Communication

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Results for the heat transfer of a fin with exponential-law temperature-dependent thermal conductivity and power-law temperature-dependent heat transfer coefficients

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Abstract: In this article, thermal behavior analysis of nonlinear fin problem with power-law heat transfer coefficient is studied to determine temperature distribution. This new supposition for the thermal conductivity, exponential-law temperature dependent, makes it to be nonlinear that is a general case in some sense. It is shown that the governing fin equation, that is, a nonlinear second-order differential equation, is exactly solvable with proper boundary conditions. To this purpose, the order of differential equation is reduced and then is converted into a total differential equation by multiplying a proper integration operant. An exact analytical solution is given to advance physical meaning, and the existence of unique solution for some specific values of the parameters of the model is demonstrated. The results are shown graphically. It is observed that fin efficiency is decreasing with respect to the power-law mode for heat transfer.

Keywords: fin efficiency, nonlinear heat transfer equation, exact closed-form solution, fin effectiveness, temperature distribution, unique solution

1 Preliminaries and problem formulation

It is noticeable that thermal studying of both solid and porous fins with regard to the differences in profiles and thermo-physical properties have been mainly focused by researchers [1,2]. In many engineering applications such as conveying flow of electricity through a conductor, nuclear rods and many other heating accessories for thermal producers, fins should be considered where conductive rate at temperature makes the model nonlinear and exponentially challenging to reach exact solution [3–11]. Kern and Kraus [12] represented its extensive surfaces and industrial applications. Also, it is difficult to obtain the accurate closed form solutions of these kinds of nonlinear problems especially when, heat transfer and thermal conductivity factors are variable and large temperature differences exist. Numerous numerical techniques and analytical methods have been carried out to solve these problems. Aziz and Hug [13] and Benzie [14] are pioneers in solving such problems, but when the factors vary linearly, techniques of perturbation ideas were applied. The differential equation and boundary condition of a fin with linear temperature-dependent heat transfer coefficient are in the following form [15]:

$$(1 + \beta\theta) \frac{d^2\theta}{dx^2} - M^2\theta^{n+1} + \beta \left(\frac{d\theta}{dx} \right)^2 = 0, \quad (1)$$

$$\text{BCs: } \{\theta'(0) = 0, \theta(1) = 1\}.$$

This problem has been solved in the case of fixed heat transfer coefficient and thermal conductivity is changing linearly with respect to the temperature ($n = 0$ in Eq. (1)) by using semi-analytical methods such as polynomial method, homotopy perturbation method (HPM), homotopy analysis method (HAM), differential transform method (DTM) and adomian decomposition method (ADM) [16–21]. Khani et al. [15] investigated the solutions of HPM, ADM

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and HAM when M rises to a large number. Lesnic and Heggs [22], Chang [23] and Chowdhury *et al.* [24] studied this equation for a fixed thermal conductivity ($\beta = 0$) in Eq. (1) using DM, ADM, HPM and HAM. Moreover, a lot of research works related to the problem Eq. (1) have been reported by focusing on its different aspects with different techniques, see refs [23–31] and references therein.

In this article, we study this nonlinear fin, but by considering exponential-law temperature-dependent for the factor of thermal conductivity. The problem on desk is presented as:

$$\frac{d}{dx} \left(k(T) \frac{dT}{dx} \right) - \frac{h(T)P}{A} (T - T_f) = 0, \quad 0 \leq X \leq L, \quad (2)$$

$$\frac{dT}{dx}(0) = 0, \quad T(L) = T_b, \quad (3)$$

where L is the length of fin, P is its perimeter and T_b is the base temperature. Among the different types of boundary conditions, Dirichlet condition, Neumann condition and the Robin condition, we assumed Neumann condition which means it lacks heat transfer at the tip of the fin. Also, Dirichlet condition prescribes temperature T_b to the base of the fin.

