta^2 - 4\delta\zeta > 0$. and $\zeta \neq 0$, then

$$\begin{aligned} \Phi_6(x, y, t) &= 2k\theta\ln(A)^2 \left(\lambda + \mu \left(\frac{\theta}{2\zeta} \right. \right. \\ &\quad + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2\zeta} \tanh_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2} \xi \right) \Bigg) \\ &\quad + \theta \left(-\frac{\theta}{2\zeta} \right. \\ &\quad + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2\zeta} \tanh_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2} \xi \right) \Bigg)^2 \Bigg), \end{aligned} \quad (62)$$

$$\begin{aligned} \Phi_7(x, y, t) &= 2k\theta\ln(A)^2 \left(\lambda + \mu \left(-\frac{\theta}{2\zeta} \right. \right. \\ &\quad + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2\zeta} \coth_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2} \xi \right) \Bigg) \\ &\quad + \theta \left(-\frac{\theta}{2\zeta} \right. \\ &\quad + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2\zeta} \coth_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2} \xi \right) \Bigg)^2 \Bigg), \end{aligned} \quad (63)$$

$$\begin{aligned} \Phi_8(x, y, t) = & 2k\theta \ln(A)^2 \left(\lambda + \mu \left(-\frac{\theta}{2\zeta} \right. \right. \\ & + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2\zeta} \tanh_A \left(\sqrt{(\theta^2 - 4\delta\zeta)} \xi \right) \\ & \pm \frac{\sqrt{pq(\theta^2 - 4\delta\zeta)}}{2\zeta} \operatorname{sech}_A \left(\sqrt{(\theta^2 - 4\delta\zeta)} \xi \right) \\ & + \theta \left(-\frac{\theta}{2\zeta} \right. \\ & + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2\zeta} \times \tanh_A \left(\sqrt{(\theta^2 - 4\delta\zeta)} \xi \right) \\ & \left. \left. \pm \frac{\sqrt{pq(\theta^2 - 4\delta\zeta)}}{2\zeta} \operatorname{sech}_A \left(\sqrt{(\theta^2 - 4\delta\zeta)} \xi \right) \right)^2 \right), \end{aligned} \quad (64)$$

$$\begin{aligned} \Phi_9(x, y, t) = & 2k\theta \ln(A)^2 \left(\lambda + \mu \left(-\frac{\theta}{2\zeta} \right. \right. \\ & + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2\zeta} \coth_A \left(\sqrt{(\theta^2 - 4\delta\zeta)} \xi \right) \\ & \pm \frac{\sqrt{pq(\theta^2 - 4\delta\zeta)}}{2\zeta} \operatorname{csch}_A \left(\sqrt{(\theta^2 - 4\delta\zeta)} \xi \right) \\ & + \theta \left(-\frac{\theta}{2\zeta} + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{2\zeta} \right. \\ & \times \coth_A \left(\sqrt{(\theta^2 - 4\delta\zeta)} \xi \right) \\ & \left. \left. \pm \frac{\sqrt{pq(\theta^2 - 4\delta\zeta)}}{2\zeta} \operatorname{csch}_A \left(\sqrt{(\theta^2 - 4\delta\zeta)} \xi \right) \right)^2 \right), \end{aligned} \quad (65)$$

$$\begin{aligned} \Phi_{10}(x, y, t) = & 2k\theta \ln(A)^2 \left(\lambda + \mu \left(-\frac{\theta}{2\zeta} \right. \right. \\ & + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4\zeta} \tanh_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \\ & - \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4\zeta} \coth_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \\ & + \theta \left(\frac{\theta}{2\zeta} \right. \\ & + \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4\zeta} \times \tanh_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \\ & \left. \left. - \frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4\zeta} \coth_A \left(\frac{\sqrt{(\theta^2 - 4\delta\zeta)}}{4} \xi \right) \right)^2 \right). \end{aligned} \quad (66)$$

