Review Article

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Advances in modelling and analysis of nanostructures: a review

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Abstract: Nanostructures are widely used in nano and micro-sized systems and devices such as biosensors, nano actuators, nano-probes, and nano-electro-mechanical systems. The complete understanding of the mechanical behavior of nanostructures is crucial for the design of nanodevices and systems. Therefore, the flexural, stability and vibration analysis of various nanostructures such as nanowires, nanotubes, nanobeams, nanoplates, graphene sheets and nanoshells has received a great attention in recent years. The focus has been made to present the structural analysis of nanostructures under thermo-magneto-electro-mechanical loadings under various boundary and environmental conditions. This paper also provides an overview of analytical modeling methods, fabrication procedures, key challenges and future scopes of development in the direction of analysis of such structures, which will be helpful for appropriate design and analysis of nanodevices for the application in the various fields of nanotechnology.

Keywords: nanostructures, size-dependent continuum models, molecular dynamics, nonlocal doublet method, structural response

1 Introduction

Nanotechnology is an emerging research field which have different domains such as nanomedicine, nanoelectronics, nanomanufacturing, and nanomachines. These domains require new devices, such as nanoenergy harvester, nanomechanical resonators, oscillators, charge detectors, nanoscale mass sensors, field emission devices, biological tissue, and electromechanical nanoactuators. These devices are generally fabricated using nanostructures. Different fundamental nanostructures are nanoscale rods, rings, beams, plates, and shells as shown in Figure 1 [1]. Due to stunning mechanical and electrical properties of carbon nanostructures such as carbon nanotubes (CNTs), graphene sheets (GSs), and fullerenes have been used as fundamental structural units of small size devices [1–6]. Various carbon nanostructures are presented in Figure 2 [7]. During application, nanostructures are subjected to mechanical loads, thermal loads, strains, and stresses. Hence analyses of mechanical behavior and properties of nanostructures are important.

The present paper, for the first time, reviews the mechanical analysis of nanoscale structures using different types of tools. The objective of this work is to sum up the state of art and high light the possible future work in the flexural, stability, and vibration analysis of various nanostructures. This is organized as follows: Section 2 presents literature on processing techniques of nanostructures. In section 3, literatures on applications of nanostructures in different fields of nanotechnology are discussed. Section 4, presents the different solution methodologies used by the researchers to study the mechanical behavior of nanoscale structures. Section 5, presents the recent research reported on the flexural, stability and vibration analysis of various nanostructures.
Advances in modelling and analysis of nanostructures: a review

2 Processing techniques of nanostructures

Main methods of single wall nanotubes (SWNTs) production are arc-discharge, laser ablation, and chemical vapour deposition (CVD). Arc-discharge and laser ablation methods produce SWNTs in few grams. Both methods consist the condensation of gaseous carbon atoms produced from the evaporation of solid carbon. Due to sophisticated equipment requirement and large amount of energy consumption in these methods, their use is limited to laboratory scale [8–10].

The CVD method can be easily scaled to industrial levels and become most important commercial method for SWNTs production. Main advantages of CVD over arc and laser methods are the scale up production to industrial level and more control on morphology and structure of the produced CNTs. This process involves heating a catalyst material to high temperatures (500–1000 °C) in a tube furnace and flow of a hydrocarbon gas through the tube reactor for particular time duration. The general CNT growth mechanism in this method involves the dissociation of hydrocarbon molecules catalysed by the transition metal, and dissolution as well as saturation of carbon atoms in the metal nanoparticle. The precipitation of carbon from the saturated metal nanoparticles results in the formation of tubular carbon solids [11].

SWNTs synthesis methods can be divided into two categories: bulk synthesis and surface synthesis. Bulk synthesis methods are methane CVD, high-pressure catalytic decomposition of carbon monoxide (HiPCO), CO CVD, and alcohol CVD. Surface synthesis of SWNTs has several advantages over bulk synthesis. Such as less defect formation, better performance, and introduction of pattern by various lithography techniques. Recent research in surface growth method involves control of diameters and orientation of SWNTs [12–16].