It is important to emphasize that we consider the exponential-law temperature-dependent thermal conductivity in this work and also, as in other nonlinear models for heat transfer of the fin, the power-law temperature-dependent heat transfer factor is assumed, in other words, we have

$$K(T) = K_b \left[\frac{\exp\left(\beta \frac{T - T_a}{T_b - T_a}\right) - 1}{\exp(\beta) - 1} \right], \quad \beta > 0, \quad (4)$$

$$h(T) = h_b \left(\frac{T - T_a}{T_b - T_a} \right)^n,$$

where K_b and h_b are their coefficients at the base temperature, respectively. The exponent n in Eq. (4) explains the mode of heat transfer. They are usually $-\frac{1}{4}, \frac{1}{4}, \frac{1}{3}, 2, 3$ [5,25]. Dimensionless parameters are as follows:

$$M^2 = \frac{h_b P L^2}{K_b A}, \quad x = \frac{X}{L}, \quad \theta = \frac{T - T_a}{T_b - T_a} \quad (5)$$

According to Eq. (1), the problem and its boundary conditions in dimensionless form could be rewritten as follows:

$$\frac{d}{dx} \left[(\exp(\beta\theta) - 1) \frac{d\theta}{dx} \right] - M^2 \theta^{n+1} = 0, \quad 0 \leq x \leq 1 \quad (6)$$

$$\frac{d\theta}{dx}(0) = 0, \quad \theta(1) = 1, \quad (7)$$

or equivalently

$$\beta \exp(\beta\theta) \cdot \left(\frac{d\theta}{dx} \right)^2 + \frac{d^2\theta}{dx^2} \cdot (\exp(\beta\theta) - 1) - M^2 \theta^{n+1} = 0, \quad (8)$$

$$0 \leq x \leq 1$$

$$\frac{d\theta}{dx}(0) = 0, \quad \theta(1) = 1. \quad (9)$$

Generally, there are many numerical and semi-analytical methods to deal with the boundary value problems arisen from the heat transfer of a fin and other kinds of problems such as MHD flow of Newtonian and non-Newtonian fluid. In ref. [32], homotopy analysis method has been applied to analyze concentration flux dependent on radiative MHD Casson flow with Arrhenius activation energy. Radiative bioconvection nanofluid squeezing flow has been discussed by a semi-numerical study with the DTM-Padé approach [33]. Abbas *et al.* [34] considered artificial neural networks for parametric analysis and minimization of entropy generation in bioinspired magnetized non-Newtonian nanofluid pumping. Also, readers are referred to see some related works refs [35,36]. On the other hands, there are some valuable studies which present exact closed-form solutions for some of these models in some especial cases [37–40]. The other main aim we seek in this work is to provide exact closed-form solution for problems (8)–(9).

2 Accurate closed form solution

We have the following relation by changing variable $u = \frac{d\theta}{dx}$:

$$\frac{d^2\theta}{dx^2} = \frac{du}{dx} = \frac{du}{d\theta} \cdot \frac{d\theta}{dx} = u \cdot \frac{du}{d\theta}. \quad (10)$$

Therefore, Eq. (8) is changed to the following equation:

$$(\exp(\beta\theta) - 1)udu + (\beta \exp(\beta\theta)u^2 - M^2\theta^{n+1})d\theta = 0. \quad (11)$$

This equation can be modified to a differentiable one by multiplying each side by

$$\exp(\beta\theta) - 1,$$

i.e.,

$$(\exp(\beta\theta) - 1)^2 udu + (\exp(\beta\theta) - 1)(\beta \exp(\beta\theta)u^2 - M^2\theta^{n+1})d\theta = 0. \quad (12)$$

Now, we look for a function such that the derivatives with respect to u and θ be

$$(\exp(\beta\theta) - 1)^2 u \quad \text{and} \quad (\exp(\beta\theta) - 1)(\beta \exp(\beta\theta)u^2 - M^2\theta^{n+1}),$$

respectively. Then the solution is easily obtained as:

$$M^2\theta^{n+2}(-\beta\theta)^{-n-2}\Gamma(n+2, -\beta\theta) + \frac{M^2\theta^{n+2}}{n+2} - u^2e^{\beta\theta} + \frac{1}{2}u^2e^{2\beta\theta} + \frac{u^2}{2} = C, \quad (13)$$

where C is the integral constant, after replacing u by $\frac{d\theta}{dx}$, Eq. (13) is converted into the following equation:

$$M^2\theta^{n+2}(-\beta\theta)^{-n-2}\Gamma(n+2, -\beta\theta) + \frac{M^2\theta^{n+2}}{n+2} - \left(\frac{d\theta}{dx}\right)^2 e^{\beta\theta} + \frac{1}{2}\left(\frac{d\theta}{dx}\right)^2 e^{2\beta\theta} + \frac{1}{2}\left(\frac{d\theta}{dx}\right)^2 = C, \quad (14)$$

where parameter C is reachable by the first boundary condition as follows:

$$C = M^2\theta_0^{n+2}(-\beta\theta_0)^{-n-2}\Gamma(n+2, -\beta\theta_0) + \frac{M^2\theta_0^{n+2}}{n+2}.$$

That $\theta_0 = \theta(0)$ is the dimensionless temperature of the fin at the tip, by interchanging C into, we have:

$$\begin{aligned} & -\left(\frac{d\theta}{dx}\right)^2 e^{\beta\theta} + \frac{1}{2}\left(\frac{d\theta}{dx}\right)^2 e^{2\beta\theta} + \frac{1}{2}\left(\frac{d\theta}{dx}\right)^2 \\ & = M^2\theta_0^{n+2}(-\beta\theta_0)^{-n-2}\Gamma(n+2, -\beta\theta_0) + \frac{M^2\theta_0^{n+2}}{n+2} \\ & \quad - M^2\theta^{n+2}(-\beta\theta)^{-n-2}\Gamma(n+2, -\beta\theta) - \frac{M^2\theta^{n+2}}{n+2}. \end{aligned} \quad (15)$$

Or equally,

$$\begin{aligned} & \frac{1}{2}(\exp(\beta\theta) - 1)^2 \left(\frac{d\theta}{dx}\right)^2 \\ & = \frac{M^2}{(-\beta)^{n+2}}(\Gamma(n+2, -\beta\theta_0) - \Gamma(n+2, -\beta\theta)) \\ & \quad + \frac{M^2}{n+2}(\theta_0^{n+2} - \theta^{n+2}), \end{aligned} \quad (16)$$

where the function $\Gamma(a, z)$ is the incomplete gamma function which is defined by the integral

$$\Gamma(a, z) = \int_z^\infty t^{a-1} \exp(-t) dt.$$

Eq. (16) can be represented as

$$dx = \frac{(\exp(\beta\theta) - 1)d\theta}{\sqrt{2M} \sqrt{\frac{\Gamma(n+2, -\beta\theta_0) - \Gamma(n+2, -\beta\theta)}{(-\beta)^{n+2}} + \frac{\theta_0^{n+2}}{n+2} - \frac{\theta^{n+2}}{n+2}}}. \quad (17)$$

After integration from both sides of Eq. (17) and imposition of the notation, we have

$$\sqrt{2}Mx = \int_{\theta_0}^{\theta} \frac{(\exp(\beta z) - 1)dz}{\sqrt{\frac{\Gamma(n+2, -\beta\theta_0) - \Gamma(n+2, -\beta z)}{(-\beta)^{n+2}} + \frac{\theta_0^{n+2}}{n+2} - \frac{z^{n+2}}{n+2}}}. \quad (18)$$

In order to deal with Eq. (19) easily, let us define the right hand side as a new non-algebraic function of definite integral:

$$F(\theta; \theta_0, n, \beta) = \int_{\theta_0}^{\theta} \frac{(\exp(\beta z) - 1)dz}{\sqrt{\frac{\Gamma(n+2, -\beta\theta_0) - \Gamma(n+2, -\beta z)}{(-\beta)^{n+2}} + \frac{\theta_0^{n+2}}{n+2} - \frac{z^{n+2}}{n+2}}}. \quad (19)$$

The function $F(\theta; \theta_0, n, \beta)$ can be treated as identical to the other familiar functions by current powerful computer software such as Maple and Mathematica. Then, we can obviously rewrite the solution as the following form:

$$\sqrt{2}Mx = F(\theta; \theta_0, n, \beta) \quad (20)$$

On the other hand, θ_0 is an unknown parameter in Eq. (20). But, it can be disclosed indeed by:

$$\sqrt{2}M = F(1; \theta_0, n, \beta). \quad (21)$$

Now, the exact closed form solution is represented by Eq. (20), when θ_0 is determined through Eq. (21) for any given M , n and β . There would be no difficulty to work with non-algebraic function $F(\theta; \theta_0, n, \beta)$ as the same as other known function for everyone familiar with the software programs. It is important to announce that the multiplicity of solution to the root θ_0 in solving the nonlinear Eq. (21) proves the existence of the multiplicity of the solutions to the problem Eq. (8).