Family 3: When $\delta\zeta > 0$ and $\theta = 0$, then

$$\begin{aligned} \Phi_{11}(x, y, t) = & 2k\theta \ln(A)^2 \left(\lambda \right. \\ & \left. + \mu \left(\sqrt{\frac{\delta}{\zeta}} \tan_A(\sqrt{\delta\zeta}\xi) \right) \right) \end{aligned} \quad (67)$$

$$+ \theta \left(\sqrt{\frac{\delta}{\zeta}} \tan_A(\sqrt{\delta\zeta}\xi) \right)^2,$$

$$\begin{aligned} \Phi_{12}(x, y, t) = & 2k\theta \ln(A)^2 \left(\lambda \right. \\ & + \mu \left(\sqrt{\frac{\delta}{\zeta}} \cot_A(\sqrt{\delta\zeta}\xi) \right) \\ & \left. + \theta \left(\sqrt{\frac{\delta}{\zeta}} \cot_A(\sqrt{\delta\zeta}\xi) \right)^2 \right), \end{aligned} \quad (68)$$

$$\begin{aligned} \Phi_{13}(x, y, t) = & 2k\theta \ln(A)^2 \left(\lambda \right. \\ & + \mu \left(\sqrt{\frac{\delta}{\zeta}} \tan_A(2\sqrt{\delta\zeta}\xi) \right) \\ & \pm \sqrt{pq} \frac{\delta}{\zeta} \sec_A(2\sqrt{\delta\zeta}\xi) \\ & + \theta \left(\sqrt{\frac{\delta}{\zeta}} \tan_A(2\sqrt{\delta\zeta}\xi) \right. \\ & \left. \pm \sqrt{pq} \frac{\delta}{\zeta} \sec_A(2\sqrt{\delta\zeta}\xi) \right)^2 \Big), \end{aligned} \quad (69)$$

$$\begin{aligned} \Phi_{14}(x, y, t) = & 2k\theta \ln(A)^2 \left(\lambda \right. \\ & + \mu \left(\sqrt{\frac{\delta}{\zeta}} \cot_A(2\sqrt{\delta\zeta}\xi) \right) \\ & \pm \sqrt{pq} \frac{\delta}{\zeta} \csc_A(2\sqrt{\delta\zeta}\xi) \\ & + \theta \left(\sqrt{\frac{\delta}{\zeta}} \cot_A(2\sqrt{\delta\zeta}\xi) \right. \\ & \left. \pm \sqrt{pq} \frac{\delta}{\zeta} \csc_A(2\sqrt{\delta\zeta}\xi) \right)^2 \Big), \end{aligned} \quad (70)$$

$$\begin{aligned} \Phi_{15}(x, y, t) = & 2k\theta \ln(A)^2 \left(\lambda \right. \\ & + \mu \left(\frac{1}{2} \left(\sqrt{\frac{\delta}{\zeta}} \tan_A \left(\frac{\sqrt{\delta\zeta}}{2} \xi \right) \right. \right. \\ & \left. \left. - \sqrt{\frac{\delta}{\zeta}} \cot_A \left(\frac{\sqrt{\delta\zeta}}{2} \xi \right) \right) \right) \\ & + \theta \left(\frac{1}{2} \left(\sqrt{\frac{\delta}{\zeta}} \tan_A \left(\frac{\sqrt{\delta\zeta}}{2} \xi \right) \right. \right. \end{aligned} \quad (71)$$

$$-\sqrt{\frac{\delta}{\varsigma}} \cot_A \left(\frac{\sqrt{\delta\varsigma}}{2} \xi \right) \Big)^2 \Big).$$

Family 4: When $\delta\varsigma < 0$ and $\Theta = 0$, then

$$\begin{aligned} \Phi_{16}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda & \\ + \mu \left(-\sqrt{-\frac{\delta}{\varsigma}} \tanh_A(\sqrt{-\delta\varsigma}\xi) \right) & \\ + \theta \left(-\sqrt{-\frac{\delta}{\varsigma}} \tanh_A(\sqrt{-\delta\varsigma}\xi) \right)^2 \Big), & \end{aligned} \quad (72)$$

$$\begin{aligned} \Phi_{17}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda & \\ + \mu \left(-\sqrt{-\frac{\delta}{\varsigma}} \coth_A(\sqrt{-\delta\varsigma}\xi) \right) & \\ + \theta \left(-\sqrt{-\frac{\delta}{\varsigma}} \coth_A(\sqrt{-\delta\varsigma}\xi) \right)^2 \Big), & \end{aligned} \quad (73)$$