3 Applications

The interest for applications of nanostructures exhibits due to their superior mechanical, electrical, thermal and chemical properties than those of traditional materials [17–20]. After more than 25 years of research, applications of nanostructures are delivering in both expected and unexpected ways to benefit the society. Researchers shows its application in different fields, like nanomedicine, nanoelectronics, nanomanufacturing, and nanomachines. Nanostructures offer commitment in severe operating and loading conditions.

3.1 Electrochemical devices

Because of large surface areas of porous nanostructure arrays, these are used as electrodes for the devices which use electrochemical double-layer charge injection e.g. supercapacitors and electromechanical actuators. Capacitance of a capacitor is inversely proportional to the separation between two electrodes. This separation for a nanotube is about a nanometre and for ordinary dielectric capacitors is more than micrometre. Due to small separation and large surface area, the capacitance of CNT supercapacitor is very large as compared to ordinary capacitor [22–27]. Single-walled carbon nanotubes (SWCNTs) based macroscale electromechanical actuators are assemblies of billions of individual nanoscale actuators. In these, low operating voltages generate large actuator strains as compared with the 100 V used for piezoelectric stacks and the 1000 V used for electrostrictive actuators. These provide considerably higher work densities per cycle than any other technology [25].

Figure 2: Different types of carbon nanostructures: (a) fullerene (b) nanotube (c) graphene sheet (d) diamond structure.

nanostructures. Finally, some concluding remarks are presented in Section 6.
Table 1: Classification of nano-sensors based on the type of variable being detected [21]

<table>
<thead>
<tr>
<th>Sr No.</th>
<th>Type of variable</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mechanical</td>
<td>Position, acceleration, stress, strain, force, pressure, mass, density, viscosity, moment, torque</td>
</tr>
<tr>
<td>2</td>
<td>Acoustic</td>
<td>Wave amplitude, phase, polarization, velocity</td>
</tr>
<tr>
<td>3</td>
<td>Optical</td>
<td>Absorbance, reflectance, fluorescence, luminescence, refractive index, light scattering</td>
</tr>
<tr>
<td>4</td>
<td>Thermal</td>
<td>Temperature, flux, thermal conductivity, specific heat</td>
</tr>
<tr>
<td>5</td>
<td>Electrical</td>
<td>Charge, current, potential, dielectric constant, conductivity</td>
</tr>
<tr>
<td>6</td>
<td>Magnetic</td>
<td>Magnetic field, flux, permeability</td>
</tr>
<tr>
<td>7</td>
<td>Chemical</td>
<td>Components (identities, concentrations, states)</td>
</tr>
<tr>
<td>8</td>
<td>Biological</td>
<td>Biomass (identities, concentrations, states)</td>
</tr>
</tbody>
</table>

3.2 Field emission devices

SWNTs and MWNTs are using as field emission electron sources [28, 29] for x-ray [30] and microwave generators [31], flat panel displays [32], lamps [33], gas discharge tubes providing surge protection [34]. Nanotubes provide steady and uniform emission, long lifetimes, low emission threshold potentials [28, 33], and high current densities [35]. Nanotube field-emitting surfaces are comparatively easy to manufacture by screen printing nanotube pastes and do not distort in moderate vacuum (108 Torr). These are benefits over tungsten and molybdenum tip arrays, which require high vacuum of 1010 torr and are tougher to fabricate [36].

3.3 Sensors

Sensors are always in demand in scientific as well as industrial applications, and a search for advance materials in sensor applications continues at all times. Sensors using nanostructures have a smaller size and lower weight, leading to higher sensitivity, better specificity and exceptional stability [21, 37–40].

The working principle of a nanosensor is to access data from atomic scales and transfer this at macroscale as analysable data. According to the type of variables to be detected by nanosensors, they are classified into six groups: mechanical, electrical, optical, magnetic, chemical, and thermal. These are used to find different types of properties as listed in Table 1. Nanosensors have a wide range of applications, including liquid flow rate measurement, early disease detection, detection of gene mutations, DNA sequencing, gas detection, accurate monitors of material states, and monitoring glucose levels for diabetic subjects [41–48].

