Research Article

Luiz Manuel Braga da Costa Campos, Manuel José dos Santos Silva*, and Agostinho Rui Alves da Fonseca

On the multipath effects due to wall reflections for wave reception in a corner

<https://doi.org/10.1515/noise-2021-0004>

Received Jul 05, 2020; accepted Nov 20, 2020

Abstract: Multipath effects occur when receiving a wave near a corner, for example, the noise of an helicopter or an aircraft or a drone or other forms of urban air mobility near a building, or a telecommunications receiver antenna near an obstacle. The total signal received in a corner consists of four parts: (i) a direct signal from source to observer; (ii) a second signal reflected on the ground; (iii) a third signal reflected on the wall; (iv) a fourth signal reflected from both wall and ground. The problem is solved in two-dimensions to specify the total signal, whose its ratio to the direct signal specifies the multipath factor. The amplitude and phase of the multipath factor are plotted as functions of the frequency over the audible range, for various relative positions of observer and source, and for several combinations of the reflection coefficients of the ground and wall. It is shown that the received signal consists of a double series of spectral bands, in other words: (i) the interference effects lead to spectral bands with peaks and zeros; (ii) the successive peaks also go through zeros and “peaks of the peaks”. The results apply not only to sound, but also to other waves, *e.g.*, electromagnetic waves using the corresponding frequency band and reflection factors.

Keywords: direct signal, reflected signal, corner, amplitude and phase changes, multipath factor, sound, electromagnetic waves

1 Introduction

Aircraft noise near airports can limit the use of runways at night and other times, leads to take-off weight limits that affect economics and if not controlled could become an obstacle to air traffic growth. Aircraft noise [1] is the subject of international certification standards, with some airports or local authorities applying lower limits. The efforts to reduce noise near airports lead to a balanced approach [2] combining low self-noise aircraft with low noise operations to minimize the number of people affected within given ground contours [3]. The certification and noise monitoring depend on measuring microphones that receive the direct sound wave from the aircraft. If the microphone is near the ground, a reflected wave is added to the direct wave; if the two waves are in-phase, the amplitude is doubled corresponding to an increase of $10 \log_{10} 2 = 3$ dB for the amplitude and $20 \log_{10} 2 = 6$ dB for the power. If the microphone is in a corner, as sketched in the Figure 1, then there are three reflected waves: one from the ground, one from the wall and one reflected from both; together with the direct wave there are four waves, and if all four are in-phase, the amplitude is multiplied by 4 leading to an increase of $10 \log_{10} 4 = 6$ dB for the amplitude and $20 \log_{10} 4 = 12$ dB for the power. Thus, the norms on noise measurement [4, 5] specify a 6 dB increase in power near the ground and 12 dB near the corner.

These noise corrections are extreme worst case scenarios because: (i) if waves are out-of-phase there is less amplification and there may be even cancellation; (ii) if the ground and wall are not perfectly reflecting then wave transmission or absorption reduces the amplitude. In addition, urban morphology [6–8] is not reduced to infinite plane and orthogonal corner reflectors. Further changes to the received sound field arise due to atmospheric wind and turbulence [9, 10]. More fundamentally, the effect of a reflector is to lead to interference between the direct and reflected waves, resulting on amplification, attenuation or even cancellation depending on the frequency (or wave length) and position of the observer and source relative to the obstacle. In the case of an orthogonal corner, sketched in the

***Corresponding Author: Manuel José dos Santos Silva:** CCTAE, IDMEC, LAETA, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1, 1049-001, Lisboa, Portugal; Email: manuel.jose.dos.santos.silva@tecnico.ulisboa.pt

Luiz Manuel Braga da Costa Campos: CCTAE, IDMEC, LAETA, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1, 1049-001, Lisboa, Portugal; Email: luis.campos@tecnico.ulisboa.pt

Agostinho Rui Alves da Fonseca: CCTAE, IDMEC, LAETA, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais 1, 1049-001, Lisboa, Portugal; Email: agostinho.fonseca@tecnico.ulisboa.pt

Figure 1: Observer O at (a, b) receiving from the source S at (x, y) four signals: (i) one direct from the source to observer at distance r ; (ii) one with the reflection on the ground making the distance $r_{11} + r_{12}$; (iii) one with the reflection on the wall making the distance $r_{21} + r_{22}$; (iv) one with the reflection on the ground followed by the reflection on the wall making the distance $r_{31} + r_{32} + r_{33}$.

Figure 2: Sound source S and observer O near a corner, with elevation angle for the latter α larger than for the former β , that is, $\beta < \alpha$, showing only the reception path with two intermediate reflections.

Figure 2, the position can be specified by Cartesian (x, y) or polar (r, θ) coordinates for the source and observer. The effect of reflections can be calculated for sound pulses [11] or for sinusoidal waves, which can form any spectrum by superposition. The present paper considers a sound source

and an observer/receiver at arbitrary positions relative to an orthogonal corner taking into account the interference between the direct and the three reflected waves, for any frequency, allowing for different reflection coefficients from the ground and the wall.

In general, the problem of multipath propagation and interference applies to all waves, in particular, acoustic [12–17] and electromagnetic waves [18–22]. The situation is illustrated in the two dimensional case in Figure 1, showing that the observer receives four signals: (i) one direct signal from the source; (ii) one signal reflected from the ground; (iii) one signal reflected from the wall; (iv) a fourth signal reflected from both wall and ground. The positions of the reflection points are determined by the condition of equal angles of incidence and reflection; once the positions of the reflection points are determined, the lengths of all the ray paths can be calculated. Together with the reflection coefficients, this specifies the total received signal; normalizing with regard to the direct signal specifies the multipath factor accounting for the interference among the four waves. The multipath factor is generally complex, with the modulus specifying the amplitude change and the argument specifying the phase change.

The problem is solved in two dimensions (Figure 1) by determining the total received field (subsection 2.1), which consists of the direct plus three reflected waves. The waves reflected on the ground (subsection 2.2) and on the wall (subsection 2.3) are specified by the positions of the respective reflection points and by the lengths of the two resulting ray paths; for the fourth wave reflected on the ground and then on the wall (subsection 2.4), the positions of the two reflection points are coupled, and specify the three lengths of three ray paths. Concerning the fourth signal there are three cases: (i) if the elevation angle of the observer is above that of the source ($\alpha > \beta$ in Figure 2), then the first reflection is on the ground and the second on the wall (subsection 2.4); (ii) in the reverse case ($\beta > \alpha$ in Figure A1), the first reflection is on the wall and the second on the ground (appendices A.1 and A.2); (iii) in the intermediate case of observer and source on the same elevation angle ($\beta = \alpha$ in Figure A2), the double reflection on the corner is treated as the limit of the preceding (appendices A.2 and A.4). The total signal is the sum of all four signals taking into account the reflection coefficients (appendix C) on the ground and wall. The total signal is normalized to the direct signal, to specify the multipath factor, whose amplitude and phase are plotted for: (subsection 3.1) two relative positions of source and observer (Figures 3 and 4); (subsection 3.2) three combinations of the reflection factors on the ground and wall (Figures 5 to 7). The results may be recast in terms of source distance and direction (subsection 3.3), and be simplified for a source in the far field. Thus, as an alternative to the preceding, for a fixed frequency, the amplitude and phase changes may be evaluated (subsection 3.4) as functions of source distance to the corner (Figure 8) or as functions of the direction of arrival of the signal (Figure 9). The contour

maps for the amplitude (Figures 10 and 12) and phase (Figures 11 and 13) of the multipath factor apply not only to acoustic, but also to electromagnetic waves (section 4).

2 Direct, singly- and doubly-reflected signals

The total signal received from a distant source by an observer in a corner (Figure 1) consists of a direct signal (subsection 2.1), plus reflections on the ground (subsection 2.2) and on the wall (subsection 2.3) plus a double reflection on both (subsection 2.4).