$$\begin{aligned} \Phi_{18}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda & \\ + \mu \left(-\sqrt{\frac{\delta}{\varsigma}} \cot_A(2\sqrt{\delta\varsigma}\xi) \right) & \\ \pm \iota \sqrt{-pq\frac{\delta}{\varsigma}} \csc_A(2\sqrt{\delta\varsigma}\xi) & \\ + \theta \left(-\sqrt{\frac{\delta}{\varsigma}} \cot_A(2\sqrt{\delta\varsigma}\xi) \right) & \\ \pm \iota \sqrt{-pq\frac{\delta}{\varsigma}} \csc_A(2\sqrt{\delta\varsigma}\xi) \Big)^2 \Big), & \end{aligned} \quad (74)$$

$$\begin{aligned} \Phi_{19}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda & \\ + \mu \left(-\sqrt{\frac{\delta}{\varsigma}} \cot_A(2\sqrt{\delta\varsigma}\xi) \right) & \\ \pm \sqrt{-pq\frac{\delta}{\varsigma}} \csc_A(2\sqrt{\delta\varsigma}\xi) & \\ + \theta \left(-\sqrt{\frac{\delta}{\varsigma}} \cot_A(2\sqrt{\delta\varsigma}\xi) \right) & \\ \pm \sqrt{-pq\frac{\delta}{\varsigma}} \csc_A(2\sqrt{\delta\varsigma}\xi) \Big)^2 \Big), & \end{aligned} \quad (75)$$

$$\Phi_{20}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda \quad (76)$$

$$\begin{aligned} + \mu \left(-\frac{1}{2} \sqrt{-\frac{\delta}{\varsigma}} \tanh_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) \right. & \\ + \sqrt{-\frac{\delta}{\varsigma}} \coth_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) & \\ + \theta \left(-\frac{1}{2} \sqrt{-\frac{\delta}{\varsigma}} \tanh_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) \right. & \\ + \left. \left. \sqrt{-\frac{\delta}{\varsigma}} \coth_A \left(\frac{\sqrt{-\delta\varsigma}}{2} \xi \right) \right)^2 \right). & \end{aligned}$$

Family 5: When $\Theta = 0$ and $\varsigma = \delta$, then

$$\begin{aligned} \Phi_{21}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda & \\ + \mu (\tan_A(\delta\xi)) + \theta (\tan_A(\delta\xi))^2 \Big), & \end{aligned} \quad (77)$$

$$\begin{aligned} \Phi_{22}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda & \\ + \mu (-\cot_A(\delta\xi)) + \theta (-\cot_A(\delta\xi))^2 \Big), & \end{aligned} \quad (78)$$

$$\begin{aligned} \Phi_{23}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda & \\ + \mu (\tan_A(2\delta\xi) \pm \sqrt{pq} \sec_A(2\delta\xi)) & \\ + \theta (\tan_A(2\delta\xi) \pm \sqrt{pq} \sec_A(2\delta\xi))^2 \Big), & \end{aligned} \quad (79)$$

$$\begin{aligned} \Phi_{24}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda & \\ + \mu (-\cot_A(2\delta\xi) \pm \sqrt{pq} \csc_A(2\delta\xi)) & \\ + \theta (-\cot_A(2\delta\xi) \pm \sqrt{pq} \csc_A(2\delta\xi))^2 \Big), & \end{aligned} \quad (80)$$

$$\begin{aligned} \Phi_{25}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda & \\ + \mu \left(\frac{1}{2} \left(\tan_A \left(\frac{\delta}{2} \xi \right) - \cot_A \left(\frac{\delta}{2} \xi \right) \right) \right) & \\ + \theta \left(\frac{1}{2} \left(\tan_A \left(\frac{\delta}{2} \xi \right) - \cot_A \left(\frac{\delta}{2} \xi \right) \right) \right)^2 \Big). & \end{aligned} \quad (81)$$

