Research Article

Chun-Hui He and Chao Liu*

Fractal approach to the fluidity of a cement mortar

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Abstract: The fluidity of a cement mortar is a key factor for 3-D printing technology and cement-based materials. This paper introduces the measurement of the fluidity according China's national standard, and a mathematical model is established to reveal main factors affecting the measure accuracy. The result shows the fluidity reveals mainly the rheological property of the mortar, but it is also affected by other measuring conditions, e.g., the vibration properties of the measuring table.

Keywords: Fractal rheological model, vibrating table, fluidity, fractal derivative, fluidity equation

1 Introduction

The cement mortar's fluidity is a basic property for various applications in the 3D printing technology [1–3] and the architecture engineering [4–8]. The fluidity plays an important role in the three-dimensional printing technology, Zhang *et al.* improved the fluidity by adding lubricant and toughening agent in the printing paste [1]; He *et al.* showed that the fluidity affected the bond stress–slip relationship of 3-D printed subjects [2]; Zuo and Liu revealed the fluidity affected the mechanical and electrical properties of printed composites [3]. Cement is the dominant building material, and its fluidity is a hot topic for fast constructing of a high building with a short period. Feng *et al.* studied the effect of polycarboxylate superplasticizers on cement mortar's fluidity [4]; Espinoza-Moreno *et al.* found that addition of graphite powder in the mortar could change the fluidity [5]; many other searchers also found the addition of superplasticizer or nanoparticles, or inorganic pigments could improve the fluidity [6–10].

The fluidity will greatly affect the printing or building process and the product's mechanical, thermal, permeable, electronic properties and the vibration strength as well. He *et al.* found the microstructure of a concrete can be used for vibration attenuation and vibration absorption [11].

The fluidity is generally measured on a jump table with a given frequency and a given amplitude. A fresh mortar with a truncated conical shape is put on the jump table for vibration in a given period, then the final diameter of the expanded mortar is measured to characterize the fluidity. There are many factors affecting the fluidity, mainly, the contents of mortar's components, the friction between the table and mortar, vibrating properties, and mortar's rheological property.

2 Rheological modelling

The rheological property is the main factor affecting mortar's fluidity. The rheological property depends only the mortar fluid's properties; while the fluidity is affected by many other factors like vibrating parameters and the table's surface property. There are many rheological models, for examples, the Bingham model [12], The Herschel-Bulkley model [13], Zuo's fractal model [14, 15].

Bingham model [12]:

$$\tau(t) = \tau_0 + \mu \frac{d\varepsilon}{dt} \quad (1)$$

where τ_0 , μ , and ε are, respectively, the yield strength, the viscosity, and the strain.

The Herschel-Bulkley model [13]:

$$\tau(t) = \tau_0 + \mu \left(\frac{d\varepsilon}{dt} \right)^n \quad (2)$$

where n is an index.

Kelvin model [16]:

$$\sqrt{\tau} = \sqrt{\tau_0} + \sqrt{\mu} \frac{du}{dx} \quad (3)$$

The power-law model [17]:

$$\tau = \mu \left(\frac{d\varepsilon}{dt} \right)^n \quad (4)$$

Fractal Maxwell rheological model [18]:

$$\frac{du}{dx^\alpha} = \frac{1}{E} \frac{d\sigma}{dx^\alpha} + \frac{1}{\mu} \sigma, \quad (5)$$

*Corresponding Author: Chao Liu: School of Civil Engineering, Xi'an University of Architecture and Technology, Xi'an, China; School of Science, Xi'an University of Architecture and Technology, Xi'an, China; Email: chaoliu@xauat.edu.cn

Chun-Hui He: School of Civil Engineering, Xi'an University of Architecture and Technology, Xi'an, China

Here d/dx^α is the fractal derivative [19, 20]

Fractal rheological model [14, 15]:

$$\tau = \tau_0 + \mu \frac{du}{dx^\alpha} \quad (6)$$

Fractional Bingham model [21]:

$$\tau(t) = \tau_0 + \mu \lambda^\alpha \frac{d^\alpha \varepsilon}{dt^\alpha} \quad (7)$$

Here d^α/dt^α is the fractional derivative. There are many fractional modifications of the rheological model, for example, the fractional Kelvin-Voigt model [22]. There are many definitions on fractional derivatives [23–30], and most of which are meaningless and cannot be used for practical applications, He's fractional derivative [31, 32] and the two-scale fractal derivative [33–35] have physical understanding and have potential applications in studying mortar's properties. Additionally, according to the dimensional analysis, n in Eqs. (2) and (4) has to equal to one.