2.1 Total signal as sum of four waves

Consider the two-dimensional problem (Figure 2) of wave reception from a source S by an observer O near a corner between a horizontal ground $y = 0$ and a vertical wall $x = 0$ taken as axis of a Cartesian reference with origin at the corner. The observer,

$$O \mapsto (a, b) = s (\cos \alpha, \sin \alpha), \quad (1a)$$

and source,

$$S \mapsto (x, y) = q (\cos \beta, \sin \beta), \quad (1b)$$

are at the distance

$$r = \left| (x - a)^2 + (y - b)^2 \right|^{1/2} \quad (2)$$

where s is the distance between the corner and the observer, and q is the distance between the corner and the source. The distance r specifies the direct signal, to which are added reflected signals, illustrated in the Figure 1, for source farther from the origin than the observer. The viscosity for the sound field in air at the most audible frequencies are negligible since the Reynolds number is very large, being of the order of 10^8 . The sound is therefore considered as a weak motion of an inviscid fluid, in this case from an initial state of rest, and thermal conduction is also neglected. Since the sound wave induces small perturbations in the air, its presence can be assumed as a linear perturbation. Consequently, the product of two perturbations are neglected and the laws describing the movement are linear, using first-order approximations. Since there is no interaction between the sound waves, they can be added by superposition, to obtain the total sound field [23]. Four signals are received and the total acoustic pressure perturbation is given by

$$p_{\text{tot}} = \frac{1}{r} \exp(ikr) + \frac{R_1}{r_{11} + r_{12}} \exp[ik(r_{11} + r_{12})] \quad (3)$$

$$+ \frac{R_2}{r_{21} + r_{22}} \exp[ik(r_{21} + r_{22})] \\ + \frac{R_{31}R_{32}}{r_{31} + r_{32} + r_{33}} \exp[ik(r_{31} + r_{32} + r_{33})],$$

corresponding to four terms in (3), namely: (i) the first, where k is the wavenumber, is the direct wave from source to observer, at distance r , and is taken with unit amplitude (the complex amplitude and the temporal part would drop out when normalizing the total signal to the direct signal); (ii) the second term involves the reflection factor R_1 of the horizontal wall at the reflection point P_1 , whose coordinates $(x_1, 0)$ specify the distance from source to reflection point r_{11} and the distance from reflection point to observer r_{12} ; (iii) the third term involves the reflection factor R_2 of the vertical wall at the reflection point P_2 , whose coordinates $(0, y_2)$ specify the distance from source to reflection point r_{21} and the distance from reflection point to observer r_{22} ; (iv) the last term involves the reflection factors of the two walls, R_{31} and R_{32} , respectively at the reflection points P_{31} and P_{32} , whose coordinates $(x_{31}, 0)$ and $(0, y_{32})$ specify the distances from source to first reflection point r_{31} , between reflection points r_{32} , and from the second reflection point to observer r_{33} . The angles in Figure 1 follow the law of specular reflection stating that the angle between the normal to the surface and reflected wave is equal to the angle between the same normal and incident wave. The equation (3) is an harmonic solution of the linearised wave equation assuming that the pressure perturbation is radial (and unsteady) and represents a wave with outward spherical propagation centred at the source [23]. The physical solution can be given by the real part of (3). The reflection factors may be complex, involving amplitude and phase changes for an impedance ground and/or wall. If the reflection factor is uniform on the ground R_h and on the wall R_v , then (3) simplifies with

$$R_1 = R_{31} = R_h, \quad (4a)$$

$$R_2 = R_{32} = R_v. \quad (4b)$$

The reflection coefficients of the ground R_h and of the wall R_v may be different (for instance, grass and concrete) or equal (for instance, both concrete). A brief review about the reflection coefficients is described in the appendix C.

2.2 Signal due to reflection on the ground

The equality of the angles θ_1 of incidence and reflection on the horizontal plane,

$$\frac{x - x_1}{y} = \cot \theta_1 = \frac{x_1 - a}{b}, \quad (5)$$

specifies the position $(x_1, 0)$ at the reflection point P_1 , that is,

$$x_1 = \frac{ay + bx}{y + b}. \quad (6)$$

The latter determines the distance from the source to the reflection point,

$$r_{11} = \left| (x - x_1)^2 + y^2 \right|^{1/2} = y \left| 1 + \frac{(x - a)^2}{(y + b)^2} \right|^{1/2}, \quad (7a)$$

and the distance from the reflection point to the observer,

$$r_{12} = \left| (x_1 - a)^2 + b^2 \right|^{1/2} = b \left| 1 + \frac{(x - a)^2}{(y + b)^2} \right|^{1/2}, \quad (7b)$$

where (6) was used. These two distances, in (7a) and (7b), determine the second term in (3) and specify the signal reflected on the horizontal ground.

2.3 Signal due to reflection on the wall

The equality of the angles θ_2 of incidence and reflection on the vertical plane,

$$\frac{y - y_2}{x} = \cot \theta_2 = \frac{y_2 - b}{a}, \quad (8)$$

specifies the position $(0, y_2)$ at the reflection point P_2 , that is,

$$y_2 = \frac{ay + bx}{x + a}. \quad (9)$$

The latter determines the distance from the source to the reflection point,

$$r_{21} = \left| x^2 + (y - y_2)^2 \right|^{1/2} = x \left| 1 + \frac{(y - b)^2}{(x + a)^2} \right|^{1/2}, \quad (10a)$$

and the distance from the reflection point to the observer,

$$r_{22} = \left| a^2 + (y_2 - b)^2 \right|^{1/2} = a \left| 1 + \frac{(y - b)^2}{(x + a)^2} \right|^{1/2}, \quad (10b)$$

where (9) was used. These two distances, in (10a) and (10b), determine the third term of (3), which specifies the signal reflected from the vertical wall.

2.4 Signal due to double reflection on the ground and on the wall

The angles of incidence or reflection on the horizontal, θ_{31} , and vertical, θ_{32} , planes couple the positions of the reflection point P_{31} on the ground $(x_{31}, 0)$,

$$\frac{x - x_{31}}{y} = \cot \theta_{31} = \frac{x_{31}}{y_{32}}, \quad (11a)$$

and the reflection point P_{32} on the wall $(0, y_{32})$,

$$\frac{y_{32}}{x_{31}} = \cot \theta_{32} = \frac{b - y_{32}}{a}. \quad (11b)$$

Solving the last two equations for y_{32} gives the equality

$$\frac{x_{31}y}{x - x_{31}} = y_{32} = \frac{x_{31}b}{a + x_{31}}, \quad (12)$$

from which follows

$$(a + x_{31})y = b(x - x_{31}), \quad (13)$$

which specifies the position of the first reflection point,

$$x_{31} = \frac{bx - ay}{y + b}. \quad (14a)$$

The position of the second reflection point,

$$y_{32} = \frac{bx - ay}{x + a}, \quad (14b)$$

follows substituting (14a) in (11a) or in any equality of (12). The positions of both reflection points determine the distances: (i) from the source to the first reflection point on the ground,

$$r_{31} = \left| (x - x_{31})^2 + y^2 \right|^{1/2} = y \left| 1 + \frac{(x + a)^2}{(y + b)^2} \right|^{1/2}; \quad (15a)$$

(ii) from the first reflection point on the ground to the second reflection point on the wall,

$$\begin{aligned} r_{32} &= \left| (x_{31})^2 + (y_{32})^2 \right|^{1/2} \\ &= |bx - ay| \left| \frac{1}{(x + a)^2} + \frac{1}{(y + b)^2} \right|^{1/2}; \end{aligned} \quad (15b)$$

(iii) from the second reflection point on the wall to the observer,

$$r_{33} = \left| a^2 + (b - y_{32})^2 \right|^{1/2} = a \left| 1 + \frac{(y + b)^2}{(x + a)^2} \right|^{1/2}. \quad (15c)$$

The last three equations are valid if $\beta \leq \alpha$. In the relations (15a) and (15b) was used (14a) and in the relations (15b) and (15c) was used (14b). The calculations from (11a) to (15c) assume that the first reflection is on the ground and the second is on the wall, as indicated in the Figure 2. This is the case if the azimuth (or elevation angle) β of the source in (1b) is less than the azimuth α of the observer in (1a), $\beta \leq \alpha$. If the reverse was true, $\beta \geq \alpha$, then a similar calculation holds with reflection first on the wall and then on the ground. The third case of sound and observer on the same azimuth, $\beta = \alpha$, corresponds to reflection at the “corner”, and can be treated as the boundary $\beta \rightarrow \alpha \pm 0$ between the two cases $\beta \geq \alpha$ and $\beta \leq \alpha$. These differences affect only the doubly reflected wave, that is, the fourth term of (3).

3 Multipath effects on the amplitude and phase of the signal

The total signal normalized to the incident signal specifies the amplitude and phase changes (subsection 3.1). These are plotted over the whole audible range (subsection 3.2) for two relative positions of source and observer and three combinations of reflection factors of the ground and wall. For a distant source, the amplitude and phase changes may be simplified (subsection 3.3), and plotted in terms of direction of arrival of the signal and for different source distances in the plane (subsection 3.4).

3.1 Multipath factor due to ground and wall reflections

The multipath factor F is defined as the ratio to the pressure perturbation of the direct signal, assuming that is a spherical wave [23],

$$p_{\text{dir}} = \frac{1}{r} \exp(ikr), \quad (16)$$

of the pressure perturbation of the total signal (3), that is,

$$F = \frac{p_{\text{tot}}}{p_{\text{dir}}} = p_{\text{tot}} r \exp(-ikr), \quad (17)$$

leading to

$$\begin{aligned} F &= 1 + R_h \frac{r}{r_{11} + r_{12}} \exp[ik(r_{11} + r_{12} - r)] \\ &\quad + R_v \frac{r}{r_{21} + r_{22}} \exp[ik(r_{21} + r_{22} - r)] \\ &\quad + R_h R_v \frac{r}{r_{31} + r_{32} + r_{33}} \exp[ik(r_{31} + r_{32} + r_{33} - r)]. \end{aligned} \quad (18)$$

The pressure perturbation of the direct signal (16) is an harmonic solution of the linearised outward spherical wave equation, centred from the source, where the physical solution can be given by its real part [23]. The multipath factor (18) depends on the various distances, $r, r_{11}, r_{12}, r_{21}, r_{22}, r_{31}, r_{32}, r_{33}$, specified respectively by the relations determined in the section 2. The multipath factor is generally complex,

$$F = |F| \exp\{i \arg(F)\}, \quad (19)$$

and its amplitude and phase are plotted separately respectively at the top $|F|$ and bottom $\arg(F)$ of Figures 3 to 7, versus frequency over the audible range, $20 \text{ Hz} \leq f \leq 20 \text{ kHz}$.