Family 6: When $\Theta = 0$ and $\varsigma = -\delta$, then

$$\Phi_{26}(x, y, t) = 2k\theta \ln(A)^2 \Big(\lambda \quad (82)$$

$$+ \mu (-\tanh_A(\delta\xi)) \\ + \theta (-\tanh_A(\delta\xi))^2),$$

$$\Phi_{27}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu (-\coth_A(\delta\xi)) \right. \\ \left. + \theta (-\coth_A(\delta\xi))^2 \right), \quad (83)$$

$$\Phi_{28}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu (-\tanh_A(2\delta\xi) \pm \iota\sqrt{pq} \operatorname{sech}_A(2\delta\xi)) \right. \\ \left. + \theta (-\tanh_A(2\delta\xi) \pm \iota\sqrt{pq} \operatorname{sech}_A(2\delta\xi))^2 \right), \quad (84)$$

$$\Phi_{29}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu (-\coth_A(2\delta\xi) \pm \sqrt{pq} \operatorname{csch}_A(2\delta\xi)) \right. \\ \left. + \theta (-\coth_A(2\delta\xi) \pm \sqrt{pq} \operatorname{csch}_A(2\delta\xi))^2 \right), \quad (85)$$

$$\Phi_{30}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu \left(-\frac{1}{2} \tanh_A \left(\frac{\delta}{2} \xi \right) + \coth_A \left(\frac{\delta}{2} \xi \right) \right) \right. \\ \left. + \theta \left(-\frac{1}{2} \tanh_A \left(\frac{\delta}{2} \xi \right) + \coth_A \left(\frac{\delta}{2} \xi \right) \right)^2 \right). \quad (86)$$

Family 7: When $\Theta^2 = 4\delta\zeta$, then

$$\Phi_{31}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu \left(\frac{-2\delta(\Theta\xi Ln(A) + 2)}{\Theta^2 \xi Ln(A)} \right) \right. \\ \left. + \theta \left(\frac{-2\delta(\Theta\xi Ln(A) + 2)}{\Theta^2 \xi Ln(A)} \right)^2 \right). \quad (87)$$

Family 8: When $\Theta = k$, $\delta = mk$ ($m \neq 0$) and $\zeta = 0$, then

$$\Phi_{32}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu (A^{k\xi} - m) + \theta (A^{k\xi} - m)^2 \right). \quad (88)$$

Family 9: When $\Theta = \zeta = 0$, then

$$\Phi_{33}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu (\delta\xi Ln(A)) + \theta (\delta\xi Ln(A))^2 \right). \quad (89)$$

Family 10: When $\Theta = \delta = 0$, then

$$\Phi_{34}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu \left(\frac{-1}{\zeta\xi Ln(A)} \right) + \theta \left(\frac{-1}{\zeta\xi Ln(A)} \right)^2 \right). \quad (90)$$

Family 11: When $\Theta = \delta = 0$, then

$$\Phi_{35}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu \left(-\frac{p\Theta}{\zeta (\cosh_A(\Theta\xi) - \sinh_A(\Theta\xi + p))} \right) \right. \\ \left. + \theta \left(-\frac{p\Theta}{\zeta (\cosh_A(\Theta\xi) - \sinh_A(\Theta\xi + p))} \right)^2 \right), \quad (91)$$

$$\Phi_{36}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu \left(-\frac{\Theta(\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi))}{\zeta (\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi + q))} \right) \right. \\ \left. + \theta \left(-\frac{\Theta(\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi))}{\zeta (\sinh_A(\Theta\xi) + \cosh_A(\Theta\xi + q))} \right)^2 \right). \quad (92)$$

Family 12: When $\Theta = k$, $\zeta = mk$ ($m \neq 0$) and $\delta = 0$, then

$$\Phi_{37}(x, y, t) = 2k\theta Ln(A)^2 \left(\lambda \right. \\ \left. + \mu \left(\frac{pA^{k\xi}}{q - \mp A^{k\xi}} \right) + \theta \left(\frac{pA^{k\xi}}{q - \mp A^{k\xi}} \right)^2 \right). \quad (93)$$

where

$$\xi = kx + ly - ct.$$

4 Results and Discussions

In this study, we have constructed novel solitons solutions along with hyperbolic and trigonometric function for the (4+1)-dimensional Fokas equation and (2+1)-dimensional Breaking soliton equation using new extended direct algebraic method. This technique is measured as most recent