3 Theoretical model for mortar's expansion under vibration

Though the mortar's fluidity test has been widely used in fabrication of various functional cements, no theoretical analysis was carried out to elucidate the effect of vibrating properties and mortar's properties on the fluidity. Figure 1 shows the deformation of a mortar in the vibrating table.

Figure 1: Deformation of a mortar in the vibrating table

We assume that the test table's vibration can be described as:

$$y(t) = A \cos \omega t \quad (8)$$

where ω and A are, respectively, the frequency and amplitude of the vibrating table. Its velocity is:

$$u = \frac{dy}{dt} = -A\omega \sin \omega t \quad (9)$$

The mortar's kinetic energy is:

$$E = \frac{1}{2} m u^2 = \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t \quad (10)$$

Its average kinetic energy during the quarter cycle is:

$$\begin{aligned} \bar{E} &= \int_0^{T/4} \left\{ \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t \right\} \sin \omega t dt \\ &= \int_0^{\pi/2} \left\{ \frac{1}{2} m \omega A^2 \sin^3 s \right\} dt = \frac{1}{2} m \omega A^2 \frac{2}{3} \frac{\pi}{2} = \frac{\pi}{6} m \omega A^2 \end{aligned} \quad (11)$$

The mass conservation law of the mortar during the vibration process requires:

$$\frac{1}{4} \pi d^2 \delta = V_0 \quad (12)$$

where d and δ are, respectively, the diameter and the height of the mortar under vibration, V_0 is the mortar's volume. Eq. (12) is obtained under the assumption that the solvent involved in the mortar is not evaporated, so the density of the mortar keeps unchanged.

The energy conservation law reads:

$$4n\bar{E} + mgh = W_f + W_\mu \quad (13)$$

where $4\bar{E}$ is the kinetic energy in a vibrating cycle, n is the total vibrating times in the given period, m is the mortar's mass, h is given as:

$$h = \frac{1}{2} (h_0 - \delta) \quad (14)$$

h_0 is the original height of the mortar before vibration. W_f and W_μ are, respectively, dissipation works due to the friction and viscosity:

$$W_f = \int_0^{2\pi} \int_{d_0/2}^{d/2} f dr d\theta \quad (15)$$

$$W_\mu = \int_0^{2\pi} \int_{d_0/2}^{d/2} \tau dr d\theta \quad (16)$$

For simplicity, we adopt the coulomb friction for the friction between the mortar and the table, it can be expressed as:

$$f = kmg \quad (17)$$

where k is the friction coefficient. So, the friction dissipation work is:

$$W_f = \pi k (d - d_0) mg \quad (18)$$

According to Zuo's fractal rheological model [14], we have:

$$\tau = \tau_0 \exp(-\zeta \omega^\alpha) \quad (19)$$

where τ_0 is the viscous force when $\omega = 0$, ζ is Zou's rheological coefficient, α is the mortar's fractal-related parameter. After a simple calculation, Eq. (19) becomes:

$$W_\mu = \pi \tau_0 (d - d_0) \exp(-\zeta \omega^\alpha) \quad (20)$$

Now Eq. (13) becomes:

$$\frac{2}{3}\pi n m \omega A^2 + \frac{1}{2}mg(h_0 - \delta) = \pi k(d - d_0)mg \quad (21)$$

$$+ \pi \tau_0(d - d_0) \exp(-\zeta \omega^\alpha)$$

In view of Eq. (12), the fluidity of a cement mortar can be expressed as:

$$\frac{2}{3}\pi n m \omega A^2 + \frac{1}{2}mg \left(h_0 - \frac{4V_0}{\pi d^2} \right) \quad (22)$$

$$= \pi k(d - d_0)mg + \pi \tau_0(d - d_0) \exp(-\zeta \omega^\alpha)$$

Eq. (22) reveals that the fluidity depends upon the vibration property, mortar's geometry, friction, and rheological property. In Eq. (22) Zou's rheological coefficient is determined experimentally, that makes Eq. (22) inconvenient for practical applications. Here is suggested another approach to find the fluidity equation by the fractal modification of the rheological model.

4 Fractal fluidity model

Fractal theory is a useful tool to analysis of various functional concretes, Tang *et al.* gave a complete review on fractal approach to cement-based materials [36], and it is Yu-ting Zuo who was the first scientist to apply the fractal theory to study the fluidity, and a fractal rheological model was successfully suggested [5, 6]. Here we apply the basic ideas of the two-scale fractal theory [34] to study the main factors affecting the expanded mortar's diameter.