In all five figures, the source is far from the corner, with

$$x = 700 \text{ m}, \quad y = 30 \text{ m}. \quad (20)$$

The observer position and the impedance of horizontal and vertical walls are indicated in the Table 1.

The Figure 3 is the baseline case with observer at position $a = 3$ m and $b = 2$ m, and with uniform rigidity of horizontal and vertical walls equal to one. When the re-

flection factors equal to one, the wave is totally reflected because the acoustic pressure of both waves is the same. In Figure 4 the observer position is changed, in Figure 5

Figure 3: Modulus (top) and phase (bottom) of the multipath factor (18) versus frequency in the audible range (left), $20 \leq f \leq 20000$ Hz, or in the sub-range (right), $20 \leq f \leq 1000$ Hz, for a fixed observer at the position $(a, b) = (3, 2)$ m and fixed source at the position $(x, y) = (700, 30)$ m, and rigid ground and wall, $R_h = R_v = 1$.

Figure 4: The same as Figure 3, but for a modified observer at the position $(a, b) = (2, 6)$ m.

Figure 5: The same as Figure 3, but for halved reflection factor on the ground, $R_h = 0.5$.

Figure 6: The same as Figure 3, but for halved reflection factor on the wall, $R_v = 0.5$.

the observer position returns to the baseline position, but the ground has reflection coefficient one-half. Instead, in Figure 6, the reflection coefficient is one-half on the wall. In Figure 7, both walls have reflection coefficient one-half.

Changing one set of parameters from the baseline shows separately the effect of observer position, or the effect of halving the reflection coefficient of the ground only or of the wall also.

Figure 7: The same as Figure 3, but for halved reflection factor on the the ground and on the wall, $R_h = R_v = 0.5$.

Table 1: Values of the observer position and reflection factors of the walls in Figures 3 to 7.

Number of the figure	a [m]	b [m]	R_h	R_v
3	3	2	1	1
4	2	6	1	1
5	3	2	0.5	1
6	3	2	1	0.5
7	3	2	0.5	0.5

3.2 Effect of observer position, and ground and wall reflection coefficients

The Figures 3 to 7 have all the same format, with the modulus or amplitude of the multipath factor at the top and its argument or phase at the bottom; the spectrum is quite dense over the audible range, and in fact was drawn using symbolic expressions. Since the spikes which form the spectrum are very narrow, a part of the full spectrum at left is amplified at right, namely to the range $20 \leq f \leq 1000$ Hz. It is seen in Figure 3 that the interference between direct and reflected signals leads to nulls and peaks; furthermore, the succession of peaks has itself peaks and nulls, like a phenomenon of beats; on the right-hand side, it can be seen clearly the individual peaks, and on the left-hand side only the “peaks of the peaks”. The complex amplitude of F is a composition (square root, sum and squares of real and

imaginary parts) of cosine and sine functions, all harmonic functions.

The Figure 3 concerns an observer below the bisector of the corner and Figure 4 an observer above. In Figure 5, the observer returns to the baseline position of Figure 3, but the reflection coefficient of the ground is halved; this leads to a “hollow” in the amplitude (top) and a narrower modulation on the phase (bottom). In Figure 6, the reflection coefficient on the wall is halved instead of on the ground whereas the amplitude has larger hollow and the phase is narrower. In Figure 7, the reflection coefficient is halved both on the ground and on the wall, and the amplitude is even more “hollow” and the phase modulation even “narrower” than in Figure 6. In fact, the Figure 6 is intermediate between Figures 5 and 7. Relative to this, the Figure 7 shows a smaller maximum amplitude and a smaller maximum phase due to the attenuation effect of the absorbing ground and wall.

3.3 Case of source in the far field and observer in the near field

The distances appearing in (18) may be expressed in polar coordinates (1a) and (1b), and simplified for the observer in near field and the source in far field, for example: (i) the distance (2) from the source (1b) to the observer (1a) is

$$r = \left| q^2 + s^2 - 2qs \cos(\beta - \alpha) \right|^{1/2} \quad (21)$$

$$= q - s \cos(\beta - \alpha) + O\left(\frac{s^2}{q}\right);$$

(ii) the distances from the source (7a) and observer (7b) to the reflection point on the ground are respectively

$$r_{11} = q - s \cot \beta \sin(\alpha + \beta) + O\left(\frac{s^2}{q}\right), \quad (22)$$

$$r_{12} = s \sin \alpha \csc \beta + O\left(\frac{s^2}{q}\right) \quad (23)$$

for single reflection; (iii) the distances from the source (10a) and observer (10b) to the reflection point on the wall are respectively

$$r_{21} = q - s \tan \beta \sin(\alpha + \beta) + O\left(\frac{s^2}{q}\right), \quad (24)$$

$$r_{22} = s \cos \alpha \sec \beta + O\left(\frac{s^2}{q}\right), \quad (25)$$

for single reflection; (iv) in the case of double reflection, the distance from the source to the reflection point on the ground (15a) is

$$r_{31} = q - s \cot \beta \sin(\alpha - \beta) + O\left(\frac{s^2}{q}\right), \quad (26a)$$

from this last point to the reflection point on the wall (15b) is

$$r_{32} = s \sec \beta \csc \beta \sin(\alpha - \beta) + O\left(\frac{s^2}{q}\right) \quad (26b)$$

and from the wall to the observer (15c) is

$$r_{33} = s \cos \alpha \sec \beta + O\left(\frac{s^2}{q}\right). \quad (26c)$$

The last three relations are valid if $\beta \leq \alpha$.

Note that the far field approximation requires that all distances can be approximated to $O(s)$, implying that: (i) if the leading term is $O(q)$, then the next approximation $O(s/q)$ is needed, for instance to specify the $O(s)$ terms in (21), (22), (24) and (26a); (ii) if the leading term is $O(s)$, as in (23), (25), (26b) and (26c) the next term would be $O(s^2/q)$ and can be omitted. The last three results hold if $\alpha \geq \beta$, and the opposite case is considered in appendix A.3. Substituting the simplified distances, the multipath factor (18) can be written explicitly as

$$\begin{aligned} F = 1 + R_h \left[1 - \frac{s}{q} A(\alpha, \beta) \right] \exp[iksA(\alpha, \beta)] & \quad (27) \\ + R_v \left[1 - \frac{s}{q} B(\alpha, \beta) \right] \exp[iksB(\alpha, \beta)] \\ + R_h R_v \left[1 - \frac{s}{q} C(\alpha, \beta) \right] \exp[iksC(\alpha, \beta)] \end{aligned}$$

where

$$A(\alpha, \beta) = \cos(\alpha - \beta) + \sin \alpha \csc \beta - \cot \beta \sin(\alpha + \beta), \quad (28a)$$

$$B(\alpha, \beta) = \cos(\alpha - \beta) + \cos \alpha \sec \beta - \tan \beta \sin(\alpha + \beta), \quad (28b)$$

$$C(\alpha, \beta) = \cos(\alpha - \beta) + \cos \alpha \sec \beta - \sin(\alpha - \beta) \csc \beta (\cos \beta - \sec \beta) \quad (28c)$$

and noting that the last expression is valid if $\alpha \geq \beta$. The relation (27) assumes the approximation $s^2 \ll q^2$. For a source in the far field at lower elevation angle than the observer in the near field, $\alpha \geq \beta$, the direct wave has amplitude and phase corrections for single reflections on the ground and on the wall, and double reflection on both.

3.4 Effect of direction of arrival of the signal and source position

The amplitude and phase changes are indicated in the Figures 8 and 9 for the baseline observer position and rigid walls (first line of Table 1), but for a fixed frequency $f = 1$ kHz corresponding at the sound speed $c = 343.21 \text{ ms}^{-1}$ to the wavenumber $k = 2\pi f/c = 18.31 \text{ m}^{-1}$. In the Table 2 and Figure 8, the source is kept in the same grazing direction,

$$\beta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{30}{700}\right) = 2.45^\circ, \quad (29a)$$

but the source position

$$(x, y) = q(\cos \beta, \sin \beta) = q(0.999, 0.0428) \text{ m} \quad (29b)$$

varies with the distance,

$$700 \text{ m} < q < 1400 \text{ m}, \quad (29c)$$

and so varies the Helmholtz number for the frequency $f = 1$ kHz,

$$12815 \leq kq \leq 25630; \quad (29d)$$

higher and lower frequencies, $f = 10$ kHz and $f = 100$ Hz respectively, are also considered in the Table 2 and Figure 8. In the Figure 9, the source distance is kept at

$$q = \left| x^2 + y^2 \right|^{1/2} = \left| 700^2 + 30^2 \right|^{1/2} = 700.64 \text{ m} \quad (30a)$$

corresponding to a Helmholtz number

$$kq = 12827 \quad (30b)$$

Figure 8: Modulus (top) and phase (bottom) of the multipath factor (18) versus source distance, $4 \text{ m} < q < 1400 \text{ m}$, in a fixed direction, $\beta = 2.45^\circ$, with rigid ground and wall, $R_h = R_v = 1$, and observer position at $(a, b) = (3, 2) \text{ m}$, adding to the frequency $f = 1000 \text{ Hz}$ two others, one with larger order of magnitude, $f = 10000 \text{ Hz}$, and one with smaller order of magnitude, $f = 100 \text{ Hz}$.