3.4 Probes

CNTs and carbide nanorods have high aspect ratios, such as nanometer order diameter and micrometer order length and a small tip radius with sharp cone. This shape is suitable for probe tips of a scanning probe microscope (SPM) [49–53]. These opens out the possibility of probing the deep crevices that occur in microelectronic circuits [51, 52], and improves the lateral resolution [53]. CNTs can be elastically buckled [50, 51] which makes them robust and also prevents damage to soft biological and organic samples by limiting the maximum force applied on the samples [53].

3.5 Composite structures

Various nanostructures are used to enhance the electrical, mechanical, and thermal properties of polymer-based composite materials, due to their exceptional properties and perfect atom arrangement [54–62]. Lau and his research team focused their study on the synthesis of coiled CNTs and their application to alter the mechanical properties of composite structures [63–65]. They investigated the applicability of coiled CNTs and randomly-oriented nanoclay-supported CNTs to improve the mechanical properties of epoxy resin under the cryogenic environment [66]. Further they investigated the pullout behavior of coiled CNTs by considering the thermal residual stresses under the cryogenic environment [67].

4 Solution methodologies

Number of experimental and theoretical studies dealing with mechanical responses of nanostructures can be seen in literature. It is very expensive, time consuming, and difficult to conduct and control the experiment at nanoscales
and hence progress in experimental studies is limited. That’s why research is focused on theoretical modeling. Theoretical modeling of nanostructures can be classified in atomistic modeling, continuum mechanics modeling, and hybrid atomistic-continuum mechanics modeling [68–70].

Atomistic modeling deals with analysis at atomic and molecular level. It includes classical molecular dynamics (MD), density functional theory techniques and tight binding molecular dynamics [71–76]. Atomistic modeling for a small scale structure with a large number of atoms/molecules demands huge computational effort so this becomes tedious and time consuming. So continuum mechanics modeling comes into picture and played an important role in mechanical analysis of nanostructures. In this type of modeling, nanostructures are treated homogeneous and continuum while their internal atomic structures are not taken into account. Hence computational cost and time using continuum mechanics approach is reduced by wiping out of unnecessary simulations. But accurate detection of crystal lattice structure using continuum mechanics approach is doubtful. Then atomistic-continuum modeling came in to picture and eliminates the drawbacks of atomistic modeling and continuum modeling. A linkage between two modeling is established by equating the molecular potential energy with mechanical strain energy of a given volume of the nanostructure of a continuum model [77–79].

Continuum mechanics can be categorized into classical (local) continuum mechanics and nonlocal continuum mechanics. In Classical continuum mechanics, various bulk material theories like classical (local) beam, plate, and shell are used for mechanical analysis of nanostructures for large scale systems [80–82]. Although a lot of research efforts have been carried out using classical continuum mechanics, their application at the nanoscale is questionable. Because small scale effects such as Van der Waals (vdW) forces, surface effects, lattice spacing, electric forces, and chemical bond are neglected in classical continuum mechanics. Both experimental and atomistic simulation results represent that at the nanoscale, these small scale effect may not be neglected [83]. This is because at the small size the lattice spacing between the atoms becomes more important and discrete structure (internal) of the material can no longer be homogenized into a continuum [70]. To take care of small-scale effects, nonlocal continuum theories have been determined. These incorporate a size-dependent parameter in the modeling of the continuum to account small scale effect. Various nonlocal continuum theories are strain gradient theory, couple stress theory, micropolar theory, and nonlocal elasticity theory [84, 85].

4.1 Molecular dynamics

Molecular dynamics (MD) is a computer simulation technique to investigate the physical movements of atoms and molecules, in which atoms and molecules interact for a fixed period of time, giving a dynamic view of the considered system. The movement of atoms and molecules are obtained by numerically integrating Newton’s equations of motion. The forces between the particles are calculated using interatomic potentials or molecular mechanics force fields [86–90].