scheme in this field and that is not utilized to these equations earlier. For physical analysis, 3-D, 2-D and contour plots of some of these solutions are comprised with suitable parameters. Likewise, 3D plots provide us to model and exhibit accurate physical behavior. The obtained solutions discover their applications in communication to convey information because optical solitons have the capability to spread over long distances without reduction and without changing their forms. From results we can say that the optical solitons is one of the most exciting and interesting regions of research in nonlinear optics. Through this study, we consider the optical solitons solutions to the nonlinear FE and KSE models using the new extended direct algebraic method. In this paper, we only included specific figures to avoid overloading the document. All the developed results are novel and distinct from that reported results. We can understand from all the graphs that the new EDAM is very effectual and more specific in assessing the under consideration nonlinear models. For graphical representation for (1), the physical behavior of (13) using the proper values of parameters $\alpha = 2.3$, $\alpha = 1.3$, $\gamma_2 = 1.5$, $\theta = 1.3$, $\lambda = 0.4$, $\mu = 0.7$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 3$, $\delta = 2$, $\zeta = 1.9$, $\tau = 0.85$, $c = 4$, $z = 3$, $\chi = 2.6$, $\epsilon = 2$, $y = 2.6$ and $t = 1$ are shown in Figure 1, the physical behavior of (18) using the appropriate values of parameters $\alpha = 2.2$, $\gamma = 1.6$,

$\theta = 2.3$, $\lambda = 3.4$, $\mu = 3.7$, $w = 2.2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.4$, $\delta = -2$, $\zeta = 1.9$, $\tau = 0.85$, $c = 2.9$, $z = 3$, $\chi = 1.5$, $\epsilon = 3$, $y = 2.5$ and $t = 1$ are shown in Figure 2, the physical behavior of (29) using the proper values of parameters $\alpha = 2.3$, $\alpha = 1.1$, $\gamma_2 = 1.5$, $\theta = 2.1$, $\lambda = 3.4$, $\mu = 1.7$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.3$, $\delta = 2$, $\zeta = 1.9$, $\tau = 0.85$, $c = 4$, $z = 3$, $\chi = 2.6$, $\epsilon = 3$, $y = 2.6$ and $t = 1$ are shown in Figure 3, the absolute behavior of (40) using the proper values of parameters $\alpha = 2.2$, $\gamma_2 = 1.4$, $\theta = 2.3$, $\lambda = 3.3$, $\mu = 1.7$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.6$, $b = 2$, $\delta = -2$, $\zeta = 2$, $\tau = 0.82$, $c = 3.1$, $z = 3$, $\chi = 1.8$, $\epsilon = 1.9$, $y = 2.7$ and $t = 1$ are shown in Figure 4. The Graphical representation for (2), the absolute behavior of (58) using the suitable values of parameters $\theta = 2.3$, $\lambda = 3.3$, $\mu = 1.8$, $\theta = 1.5$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.6$, $\delta = -2$, $\zeta = 2$, $c = 2$, $y = 2.7$, $l = 1.5$ and $t = 1$ are shown in Figure 5, the absolute behavior of (63) with the suitable values of parameters $\theta = 2.5$, $\lambda = 3.1$, $\mu = 1.5$, $\theta = 1.4$, $p = 0.98$, $q = 0.95$, $k = 2.1$, $A = 2.8$, $\delta = -2$, $\zeta = 2$, $c = 2.3$, $y = 1.7$, $l = 1.9$ and $t = 1$ are shown in Figure 6, the absolute behavior of (71) using the proper values of parameters $\theta = 1.5$, $\lambda = 2.2$, $\mu = 2.9$, $\theta = 1.7$, $p = 0.98$, $q = 0.95$, $k = 2.2$, $A = 2.9$, $\delta = 2$, $\zeta = 2.1$, $c = 3.1$, $y = 2.7$, $l = 3.8$ and $t = 1$ are shown in Figure 7, the absolute behavior of (82) using the proper values of

Figure 1: (A) 3D graph of (13) with $\alpha = 2.3$, $\gamma = 1.5$, $\theta = 1.3$, $\lambda = 0.4$, $\mu = 0.7$, $\theta = 0.7$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 3$, $\delta = 2$, $\zeta = 1.9$, $\tau = 0.85$, $c = 4$, $z = 3$, $\epsilon = 2$, $y = 2.6$. (A-1) 2D plot of (13) with $t = 1$. (A-2) Contour graph of (13).