Figure 9: The same as Figure 8 with the source at fixed distance, $q = 700.64 \text{ m}$, and variable elevation, $0 \leq \beta \leq \pi/2 \text{ rad}$, for the fixed frequency, $f = 1000 \text{ Hz}$, showing the result of: (i) the exact multipath factor (18) as thin line; (ii) the far field approximation (27) as thick line.

Table 2: Mean value of the modulus (middle column) and phase (right column) of the multipath factor (18) versus source distance, $700 \text{ m} < q < 1400 \text{ m}$, in a fixed direction, $\beta = 2.45^\circ$, with rigid ground and wall, $R_h = R_v = 1$, and observer position at $(a, b) = (3, 2) \text{ m}$, adding to the frequency $f = 1000 \text{ Hz}$ four others, two with larger order of magnitude and two with smaller order of magnitude.

Frequency [Hz]	$ F $	$\arg(F)$ [deg]
100	2.7553	-36.4352
500	1.8873	-2.9828
1000	0.0184	-100.2046
5000	0.0854	-142.0319
10000	1.9116	-59.9268

for the frequency $f = 1 \text{ kHz}$, and the direction changes over the whole corner,

$$0 \leq \beta \leq \pi/2 \text{ rad}, \quad (30c)$$

so that the source position is

$$(x, y) = 700.64 (\cos \beta, \sin \beta) \text{ m}. \quad (30d)$$

The Table 2 shows only the mean values because the changing of the distance q of the far away source between the values 700 and 1400 meters has little effect, for any fixed frequency f , independently of its value. Although the amplitude and phase of the multipath effect are almost independent of source distance q , they are quite sensitive to frequency, as it can be seen in Table 2. The amplitude of the multipath factor (second column of Table 2) slightly increases with source distance for $f = 100 \text{ Hz}$, $f = 500 \text{ Hz}$ and $f = 10000 \text{ Hz}$, but slightly decreases for $f = 1000 \text{ Hz}$ and $f = 5000 \text{ Hz}$ showing that the behaviour of the multipath factor is strongly dependent on the frequency, in contrast to the source distance. Regarding the phase (third column of 2), for all the values of the frequency, the source distance does not significantly influence the phase. The Table 2 shows that the phase is negative for the five values of the frequency. However, that is not always the true because there are some frequencies for which the phase is positive as it can be seen in the appendix B, specifically in the Figure A4 (in that figure, the source distance is fixed). That means that the frequency also strongly influences the phase of the multipath factor, in contrast with the source distance.

The conclusion that the multipath factor varies only slightly with the source distance does not hold for all the values, as shown in Figure 8. When the source is close to the observer, the effects of varying the source distance are significant, especially for the absolute value of F and for higher frequencies, and they become negligible for frequencies above a given value.

The multipath factor can be calculated from (27) using the equations (28a) to (28c) which: (i) holds for amplitude with an error $(s/q)^2 = 13/q^2 \leq 13/700^2 = 2.65 \times 10^{-5}$; (ii) holds for phase with an accuracy $O((s/q)^2)$, corresponding to an error $kq(s/q)^2 = ks^2/q \leq 0.343$; (iii) may fail at grazing incidences close to $\theta = 0$ and $\theta = \pi/2$ when some of the approximations, from (22) to (26c), may cease to hold. Thus, the asymptotic approximation (27) is more accurate for amplitude than for phase, and should be used at intermediate elevation. As an alternative, substituting (1a) and (1b) in the exact expressions in (18) specifies the multipath factor, correct to all orders of s/q , and valid in all directions, $0 \leq \theta \leq \pi/2$, including grazing directions. The Figure 9 shows that both the amplitude (top) and phase (bottom) of the multipath factor are strongly affected by source direction, in contrast with source distance, which has little effect.

The Figure 9 also shows the exact multipath factor (thin line) in comparison with the far field approximation (solid line). The far field approximation (27) is extremely accurate for the amplitude (Figure 9, top) since the solid line overlaps the thin line of the exact expression (18) of the multipath factor. Concerning the phase (Figure 9, bottom), the far field approximation (solid line) follows closely the exact theory (thin line) except for local peaks. The amplitude and phase of the multipath factor are shown for fixed frequency $f = 1 \text{ kHz}$ for all source directions in Figure 9, and conversely over the audible range, $20 \leq f \leq 20000 \text{ Hz}$, for four source directions in Figures A3 to A6 in the appendix B.

4 Conclusion

The maximum amplification of a signal due to reflections from walls near the receiver is shown in the Table 3 for N waves in phase, both for amplitude, $\text{dB} = 10 \log N$, and for power, $\text{dB} = 20 \log N$, that are proportional respectively to the modulus $|F|$ of the multipath factor F and to its square $|F|^2$. The reception near an infinite plane consists of one direct and one reflected wave; if the two waves are in phase, the amplitude is doubled, $10 \log 2 = 3 \text{ dB}$, and the power multiplied by four, $20 \log 2 = 6 \text{ dB}$. In the case of an orthogonal corner formed by two infinite planes, the reception consists of: (i) a direct wave; (ii) two waves, each one reflected once on each plane; (iii) one wave reflected twice, once on each plane. There is a total of four waves, and if they are all in phase, the maximum amplitude is multiplied by four, $10 \log 4 = 6 \text{ dB}$, and the power is multiplied by sixteen, $20 \log 4 = 12 \text{ dB}$. This applies both in: (i) the two-

Table 3: Maximum amplification from wall reflections.

Receiver near a:	plane	2-D corner	3-D corner
Direct wave	1	1	1
Reflected once	1	2	3
Reflected twice	0	1	3
Reflected three times	0	0	1
Total number of waves N	2	4	8
Maximum amplification:			
- for amplitude: $\text{dB} = 10 \log N$	3 dB	6 dB	9 dB
- for power: $\text{dB} = 20 \log N$	6 dB	12 dB	18 dB

Table 4: Maximum increase or decrease of the sound power level (SPL), for a certain frequency, due to reflections on the ground and wall of the wave originated from the source at $(x, y) = (700, 30)$ m, for the cases of Figures 3 to 7.

Number of the figure	a [m]	b [m]	R_h	R_v	Maximum Minimum		
					$\Delta\text{SPL}_{\text{ground}}$ [dB]	$\Delta\text{SPL}_{\text{wall}}$ [dB]	$\Delta\text{SPL}_{\text{ground+wall}}$ [dB]
3	3	2	1	1	6.0195 -72.1629	5.9835 -41.3899	12.0023 -86.1743
4	2	6	1	1	6.0174 -62.6480	5.9958 -44.8957	11.9992 -69.3974
5	3	2	0.5	1	3.5211 -6.0185	5.9835 -41.3899	9.5039 -96.2305
6	3	2	1	0.5	6.0195 -72.1629	3.4971 -5.9469	9.5159 -57.3175
7	3	2	0.5	0.5	3.5211 -6.0185	3.4971 -5.9469	7.0175 -15.2550

dimensional case considered here with all waves in a plane perpendicular to the corner; (ii) the three-dimensional case with incident and reflected waves in a plane oblique to the two-dimensional corner. In the case of a three-dimensional corner consisting of three orthogonal planes, the reception consists of: (i) one direct wave; (ii) three waves, each one reflected once at one plane; (iii) three waves, each one reflected twice on a pair of planes; (iv) one wave reflected three times, that is, once on each plane. If all eight waves are in phase, the maximum amplitude is multiplied by eight, $10 \log 8 = 9$ dB, and the power is multiplied by sixty-four, $20 \log 8 = 18$ dB.