4.2 Surface elasticity

In order to avoid the large-scale computational efforts arisen from the molecular simulations, the continuum mechanics is still employed to model the effective and localized responses of materials, such as the nanowires, nanotubes, nanofibers, etc., which usually exhibit significantly distinct behavior from the macro-sized or even micro-sized structures. The mechanical difference caused by the surface effects due to the large surface-to-volume ratios is simulated using the surface-elasticity model described by Gurtin and Murdoch [91, 92], who treated the surfaces of the nanomaterials as zero-thickness smooth layers with distinct physical properties. The surface properties are also dependent on the internal crystalline directions of the bulk materials. For instance, Miller and Shenoy [93] employed the atomistic simulations to generate the nano Aluminum surface properties that were extensively employed. Later on, Steigmann and Ogden [94] extended the theory by considering the flexural resistance. Because of the easy mathematical characterization, the surface-elasticity models are continuously implemented in the recent two decades, including the nano-beams [95], nanoplates [96], nano-films [97], and functionally graded nanomaterials [98], and even heterogeneous materials with nanoscale inhomogeneities [99, 100]. A detailed review work is conducted by Wang et al. [101] and Chen et al. [102].

4.3 Nonlocal elasticity theory

This theory was provided by the Eringen [103–105] in 1972 and for the first time, Peddieson et al. [106] recommended this theory to analyze the mechanical behavior of nanoscale structures. According to this theory, stress tensor at a reference spot in an elastic continuum is dependent on the strain field at all spots in the domain. This assumption represents the long range intermolecular in-
terations and leads to a size-dependent theory of elasticity, which represents good agreement with the atomic theory of lattice dynamics and experimental examination on phonon dispersion. Neglecting the body forces, the most general form of the constitutive relation for linear homogeneous isotropic elastic solids according to nonlocal elasticity theory are

\[
\sigma_{ij}^{nl}(\xi) = \int_{V} K(|\xi' - \xi|, \mu)\sigma_{ij}^{l}(\xi') \, dV(\xi')
\]

(1)

The terms \(\sigma_{ij}^{nl}, \sigma_{ij}^{l}, K(|\xi' - \xi|, \mu), |\xi' - \xi|, \) and \(\mu\) are nonlocal stress tensor, local stress tensor, nonlocal modulus, distance in the euclidean form, and nonlocal parameter (this is dependent on the internal and external characteristic lengths), respectively.

The local stress \(\sigma_{ij}^{l}\) at a spot \(\xi'\) is associated to the strain \(\epsilon_{ij}\) at that spot by generalized Hooke’s law

\[
\sigma_{ij}^{l}(\xi') = C_{ijkl} \epsilon_{kl}(\xi')
\]

(2)

Where \(C_{ijkl}\) is the fourth-order elasticity tensor. For the ease of simplicity, equivalent differential form of the integral constitutive equations (1) and (2) was proposed which is as follows

\[
(1 - \mu^2 \nabla^2)\sigma_{ij}^{nl}(\xi) = \sigma_{ij}^{l}(\xi)
\]

(3)

4.4 Nonlocal strain gradient theory

In literature, researchers observed disappearance of size effect after a certain length and only stiffness softening by using the nonlocal elasticity theory. But few researchers observed stiffness hardening, especially at higher lengths. Furthermore few researchers got the same results by nonlocal elasticity theory as given by classical theory [1]. To overcome these issues of using nonlocal elasticity theory, Lim et al. [107] presented a higher-order nonlocal strain gradient theory (NSGT) using two independent short length-scale parameters. According to NSGT, stress at a particular point is a function of strain as well as higher order strain gradient at all points in the domain. This theory takes into account both the inter-atomic forces and higher-order micro-structure deformation mechanism. The new theory observed both increase and decrease in stiffness of material at small-scale levels, also predicts scale effects in a long range of lengths. Afterwards, lot of research has been carried out to investigate the mechanical response of nanostructures by using NSGT [105, 108]. Researchers found that wave dispersion obtained by NSGT was very close with that predicted by molecular dynamic simulation (MDS) for the nanobeams [109].

According to NSGT, a new stress tensor is proposed as follows [110]:

\[
\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla \sigma_{ij}^{(1)}
\]

(4)

Here, \(\sigma_{ij}^{(0)}\) and \(\sigma_{ij}^{(1)}\) are the zero order nonlocal stress and the higher order nonlocal stress, respectively, and given as:

\[
\sigma_{ij}^{(0)}(\xi) = \int_{V} K_{0}(|\xi' - \xi|, \mu_{0})\sigma_{ij}^{l}(\xi') \, dV(\xi')
\]

(5)

\[
\sigma_{ij}^{(1)}(\xi) = l^2 \int_{V} K_{1}(|\xi' - \xi|, \mu_{1}) \nabla \sigma_{ij}^{l}(\xi') \, dV(\xi')
\]

(6)

The terms \(K_{0}\) and \(K_{1}\) are the modulus and \(mu_{0}\) and \(mu_{1}\) are the scale coefficients, while \(l\) is strain gradient coefficients.