The average expanding velocity per cycle of the mortar vibrating in the horizontal direction is:

$$v = \frac{d/2 - d_0/2}{nT} \quad (23)$$

Where:

$$T = \frac{2\pi}{\omega} \quad (24)$$

In Eq. (23), we assume that the mortar expands $(d/2 - d_0/2)/n$ each cycle. Eq. (23) becomes:

$$v = \frac{(d - d_0)\omega}{\pi n} = \frac{2(r - r_0)\omega}{\pi n} \quad (25)$$

According to the definition of fractal derivative, the radial viscous force can be calculated as:

$$\tau_r = \mu \frac{dv}{dr^\alpha} = \mu \Gamma(1 + \alpha) \lim_{r-r_0 \rightarrow \Delta r} \frac{v(r) - v(r_0)}{(r - r_0)^\alpha} \quad (26)$$

The fractal derivative has the following properties [35]:

$$\frac{dv}{dr^\alpha} = \begin{cases} v, \alpha = 0 \\ \frac{d^n v}{dr^n}, \alpha = n; (n = 1, 2, 3, \dots) \end{cases} \quad (27)$$

and

$$\frac{d}{dt^\alpha} t^m = \frac{\Gamma(1 + m)\Gamma(1 + \alpha - N)}{\Gamma(1 + m - N)} t^{m-\alpha}, \alpha > N \quad (28)$$

Eq. (26) can be approximated as:

$$\tau_r = \mu \frac{dv}{dr^\alpha} \approx \mu \Gamma(1 + \alpha) \frac{2\omega}{\pi n(r - r_0)^{\alpha-1}} \quad (29)$$

Similarly, the axial viscous force is:

$$\tau_y = \mu \frac{du}{dy^\alpha} \approx \mu \Gamma(1 + \alpha) \frac{A\omega}{4n(h_0/2 - h/2)^\alpha} \quad (30)$$

The dissipation works in radial and axial directions are, respectively, as:

$$W_r = \int_0^{2\pi} \int_{r_0}^r \tau_r dr d\theta = \frac{2\mu\omega\Gamma(1 + \alpha)}{\pi(2 - \alpha)n} (r - r_0)^{2-\alpha} \quad (31)$$

$$W_y = \int_0^{2\pi} \int_{h/2}^{h_0/2} \tau_y dr dy \quad (32)$$

$$= 2\pi\mu\Gamma(1 + \alpha) \frac{A\omega}{4n(h_0/2 - h/2)^{\alpha-1}}$$

So, Eq. (23) can be updated as:

$$\frac{2}{3}\pi n m \omega A^2 + \frac{1}{2}mg \left(h_0 - \frac{4V_0}{\pi d^2} \right) \quad (33)$$

$$= \pi k(d - d_0)mg + \frac{2\mu\omega\Gamma(1 + \alpha)}{\pi(2 - \alpha)n} (r - r_0)^{2-\alpha}$$

$$+ 2\pi\mu\Gamma(1 + \alpha) \frac{A\omega}{4n(h_0/2 - h/2)^{\alpha-1}}$$

where h can be calculated as:

$$h = \frac{1}{2} \left(h_0 - \frac{4V_0}{\pi d^2} \right) \quad (34)$$

Finally, we obtain the following fluidity equation:

$$\frac{2}{3}\pi n m \omega A^2 + \frac{1}{2}mg \left(h_0 - \frac{4V_0}{\pi d^2} \right) \quad (35)$$

$$= \pi k(d - d_0)mg + \frac{2\mu\omega\Gamma(1 + \alpha)}{\pi(2 - \alpha)n} \left(\frac{d}{2} - \frac{d_0}{2} \right)^{2-\alpha}$$

$$+ 2\pi\mu\Gamma(1 + \alpha) \frac{A\omega}{4n \left[\frac{1}{2}h_0 - \frac{1}{4} \left(h_0 - \frac{4V_0}{\pi d^2} \right) \right]^{\alpha-1}}$$

For given test conditions, all parameters except α in Eq. (33) can be determined experimentally, so the fluidity experiment can be used to determine the value of α in Zuo's fractal rheological model [15].

5 Conclusion

This paper studies the experiment for measuring the fluidity of a cement mortar, the theoretical analysis shows that the measured fluidity depends upon not only the mortar's rheological property, but also measuring conditions. When all measuring conditions are fixed, the fluidity experiment can be used for determination of the fractal dimension of α in Zuo's fractal rheological model [15], an inexplicit formulation for calculation of the fractal order is given Eq. (35), it can be solved by Newton's iteration method or Chun-Hui He's iteration algorithm.

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