If the ground is the only surface considered, without any other surfaces, the multipath factor is given by only the first two terms of (18), while the addition of a wall induces the sum of the last two terms of (18) to the multipath factor. The Figures 3 to 7 show five particular cases whose the greatest and lowest changes in decibels are indicated in Table 4. The decibels are $10 \log_{10} |F|$ for the sound pressure level in Figures 3 to 7 and $10 \log_{10} (|F|^2) = 20 \log_{10} |F|$ for the sound power level (SPL). In the Table 4, ΔSPL is the difference of the SPL at observer position between the cases of only direct wave received and direct wave plus reflected waves received by the observer. $\Delta\text{SPL}_{\text{ground}}$ is the change, in decibels, due to a wave reflected on the ground, $\Delta\text{SPL}_{\text{wall}}$ is the change, in decibels, due to a wave reflected

on the wall and $\Delta\text{SPL}_{\text{ground+wall}}$ is the change, in decibels, due to the three reflected waves, as depicted in the Figure 1. The maximum increase occurs when the ground and wall are both considered because in that case there are three “new” waves due to reflections on surfaces travelling to the observer position, besides the direct wave. The increase can reach approximately 12 dB if the two surfaces (ground plus wall) totally reflect the wave and if the observer is near the corner, but if it is considered only one surface, again one that totally reflects the wave, the increase can be, at maximum, 6 dB, justifying therefore the norms on noise measurement [4, 5].

These maximum increases of power in decibels can occur for several frequencies. However, the Figures 3 to 7 show that, for some frequencies, $|F|$ is less than 1 (for some frequencies, almost equal to 0) because of the destructive interference from the superposition of the waves, resulting in a decrease of decibels. The pressure reflection coefficient on the ground for spherical waves is $R_h = |R_h| \exp(i\phi)$, where ϕ represents the phase change on reflection, and with ϕ being equal to 0, usually set for an acoustically hard boundary [24] (the same was used for the reflection coefficient of the wall). Consequently, the phase difference between a direct wave and a reflected wave is caused only by the path length difference of the waves, that have the same frequency, and since the path difference always exist (except for the case

$\alpha = \beta$), there is always some destructive interference and therefore it is not possible to reach the maximum theoretical value of SPL when adding two or more waves (the worst case scenario when adding two waves with the same frequency would be if they have also the same phase). The increase or decrease of power in decibels depend on the reflection coefficients and the observer position, despite being more influenced by the former. The results of the Table 4 are valid for one single wave originating from the source with one frequency. The sound spectrum can consist of a superposition of several harmonics of distinct frequencies, leading therefore to a more significant increase of power in decibels at the same observer position.

A three-dimensional plot for each of the modulus $|F|$ and phase $\arg(F)$ of the multipath factor as a function of the observer coordinates a and b would be difficult to visualize due to the large number of closely spaced peaks and nulls and to the wide range of values. A better way to visualize the modulus and phase of the multipath factor is to plot the isolines of $|F|$ and $\arg(F)$, that are closed curves where the function has a constant value, knowing at first the values of 1000×1000 different coordinates uniformly spaced in the 2D region. The Figure 10 shows the isolines for four different

values of the modulus of F : 0, 4, 8 and 11 decibels (there are also regions with 12 decibels, however it would be hard to visualize them). The walls are rigid, $R_h = R_v = 1$, the source point is at the coordinates (700, 30) m, that is, at the upper right corner in each plot, and the selected frequency is $f = 2003.9$ Hz, equal to the frequency of the second line in Table 4, corresponding therefore to the worst case scenario when the observer is at the coordinates (3, 2) m with the same remaining conditions. In the Figure 10, the axis x and y stand for the distances to the vertical and horizontal walls respectively, and not to the source position. There are many more points (for example more 337410 points forming more 5540 isolines between the first and last plots in Figure 10) where the presence of walls does not change the modulus of F (resulting in isolines of 0 dB) than the points where there is an increase of 11 dB due to the same reason. The isolines of 0 dB are plotted in whole 2D region, however the isolines of 11 dB exist only in the region near the vertical wall, specifically near the corner. The same applies to the phase of the multipath factor, as depicted in the Figure 11, where the points for lower phase values are much more numerous (for example, more 252361 points forming more 10475 isolines between the cases of 40 and 160 degrees

Figure 10: Map of the modulus of the multipath factor F as a function of observer position in the plane for a fixed source position at the upper right corner in each plot, for rigid walls, $R_h = R_v = 1$ and for the frequency $f = 2003.9$ Hz. The variables x and y in the axis labels stand for the distances to the vertical and horizontal walls respectively.

Figure 11: Map of the phase of the multipath factor F as a function of observer position for the same conditions as in the Figure 10.

Figure 12: The same as Figure 10, but for semi-rigid walls, $R_h = R_v = 0.5$.

Figure 13: The same as Figure 11, but for semi-rigid walls, $R_h = R_v = 0.5$.

in Figure 11) than the points for greater phase values. The isolines for large phase values, for instance 160 degrees, are plotted, not only near the corner, as it happens with the modulus (fourth plot of Figure 10), but also for regions far away from the corner. This means that being near a corner influences more the modulus of the multipath factor than its phase. The Figures 12 and 13 show the isolines of the modulus and the phase respectively of the multipath factor F , but for lower values of the reflection coefficients of both walls, specifically $R_h = R_v = 0.5$. The frequency is the same, $f = 2003.9$ Hz, because that value also corresponds to the fourth line of Table 4, leading to the worst case scenario when the observer is at the position (3, 2) m. The remarks are the same; the only difference is that in this case, the maximum values of the modulus and phase of F are not as much increased as for the maximum values when the reflection coefficients of both walls are equal to unity, as depicted in the Figures 10 and 11. The plots of the phase in Figures 11 and 13 show the isolines only for positive values; the plots would be practically the same if the isolines were drawn for the negative phases.

The present theory assumes perfectly flat walls. Real walls are rough, and if the average height of irregularities is ε , the walls may be considered smooth if the wavelength λ is much larger, $\lambda \gg \varepsilon$. Considering audible frequency

range from 20 Hz to 20 kHz,

$$f = 2 \times 10^1 \text{ s}^{-1} - 2 \times 10^4 \text{ s}^{-1}, \quad (31a)$$

for sound propagation in the atmosphere at sea level with the sound speed

$$c = 340 \text{ ms}^{-1}, \quad (31b)$$

the wavelength is

$$\varepsilon \ll \lambda = \frac{c}{f} = 1.7 \times 10^{-2} \text{ m} - 17 \text{ m} \quad (32)$$

and the wall may be considered smooth if the average roughness ε is much smaller than the smallest wavelength $\lambda_{min} = 1.7$ cm, say $\varepsilon < 2$ mm. The theory applies to acoustic and other waves, for example electromagnetic waves, always in terms of wavelength, not frequency. The same range of wavelengths,

$$1.7 \times 10^{-2} \text{ m} < \lambda < 1.7 \text{ m}, \quad (33a)$$

for electromagnetic waves propagating at the speed of light

$$c_0 = 3 \times 10^8 \text{ ms}^{-1}, \quad (33b)$$

that is much higher than the sound speed (31b), leads to much higher frequencies,

$$f = \frac{c_0}{\lambda} = 1.76 \times 10^8 \text{ Hz} - 1.76 \times 10^{11} \text{ Hz} \quad (34)$$

$$= 17.6 \text{ Mhz} - 17.6 \text{ GHz.}$$

Thus, for the same average surface roughness $\varepsilon = 2 \text{ mm}$, the present theory applies to electromagnetic waves in the range of frequencies (34) spanning the high frequencies indicated in the Table 5. The theory also applies to lower bands of electromagnetic waves with longer wavelength, spanning the medium and low frequencies, also indicated in the Table 5. The theory could not apply, unless the roughness was smaller, to higher frequencies and shorter wavelengths, for instance, to the frequencies indicated in the Table 6.

Table 5: Ranges of frequencies in which the theory in this paper can be applied.

Name	Initials	Frequency band
Super High Frequencies	SHF	3 GHz – 30 GHz
Ultra High Frequencies	UHF	300 MHz – 3 GHz
Very High Frequencies	VHF	30 MHz – 300 MHz
High Frequencies	HF	3 MHz – 30 MHz
Medium Frequencies	MF	300 kHz – 3 MHz
Low Frequencies	LF	30 kHz – 300 kHz
Very Low Frequencies	VLF	3 kHz – 30 kHz
Ultra Low Frequencies	ULF	300 Hz – 3 kHz
Super Low Frequencies	SLF	30 Hz – 300 Hz
Extremely Low Frequencies	ELF	3 Hz – 30 Hz

Table 6: Ranges of frequencies in which the theory in this paper cannot be applied.

Name	Initials	Frequency band
Extremely High Frequencies	EHF	30 GHz – 300 GHz
Far Infra-Red	FIR	300 GHz – 3 THz
Mid Infra-Red	MIR	3 THz – 30 THz
Near Infra-Red	NIR	30 THz – 300 THz
Visible and Ultraviolet	UV	300 THz – 30 PHz
Soft X-rays	SX	30 PHz – 3 EHz
Hard X-rays	HX	3 EHz – 30 EHz
Gamma rays	γ	30 EHz – 300 EHz

The contour plots in Figures 10 to 13 assume a frequency $f = 2003.9 \text{ Hz}$ corresponding to sound waves with wavelength $\lambda = 340/2003.9 \text{ m} = 0.170 \text{ m}$. They apply also to other waves with the same wavelength, for example electromagnetic waves with frequency $f = 3 \times 10^8/1.7 \text{ Hz} = 17.6 \text{ MHz}$ in the HF band. Although the theory applies

equally well to electromagnetic waves [18–22], for brevity we concentrate on the acoustic literature. The theory is directly applicable to noise mapping in urban environments [1–8] due to surface transport and aircraft. In the latter case of aircraft, the effects of atmospheric propagation have to be considered [9, 10]. A spherical wave incident on a plane gives rise to a reflected wave considered here, and a surface lateral wave [14, 24–28], that has been neglected here as a smaller second-order effect away from the wall. Here, the simplest approach was chosen based on the superposition of spherical waves in general acoustics [13–17, 23]. The present approach demonstrates that the interference of reflected spherical waves together with the direct wave can lead to amplitudes much smaller than the maximum and complex interference patterns. The results are presented for the whole audible range of monochromatic frequencies and can be superimposed via a Fourier integral to any spectrum of the incident signal. The walls may be considered smooth for wavelengths much larger than the surface roughness. For example, if the surface roughness does not exceed a few millimetres, the theory applies to the whole audible acoustic spectrum and to electromagnetic waves in the ultra high frequency (UHF) band and below.