Figure 2: (B) 3D graph of (18) with $\alpha = 2.2$, $\gamma = 1.6$, $\theta = 2.3$, $\lambda = 3.4$, $\mu = 3.7$, $w = 2.2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.4$, $\delta = -2$, $\zeta = 1.9$, $\tau = 0.85$, $c = 2.9$, $z = 3$, $\chi = 1.5$, $\epsilon = 3$, $y = 2.5$. (B-1) 2D plot of (18) with $t = 1$. (B-2) Contour graph of (18).

Figure 3: (C) 3D graph of (29) with $\alpha = 2.3$, $\gamma = 1.5$, $\theta = 2.1$, $\lambda = 3.4$, $\mu = 1.7$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.3$, $\delta = 2$, $\zeta = 1.9$, $\tau = 0.85$, $c = 4$, $z = 3$, $\chi = 2.6$, $\epsilon = 3$, $y = 2.6$. (C-1) 2D plot of (29) with $t = 1$. (C-2) Contour graph of (29).

Figure 4: (D) 3D graph of (40) with $\alpha = 2.2$, $\gamma = 1.4$, $\theta = 2.3$, $\lambda = 3.3$, $\mu = 1.7$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.6$, $b = 2$, $\delta = -2$, $\zeta = 2$, $\tau = 0.82$, $c = 3.1$, $z = 3$, $\chi = 1.8$, $\epsilon = 1.9$, $y = 2.7$. (D-1) 2D plot of (40) with $t = 1$. (D-2) Contour graph of (40).

Figure 5: (E) 3D graph of (58) with $\theta = 2.3$, $\lambda = 3.3$, $\mu = 1.8$, $\Theta = 1.5$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.6$, $\delta = -2$, $\zeta = 2$, $c = 2$, $y = 2.7$, $l = 1.5$. (E-1) 2D plot of (58) with $t = 1$. (E-2) Contour graph of (58).

Figure 6: (F) 3D graph of (63) with $\theta = 2.5$, $\lambda = 3.1$, $\mu = 1.5$, $\Theta = 1.4$, $p = 0.98$, $q = 0.95$, $k = 2.1$, $A = 2.8$, $\delta = -2$, $\zeta = 2$, $c = 2.3$, $y = 1.7$, $l = 1.9$. (F-1) 2D plot of (63) with $t = 1$. (F-2) Contour graph of (63).

Figure 7: (G) 3D graph of (71) with $\theta = 1.5$, $\lambda = 2.2$, $\mu = 2.9$, $\Theta = 1.7$, $p = 0.98$, $q = 0.95$, $k = 2.2$, $A = 2.9$, $\delta = 2$, $\zeta = 2.1$, $c = 3.1$, $y = 2.7$, $l = 3.8$. (G-1) 2D plot of (71) with $t = 1$. (G-2) Contour graph of (71).

Figure 8: (H) 3D graph of (82) with $\theta = 2.4$, $\lambda = 2.2$, $\mu = 1.2$, $\Theta = 1.7$, $p = 0.98$, $q = 0.95$, $k = 2.4$, $A = 2.1$, $\delta = 2$, $\zeta = 2$, $c = 3.3$, $y = 2.8$, $l = 1.1$. (H-1) 2D plot of (82) with $t = 1$. (H-2) Contour graph of (82).

parameters $\theta = 2.4$, $\lambda = 2.2$, $\mu = 1.2$, $\Theta = 1.7$, $p = 0.98$, $q = 0.95$, $k = 2.4$, $A = 2.1$, $\delta = 2$, $\zeta = 2$, $c = 3.3$, $y = 2.8$, $l = 1.1$ and $t = 1$ are shown in Figure 8.

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5 Conclusion

In this paper, we constructed novel optical solitons solutions for two different models via the new EDAM, in the form of bright, dark, mixed bright-dark solitons as well as hyperbolic and trigonometric functions solutions. By choosing the suitable values of parameters and to better understand the physical structures of the solutions, 3-D and 2-d graphs have been plotted. From the acquired results and figures, it is observed that all solutions demonstrated wave behavior. Also, these solutions yield traveling dark wave behaviors to the considered models, physically. Therefore, from acquired solutions and previous results in different research literature, we have determined that our solutions are general and powerful as compared in before results deliberated in [1–41]. It shows that the performance of our technique is best mathematical tool to solve the nonlinear wave problems.

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