In NSGT, constitutive relation of stress and strain components can be written as [107]:

\[
[1 - \mu_{1}^2 \nabla^2] [1 - \mu_{0}^2 \nabla^2] \sigma_{ij} = C_{ijkl}[1 - \mu_{1}^2 \nabla^2] \epsilon_{kl} - C_{ijkl} l^2 [1 - \mu_{0}^2 \nabla^2] \nabla \epsilon_{kl}
\]

(7)

where \(\nabla^2\) is a Laplacian operator.

4.5 Nonlocal doublet mechanics

Doublet mechanics (DM) is a micro mechanics proposed by Granik in 1978 [111]. Unlike the other size dependent theories, DM directly depends on the micro/nano structure of the solid. In DM, micro stress and micro-strain relations are obtained for displacements of each particle in the domain. Then these micro stress-strains relations are connected to macro stress-strain relations and lead to the governing equations of the problem. These equations include material properties, macro deformations, and intrinsic length scale. In this, scale parameter for the materials are the real atomic distance and small scale displacements are expanded in Taylor series. Also, various numbers of terms in the Taylor series expansion are used to control the order of the equations. However, the accuracy of the theory is not dependent on the number of terms considered in the Taylor series. The number of terms of Taylor series taken in to account is only related to the discreteness of the domain. If we consider the only first term in the Taylor series expansion, then governing equations will reduce to classical elasticity theory [112–114].
5 Analysis

A review of the current state of the art is written with an emphasis to present flexural analysis, stability analysis, and vibration analysis of various nanostructures using different analytical methods proposed by several researchers without recognizing the detailed mathematical implication of different analytical and numerical methodologies.

5.1 Flexural analysis

5.1.1 Molecular dynamics

Iijima et al. [71] performed molecular dynamic simulations (MDS) using Tersoff-Brenner interaction between the carbon atoms and vdW interaction between the neighbor walls of MWNTs. They found that the bending of SWCNTs and multi-walled carbon nanotubes (MWCNTs) are fully reversed up to 110°, despite the formation of kinks. This is due to the high flexibility of hexagonal structure, which maintains bond under high values of strain. Simulation with diameters, lengths, and helicities as varying parameters showed that the critical local curvature is independent on the length of the tube. Shibutani and Ogata [115] evaluated the bending and torsional deformations of SWNTs considering the Tersoff-Brenner type interatomic potential and found a hysteresis loop in bending due to the transformation from four hexagons to double pairs of heptagons and pentagons. While loop was absent in torsion deformation.

Using controlled SPM manipulation, Duan et al. [116] found “abrupt” and “gradual” two modes of CNTs buckling during the bending. Furthermore, these two modes are dependent on the diameter and thickness of CNTs. Wang et al. [117] solved fourth-order nonlinear ordinary differential equations using continuation algorithm of the buckling of nanobeams under concentrated loads [106, 120, 121]. Then Wang and Liew [122] developed nonlocal Timoshenko beam theory (TBT) and Euler–Bernoulli beam theory (EBT) by considering concentrated loads as a Dirac delta function. Their results reported the effect of nonlocal elasticity on the bending behavior of nanobeams for the first time. Then Researchers formulated various nonlocal beam theories such as EBT, TBT, Reddy beam theory (RBT), and Levinson beam theory (LBT) and applied them to study the mechanical behavior of CNTs under loading. Based on the modified couple stress theory, Ma et al. presented a Timoshenko beam model to explain the static bending and free vibration problems. Yang et al. studied the nonlocal effect on the pull-in instability of nano-switches under electrostatic and inter-atomic forces [123–126]. Aydogdu [127] developed a generalized nonlocal beam theory, from which earlier formulated theories can be obtained as special cases. He investigated the effect of nonlocality and length on the structural response of nanobeams.