The theory in the general forum presented allows for different reflection factors from each wall. The calculation of reflection factors is a major subject in its own right, briefly reviewed in annex C.

Acknowledgement: The present work was performed in the context of the project Friendcopter supported by the Aeronautics Programmes of the European Union. This work was also supported by the Fundação para a Ciência e Tecnologia (FCT), Portugal, through Institute of Mechanical Engineering (IDMEC), under the Associated Laboratory for Energy, Transports and Aeronautics (LAETA), whose grant numbers are UIDB/50022/2020 and SFRH/BD/143828/2019.

References

- [1] Zaporozhets O, Tokarev V, Attenborough K. Aircraft Noise: Assessment, prediction and control. 1st ed. Abingdon: Spon Press; 2011. ISBN: 978-0415240666. <https://doi.org/10.1201/b12545>.
- [2] Doc IC. 9829, Guidance on the Balanced Approach to Aircraft Noise Management, 2nd ed.; 2008. ISBN:978-9292310370
- [3] Doc IC. 9911, Recommended Method for Computing Noise Contours Around Airports, 1st ed.; 2008. ISBN:978-9292312251
- [4] ISO 20906:2009, Acoustics – Unattended monitoring of aircraft sound in the vicinity of airports, 1st ed.; 2009.
- [5] ISO 1996–2:2017, Acoustics – Description, measurement and assessment of environmental noise – Part 2: Determination of sound pressure levels, 3rd ed.; 2017.

- [6] Wang B, Kang J. Effects of urban morphology on the traffic noise distribution through noise mapping: A comparative study between UK and China. *Appl Acoust.* 2011;72(8):556–68.
- [7] Hao Y, Kang J. Influence of mesoscale urban morphology on the spatial noise attenuation of flyover aircrafts. *Appl Acoust.* 2014;84:73–82.
- [8] Flores R, Gagliardi P, Asensio C, Licita G. A Case Study of the Influence of Urban Morphology on Aircraft Noise. *Acoust Aust.* 2017;45(2):389–401.
- [9] Campos LM, Serrão PG. On the effect of atmospheric temperature gradients and ground impedance on sound. *Int J Aeroacoust.* 2014;13(5–6):427–48.
- [10] Campos LM, Cunha FS. On the power spectra of sound transmitted through turbulence. *Int J Aeroacoust.* 2012;11(3–4):475–520.
- [11] Albert DG, Liu L. The effect of buildings on acoustic pulse propagation in an urban environment. *J Acoust Soc Am.* 2010 Mar;127(3):1335–46.
- [12] Rayleigh JW, Lindsay RB. *The Theory of Sound* 2. 2nd ed. New York (NY): Dover Publications; 1945. ISBN: 978-0486602936.
- [13] Morse PM, Feshbach H. *Methods of Theoretical Physics* 2. New York (NY): McGraw-Hill; 1953. ISBN: 978-0070433175.
- [14] Landau LD, Lifshitz EM. *Fluid mechanics* 6. 2nd ed. Oxford: Pergamon Press; 1987. ISBN: 0080339328.
- [15] Lighthill MJ. *Waves in fluids*. 1st ed. Cambridge: Cambridge University Press; 1978. ISBN: 978-0521216890.
- [16] Pierce AD. *Acoustics: An Introduction to Its Physical Principles and Applications*. 3rd ed. Cham: Springer International Publishing; 2019. ISBN: 978-3030112134. <https://doi.org/10.1007/978-3-030-11214-1>.
- [17] Howe MS. *Acoustics of Fluid-Structure Interactions*. Cambridge: Cambridge University Press; 1998. ISBN: 0521633206. <https://doi.org/10.1017/CBO9780511662898>.
- [18] Maxwell JC. *A treatise on electricity and magnetism* 2. 1st ed. Oxford: Clarendon Press; 1873.
- [19] Heaviside O. *Electromagnetic theory* 1. 1st ed. London: The Electrician Publishing; 1893.
- [20] Stratton JA. *Electromagnetic theory*. New York (NY): McGraw-Hill book company; 1941.
- [21] Landau LD, Lifshitz EM. *The Classical Theory of Fields* 2. 4th ed. Oxford: Butterworth-Heinemann; 1987. ISBN: 9780750627689.
- [22] Jones DS. *The theory of electromagnetism*. Oxford: Pergamon Press; 1964.
- [23] Dowling AP, Williams JE. *Sound and sources of sound*. Chichester: Ellis Horwood; 1983. ISBN: 0853124000.
- [24] Attenborough K. Review of Ground Effects on Outdoor Sound Propagation from Continuous Broadband Sources. *Appl Acoust.* 1988;24(4):289–319.
- [25] Sommerfeld A. Über die Ausbreitung der Wellen in der drahtlosen Telegraphie. *Ann Phys.* 1909;28(4):665–737.
- [26] Rudnick I. The Propagation of an Acoustic Wave along a Boundary. *J Acoust Soc Am.* 1947;19(2):348–56.
- [27] Ingard U. On the Reflection of a Spherical Sound Wave from an Infinite Plane. *J Acoust Soc Am.* 1951;23(3):329–35. <https://doi.org/10.1121/1.1906767>.
- [28] Taraldsen G. A note on reflection of spherical waves. *J Acoust Soc Am.* 2005;117(6):3389–92.

A Effect of relative elevation angles of observer and source

The relative elevation angles of observer α and source β lead to three cases: (i) observer above (Figures 1 and 2), already considered in subsection 2.4; (ii) source above (Figure A1), treated next in appendix A.1; (iii) observer and source on the same elevation angle (Figure A2) which is the intermediate case between the preceding two, that will be considered in appendix A.2. The far field approximation (in subsection 3.3) for source elevation above the observer is also considered in appendix A.3. In the intermediate case of observer and source on the same elevation angle, the exact directivity takes a simple form at arbitrary distance, that will be treated in appendix A.4.

A.1 Case of observer elevation angle lower than the source elevation angle

The direct signal, using the equations (1a) to (2), the signal reflected at the ground, using the equations (5) to (7b), and the signal reflected at the wall, using the equations (8) to (10b), apply to any relative position of observer and source. The fourth signal is given by the equations (11a) to (15c) when the first reflection is on the ground and the second on the wall, that is, for source at elevation below than the observer elevation, $\beta \leq \alpha$ (Figure 2). In the opposite case of source elevation above than the observer elevation (Figure A1), the first reflection is on the wall at $(0, y_{31})$ and the second on the ground at $(x_{32}, 0)$, where the height y_{31} and horizontal distance x_{32} are determined by the coupled equations

$$\frac{y - y_{31}}{x} = \frac{y_{31}}{x_{32}}, \quad (\text{A1a})$$

$$\frac{x_{32}}{y_{31}} = \frac{a - x_{32}}{b}. \quad (\text{A1b})$$

Solving the equations above for y_{31} gives the equalities

$$\frac{x_{32}y}{x + x_{32}} = y_{31} = \frac{x_{32}b}{a - x_{32}} \quad (\text{A2})$$

from which follows

$$b(x + x_{32}) = y(a - x_{32}), \quad (\text{A3})$$

which specifies the second reflection point x_{32} on the ground,

$$x_{32} = \frac{ay - bx}{y + b} = -x_{31}; \quad (\text{A4a})$$

the first reflection point, on the wall, is obtained substituting (A3) in one of the relations (A2), leading to

$$y_{31} = \frac{ay - bx}{x + a} = -y_{32}. \quad (\text{A4b})$$

Note that the two last relations coincide with (14a) and (14b) respectively with reversed sign. From the last two relations, the three distances are determined: (i) from the source to the first reflection on the wall,

$$r_{31} = \left| x^2 + (y - y_{31})^2 \right|^{1/2} = x \left| 1 + \frac{(y + b)^2}{(x + a)^2} \right|^{1/2}; \quad (\text{A5a})$$

(ii) between reflection points on the wall and on the ground,

$$\begin{aligned} r_{32} &= \left| (x_{32})^2 + (y_{31})^2 \right|^{1/2} \\ &= |ay - bx| \left| \frac{1}{(x + a)^2} + \frac{1}{(y + b)^2} \right|^{1/2}; \end{aligned} \quad (\text{A5b})$$

(iii) from the reflection point on the ground to the observer,

$$r_{33} = \left| (a - x_{32})^2 + b^2 \right|^{1/2} = b \left| 1 + \frac{(x + a)^2}{(y + b)^2} \right|^{1/2}. \quad (\text{A5c})$$

The last three equations are valid if $\beta \geq \alpha$. Comparing the cases of observer at higher elevation angle than the source, that is, the equations (15a) to (15c), with the reverse case, that is, the equations (A5a) to (A5c), it is clear that: (i) only the distance between the reflection points r_{32} coincide; (ii) the distance from the reflection point on the ground exchanges y if it is measured the distance r_{31} from the source in (15a) by b if it is measured the distance r_{33} from the observer in (A5c); (iii) the distance from the reflection point on the wall exchanges a if it is measured the distance r_{33} from the observer in (15c) by x if it is measured the distance r_{31} from the source in (A5a). In both cases, the last term of (3) holds, with distinct expressions for the distances r_{3i} with $i = \{1, 2, 3\}$. The remaining terms of the total signal (3) are unchanged.