3 Fin efficiency and effectiveness

According to ref. [31], fin efficiency is the ratio of the real heat transfer rate to the ideal heat transfer rate if the entire fin were at the base temperature,

$$\eta = \frac{Q_{\text{actual}}}{Q_{\text{ideal}}} = \frac{\int_0^L ph(T)(T - T_a)dx}{pLh_b(T_b - T_a)} = \int_0^1 \theta^{n+1}(x)dx, \quad (22)$$

hence

$$\eta = \int_0^1 \{F^{-1}(\sqrt{2}Mx; \theta_0, n, \beta)\}^{n+1}dx, \quad (23)$$

where F^{-1} is the inverse function of F . Also, the rate of heat transferred by a fin to the rate of heat transferred without the fin is fin effectiveness that is given, in dimensionless form, by

$$\varepsilon = \omega \int_0^1 \theta^{n+1}(x)dx = \omega \int_0^1 \{F^{-1}(\sqrt{2}Mx; \theta_0, n, \beta)\}^{n+1}dx, \quad (24)$$

where ω is the fin length to the fin thickness ratio.

4 Main results

In the previous sections, exact closed form solution of the nonlinear fin problem formulated by Eqs. (8) and (9) has been developed and represented by the form of Eq. (20) and augmented to Eq. (21). The implicit solution Eq. (20) can be easily obtained by computer's software mentioned before, we have used Mathematica in this article.

As it can be shown, Figure 1 illustrates the effect of fin parameter M on the temperature. With regard to Eq. (5), it can be proven that when fin parameter increases, the mean tip end temperature and the mean temperature decline. At the end of the fin where $x = 0$, when the ratio h_b/k_b increases, the temperature along the fin has lower figure. The inverse condition would happen if this ratio decreases.

Temperature distribution along the fin has been shown in Figure 2 for different values of β for $n = 1/3$ and $M = 1$. In Figure 3, temperature distribution has

been drawn for different values of n for $\beta = -1/2$ and $M = 2$. Furthermore, Figure 4 stands for temperature distribution when both n and β change simultaneously

Figure 3: Diagram of $\theta(x)$ versus x for temperature distributions with $M = 1$ and $\beta = \frac{1}{2}$.

Figure 1: Diagram of $\theta(x)$ versus x for temperature distributions with $n = \beta = \frac{1}{4}$.

Figure 4: Diagram of $\theta(x)$ versus x for temperature distributions with $M = \frac{1}{2}$.

Figure 2: Diagram of $\theta(x)$ versus x for temperature distributions with $M = 1$ and $n = \frac{1}{3}$.

Figure 5: Diagram of fin efficiency versus M for different n with $\beta = \frac{1}{2}$.

with $M = \frac{1}{2}$. The effect of different values of thermo-geometric parameter on fin efficiency is shown in Figure 5. Regarding correlation between the mean temperature and fin efficiency, both of them have identical treatment.

5 Conclusion

The present study solves the nonlinear fin problem with exponentially temperature-dependent thermal conductivity exactly and presents exact analytical solution of the problem in implicit form. To this aim, we have reduced the order of differential equation and then converted into a total differential equation by multiplying a proper integration operant, after that we resolved it by imposing boundary conditions. The problem has been assumed that transfer coefficient is power-law temperature dependent. Depending on different values of the parameters of the model n , M and β , we have extracted at least one solution for the considered problem. The existence of multiple solutions can be a new research line for future works. Furthermore, the exact analytical expression for fin efficiency has been obtained and illustrated graphically.

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