A.2 Case of observer and source on the same azimuth

The case when the observer is on the same elevation angle than the source (Figure A2) can be treated as intermediate between observer above (Figures 1 and 2) and observer below (Figure A1). In this case, both formulas, (11a) to (15c) for $\beta \leq \alpha$, and (A1a) to (A5c) for $\beta \geq \alpha$, must hold. From (A2), it follows that the reflection point is the origin, at the corner,

$$x_{31} = x_{32} = 0 = y_{31} = y_{32}, \quad (\text{A6a})$$

when the condition

$$\frac{y}{x} = \tan \beta = \tan \alpha = \frac{b}{a} \quad (\text{A6b})$$

Figure A1: The same as the Figure 2, but with the difference that the elevation angle β for the source is larger than for the observer α , that is, $\alpha < \beta$, showing again the double reflection path as the Figure 2, where in this case the first reflection is on the wall and the second reflection is on the ground.

Figure A2: The intermediate case between the Figures 2 and A1 is that of the observer and source with the same elevation angle, that is, $\alpha = \beta$ and “double reflection” at the origin.

of source and observer on the same azimuth angle is met. The distance between the “two” coincident reflection points is zero,

$$r_{32} = 0 \quad (\text{A7a})$$

in (15b) or (A5b), the distance from the source to the origin is, regarding (15a) or (A5a),

$$r_{31} = |x^2 + y^2|^{1/2} = q, \quad (\text{A7b})$$

and from the observer to origin is, regarding (15c) or (A5c),

$$r_{33} = |a^2 + b^2|^{1/2} = s, \quad (\text{A7c})$$

where comparison with (1a) and (1b) was made. These values simplify the last term of (3), which remains valid as an expression for the total signal.

A.3 Far field approximation for all elevation angles

The far field approximations for the direct signal (21), reflected signal on the ground, (22) and (23), and reflected signal on the wall, (24) and (25), hold regardless of the relative positions of observer and source. For the fourth signal, involving double reflection, the far field approximation, (26a) to (26c), for the observer elevation above than the source elevation, is replaced in the opposite case by the equations (A5a) to (A5c), leading to

$$r_{31} = q - s \tan \beta \sin(\beta - \alpha), \quad (\text{A8a})$$

$$r_{32} = s \sec \beta \csc \beta \sin(\beta - \alpha), \quad (\text{A8b})$$

$$r_{33} = s \sin \alpha \csc \beta. \quad (\text{A8c})$$

The last three expressions are valid if $\alpha \leq \beta$ and considering that the distance from the corner to the source is much larger than the distance from the corner to the observer, that is, $s^2 \ll q^2$. These expressions affect only the last term of (3), so (27) remains valid with (28a) and (28b) unchanged, and (28c) replaced by

$$C(\alpha, \beta) = \cos(\alpha - \beta) + \sin \alpha \csc \beta + \sin(\beta - \alpha) \sec \beta (\csc \beta - \sin \beta); \quad (\text{A9})$$

thus (A9) is similar to (28c) with the exchanges

$$\cos \alpha \leftrightarrow \sin \alpha, \quad (\text{A10a})$$

$$\sec \beta \leftrightarrow \csc \beta, \quad (\text{A10b})$$

plus some sign changes as the elevation of source and observer are reversed. This result is valid for a source in the far field and observer in the near field, and can be replaced by the exact formula (3) using (1a) and (1b) in the formulas for the distances in the multipath factor (18). In the case (A6b) of equal source and observer elevation angles, $\beta = \alpha$, not only the relations (28a) and (28b) are simplified, but also (28c) and (A9) coincide,

$$A(\alpha, \alpha) = 2 \sin^2 \alpha, \quad (\text{A11a})$$

$$B(\alpha, \alpha) = 2 \cos^2 \alpha, \quad (\text{A11b})$$

$$C(\alpha, \alpha) = 2, \quad (\text{A11c})$$

and (27), for $\beta = \alpha$ and $s^2 \ll q^2$, simplifies to

$$F = 1 + R_h \left(1 - \frac{2s}{q} \sin^2 \alpha \right) \exp(i2ks \sin^2 \alpha) + R_v \left(1 - \frac{2s}{q} \cos^2 \alpha \right) \exp(i2ks \cos^2 \alpha) + R_h R_v \left(1 - \frac{2s}{q} \right) \exp(i2ks). \quad (\text{A12})$$

The correction factor (A12) will be next written exactly, to all orders in s/q .

A.4 Exact directivity for equal elevations of observer and source

The exact multipath factor is calculated next, for the observer and source at arbitrary distances with the same elevation angle, using: (i) the distance (21), for $\alpha = \beta$, from the source to the observer

$$r = |q^2 + s^2 - 2qs|^{1/2} = q - s; \quad (\text{A13})$$

(ii) the distances from the source (7a) and observer (7b) to the reflection point on the ground, for $\alpha = \beta$, in the case of single reflection,

$$r_{11} = \left| \sin^2 \alpha + \left(\frac{q-s}{q+s} \right)^2 \cos^2 \alpha \right|^{1/2} q, \quad (\text{A14a})$$

$$r_{12} = \left| \sin^2 \alpha + \left(\frac{q-s}{q+s} \right)^2 \cos^2 \alpha \right|^{1/2} s; \quad (\text{A14b})$$

(iii) the distances from the source (10a) and observer (10b) to the reflection point on the wall, for $\alpha = \beta$, in the case of single reflection,

$$r_{21} = \left| \cos^2 \alpha + \left(\frac{q-s}{q+s} \right)^2 \sin^2 \alpha \right|^{1/2} q, \quad (\text{A15a})$$

$$r_{22} = \left| \cos^2 \alpha + \left(\frac{q-s}{q+s} \right)^2 \sin^2 \alpha \right|^{1/2} s; \quad (\text{A15b})$$

(iv) in the case of double reflection, the distances from the source (15a) $\equiv$ (A5a) and observer (15c) $\equiv$ (A5c) to the coincident reflection points (15b) $\equiv$ (A5b) at the origin, for $\alpha = \beta$,

$$r_{31} = q, \quad r_{32} = 0, \quad r_{33} = s. \quad (\text{A16})$$

Substituting the equations (A14a) to (A16) in (18), for $\alpha = \beta$, specifies the exact multipath factor

$$F = 1 + R_h R_v \frac{q-s}{q+s} \exp(i2ks) + \exp[ik(s-q)] \times \left\{ R_h \left| \cos^2 \alpha + \left(\frac{q+s}{q-s} \right)^2 \sin^2 \alpha \right|^{-1/2} \times \exp \left(ik \sqrt{(q+s)^2 \sin^2 \alpha + (q-s)^2 \cos^2 \alpha} \right) + R_v \left| \sin^2 \alpha + \left(\frac{q+s}{q-s} \right)^2 \cos^2 \alpha \right|^{-1/2} \times \exp \left(ik \sqrt{(q+s)^2 \cos^2 \alpha + (q-s)^2 \sin^2 \alpha} \right) \right\}. \quad (\text{A17})$$

The approximation of (A17) to $O(s^2/q^2)$ coincides with (A12).

B Multipath factor as a function of frequency

The exact expression (18) and far field approximation (27) of the multipath factor are shown to be quite consistent in the Figure 9, for fixed observer location in the near field with $a = 3$ and $b = 2$ meters, rigid walls, $R_h = R_v = 1$, large

source distance, $q = 700$ m, and fixed frequency, $f = 1$ kHz, over all source directions, $0 \leq \beta \leq 90^\circ$. In the Figures A3 to A6, the modulus (top) and phase (bottom) of the multipath factor are shown over the full audible frequency range, $20 \leq f \leq 20000$ Hz, for four fixed angles of the source, respectively $\beta = 2.45^\circ$, $\beta = 30^\circ$, $\beta = 45^\circ$ and $\beta = 60^\circ$. The exact expression (18) and far field approximation (27) are indistinguishable in all figures, because the thin and solid

Figure A3: The same as Figure 9 with fixed grazing source direction, $\beta = 2.45^\circ$, showing the amplitude (top) and phase (bottom) of the multipath factor as a function of frequency over the audible range, $20 \leq f \leq 20000$ Hz.

Figure A4: The same as Figure A3 for low source direction, $\beta = 30^\circ$, below the diagonal.

Figure A5: The same as Figure A3 for intermediate source direction, $\beta = 45^\circ$, along the diagonal of the two walls.

Figure A6: The same as Figure A3 for high source direction, $\beta = 60^\circ$, above the diagonal.

lines are very close, and the graphs very dense. The shape of the envelopes of amplitude and phase is strongly affected by the angle of the source: (i) for a small angle, $\beta = 2.45^\circ$, corresponding to grazing incidence (Figure A3), the amplitude envelope is a rectified sinusoid (top) and the phase envelope is a sequence of parallelograms (bottom); (ii) for a low angle, $\beta = 30^\circ$, below the diagonal, both the amplitude and phase (Figure A4) have a jagged appearance

like random noise; (iii) for the intermediate angle, $\beta = 45^\circ$, bisecting the corner (Figure A5), the amplitude (top) and phase (bottom) have a solid core with many protruding peaks; (iv) for a high angle, $\beta = 60^\circ$, above the diagonal (Figure A6), the solid core has modulation and the peaks are broadened into sinusoids. As an overall conclusion for a single monochromatic source of waves (sound, light or elec-

tromagnetic) the reflection in a corner can lead to complex interference patterns both for amplitude and phase.

C Reflection factors from a variety of surfaces

When an incident sound wave of pressure I crosses an interface between two different media, for instance when a wave impinges on the ground, making an angle θ with the normal to the surface, only a transmitted wave of pressure T escapes through the interface and travels through the second medium while the wave may also be reflected from the interface to propagate in the same medium than the incident wave with the value of the pressure being R . At all times and at all points on the plane discontinuity, the pressures of the two sides of the boundary must be equal and the particle displacements, normal to the interface, must also be equal. Setting these boundary conditions, the reflection coefficient, or equivalently, the ratio between the pressures of the reflected and incident waves, depend on the specific acoustic impedances of both media and the inclination of the incident wave, leading to the expression

$$\frac{R}{I} = \frac{\rho_1 c_1 / \cos \phi - \rho_0 c_0 / \cos \theta}{\rho_1 c_1 / \cos \phi + \rho_0 c_0 / \cos \theta} \equiv R_p \quad (C1)$$

where ρ is the density, c is the sound speed and for an harmonic wave is equal to ω/k (k is the wavenumber and ω is the angular frequency), and ϕ is the angle the transmitted wave makes with the normal of interface, specified by the Snell's law [23]. The reflection coefficient (C1) depends on the plane wave impedances $\rho_1 c_1$ and $\rho_0 c_0$ on the two sides of the interface, modified to take into account the angle with the normal for the incident θ and transmitted ϕ waves, that affect the normal velocities in the impedances $p_1/u_1 = \rho_1 c_1 / \cos \phi$ and $p_0/u_0 = \rho_0 c_0 / \cos \theta$. In the case of sound incident on a plane material layer dividing a fluid with uniform acoustics properties, ρ and c , for instance a wall of the building, and making an angle θ with the normal of the layer, some sound will be reflected and some will be transmitted through the wall. Setting the continuity of the normal displacement of the waves at the wall and equating the pressure difference across the wall with the inertia of the surface material of mass m per unit area, the ratio between the pressures of the reflected and incident waves is

$$\frac{R}{I} = \frac{i\omega m \cos \theta}{2\rho c + i\omega m \cos \theta}. \quad (C2)$$

The high frequency waves are mostly reflected whereas the low frequency waves are mostly transmitted and get

trough all but the most massive walls with very little attenuation [23]. In both cases, the acoustic pressure is obtained by adding the pressure of a direct wave from the source with the pressure of a reflected wave from the surface, the latter being the pressure of incident wave multiplied by the reflection coefficient, determined in one of the last two equations. The equations (C1) and (C2) are derived for plane waves. In the case of a point source emitting a spherical wave, the plane wave approximation is inadequate, since the boundary conditions at the surface can be met only in the presence of a lateral wave, that changes significantly the sound field for grazing incidences.

The reflection of a spherical wave by a plane wall was first considered by Sommerfeld [25], using a method of virtual sources, that is relevant to outdoor sound propagation. Rudnick [26] wrote the velocity potential of sound waves as the sum of the potentials of the direct wave function due to the point source with the secondary wave function due to the reflection on the ground. Knowing that a sound wave in an homogeneous medium can be represented by elementary cylindrical waves, Rudnick [26] applied the same previously mentioned boundary conditions to evaluate the velocity potentials and obtained the reflection coefficient for spherical waves, leading to the expression

$$\frac{R}{I} = R_p + (1 - R_p) \left(1 + 2i\sqrt{w}e^{-w} \int_{-i\sqrt{w}}^{\infty} e^{-u^2} du \right) \quad (C3)$$

where the term in parentheses is usually called the boundary loss factor and w is given by

$$w = i \frac{4k_1 R_2}{(1 - R_p)^2} \left(\frac{\rho_1 c_1}{\rho_2 c_2} \right)^2 \left[1 - \left(\frac{k_1}{k_2} \cos \theta \right)^2 \right] \quad (C4)$$

with R_2 being the distance travelled by the reflected wave. As pointed out by Rudnick [26], when the receiver is very far from the source compared to the wavelength in air, w becomes very large and consequently the term in curved parentheses approaches 0, approximating the reflection coefficient to R_p that is the solution of a plane incident wave. Rudnick used a boundary between two semi-infinite homogeneous media and the conditions of continuity of pressure and normal displacement, and the final result was to express the last two equations explicitly in terms of the propagation constants of the media. Ingard [27] studied the same problem, but the boundary conditions at the wall were expressed in terms of a normal impedance independent of the angle of incidence and arrived at a very similar expression to the reflected wave to the one obtained by Rudnick. Furthermore in both cases the spherical wave reflection coefficient depends on the plane wave reflection

coefficient and the boundary loss factor that is a function of the normal impedance and the position of the field point (in the former case, it is a function of the acoustic impedance of both media and the position of the field point). The difference lies in the expression for that parameter. If the normal impedance of the wall is ρc , the reflection coefficient is

$$\frac{R}{I} = R_p + (1 - R_p) [1 + \rho_1 e^{\rho_1} \text{Ei}(-\rho_1)] \quad (\text{C5})$$

with ρ_1 given by

$$\rho_1 = ikR_2 (1 + \cos \theta) \quad (\text{C6})$$

in which R_2 and θ are again the distance travelled by the reflected wave and the angle of incidence respectively, whereas Ei is an “exponential integral” function [27]. This means that the reflection coefficient depends on several parameters and it is not only the frequency that influences the reflection, having a minor role on this characteristic. Taraldsen [28] rewrote the exact solution given by Ingard and proved that the boundary loss factor, a term in the reflection coefficient, is a function of only two complex dimensionless quantities and it is sufficient to have the general solution where the source and receiver are on the ground. Because it is not the frequency alone the main parameter that predicts the reflection of sound waves, the spectral dependence of reflection effects is not considered and, although the study of acoustic waves is made for a wide range of frequencies, the reflection coefficient is considered to be a constant for each surface, independent of the frequency, to simplify the problem.

Despite that, several experimental studies were performed to understand the outdoor acoustic propagation. Parkin and Scholes measured the pressure level of acoustic waves, originated from a jet engine (at 1.82 m above the ground) and propagating at nearly grazing incidence above different grounds, at two distances from the source [24]. The measurements were taken when the temperature difference between monitoring points at 1.2 m and 12.2 m heights was less than 0.3 °C and with wind speeds less than 1.52 ms⁻¹. The experiments show that acoustic waves, when propagating near to grassland, snow, or cultivated land, have much stronger attenuation (they are mostly transmitted to the ground) usually between 250 Hz and 800 Hz than either at frequencies below 250 Hz or frequencies in the range of 800 Hz to 2000 Hz. Thus, the sound pressure level far away from the source is much lower than at a point near the source, due only to ground effects, making a ground effect dip in the plot of sound pressure level versus frequency. Moreover, waves with frequencies below 250 Hz are enhanced compared with the same waves if the ground surface does not exist (mainly due to the reflection

on the ground). Attenborough [24] reviewed the theory of the sound interaction with the ground and developed theoretical models for the prediction of sound propagation near to ground surface, applied to the Parkin and Scholes data. The interaction has been shown to depend upon the positions of source and receiver and upon the acoustical properties of the ground. Attenborough pointed out that at frequencies less than 300 Hz and for ranges greater than 50 m over grassland, the reflection coefficient gives rise to surface waves decaying principally as the inverse square root of horizontal range and exponentially with height above the ground, and concluded that they are the major carriers of acoustic noise over long distances because they fit well to the experimental data for low frequencies [24].