Challamel and Wang [128] solved a paradox: the absence of small scale effect in some bending solution. They developed a hybrid model by taking strain energy as a function of both local and nonlocal curvatures. Zhang et al. [129] used this model to study the static and dynamic behavior of nanobeams and found the different solutions of bending for clamped beams and cantilever beams. Civalek and Demir [130] formulated a nonlocal EBT for bending analysis of microtubules using the differential quadrature method (DQM).

Thai [131] proposed a nonlocal shear deformation beam theory to study the structural response of nanobeams without using shear correction factor. Zenkour and Sobhy [132] proposed a shear and normal deformations nonlocal theory for bending of nanobeams under

5.1.2 Nonlocal elasticity theory

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thermal environment. Also, they investigated the effects of nonlocality, length of the beam, length-to-depth ratio, temperature parameters, shear and normal strains on the bending of nanobeams. Rosa and Franciosi [133] proposed a simpler method to calculate the effect of nonlocal elasticity by using Mohr analogy with EBT and TBT. Reddy and Borgi [134] formulated beam theories using Eringen’s differential model with the modified von Kármán nonlinearity and developed finite element (FE) model. They studied the effect of nonlocal parameter on the bending behavior. Khodabakhshi and Reddy [135] proposed a nonlocal integro-differential model as a more generalized Eringen’s nonlocal model for 3-Dimensional FE formulation. They solved the inconsistency in results found by several authors [106, 121, 122, 128, 136] for a cantilever when compared to other boundary conditions.

Saez et al. [137] used an integral form of Eringen nonlocality for the bending analysis of Euler–Bernoulli beam and solved the paradox which was appearing in the solution of a cantilever beam using the differential form of Eringen nonlocality. Tuna and Kirca [138] derived the exact solution of the integral form of Eringen nonlocal model for the bending analysis of Euler–Bernoulli and Timoshenko beams. Koutsoumaris et al. [139] investigated the static response of nanobeams using nonlocal integral elasticity with the modified kernel. Romano and Barretta [140] compared stress-driven and strain driven nonlocal integral models for nano-beams. Romano et al. [141] discussed issues in the solution of nonlocal elastostatic problems of beams. They found that the constitutive boundary conditions are not compatible with the equilibrium conditions imposed on the bending field. Barretta et al. [142] developed a stress-driven nonlocal integral model to study the thermoelastic behavior of nanobeams.

Jiang and Yan [143] derived explicit solutions to study the combined effects of residual surface stress, surface elasticity and shear deformation on the static bending of NWs using Timoshenko beam model. They found that stiffness may increase or decrease with residual surface stress depending on the boundary conditions, and shear deformation always makes the NWs softer compared with the Euler–Bernoulli beam model. Ansari and Sahmani [144] proposed explicit formulas to each type of beam theories (EBT, TBT, RBT, and LBT) to investigate the surface stress effects on bending and buckling behavior of nanobeams and found identical results as Jiang and Yan [143]. Juntaarasaid et al. [145] took the first step to investigate the combined effect of surface stress and nonlocal elasticity on the bending and buckling of nanowires. Mahmoud et al. [146] developed a nonlocal FE model to study the effects of beam thickness as well as nonlocality on the rigidity and bending behavior of Euler-Bernoulli nanobeams including surface effects. Preethi et al. [147] presented a non-local nonlinear FE formulation for bending and free vibration analysis of Timoshenko beam with surface stress effects.

Yang et al. [148] performed nonlinear thermal bending analyses for the simply supported (SS), Clamped-clamped (C-C), and propped cantilever shear deformable nanobeams. They used the Timoshenko beam model with von Karman geometric nonlinearity. Najer et al. [149] modeled nanoactuator as a Euler–Bernoulli clamped-free (C-F) beam and C-C beam to study its nonlinear static and dynamic responses when subjected to a DC voltage. They took into account Casimir and vdW forces, an electrostatic force with fringing effect, and von Kármán nonlinear strains. Hamilton’s principle and DQM are used to derive and discretize the governing equation respectively.

In addition to nanobeams, many researchers reported flexural analysis of nanoplates in the literature. Aghababaei and Reddy [150] reported size-dependent third-order shear deformation plate theory for the bending and vibration analyses of S-S rectangular nanoplates. Further, Reddy [151] developed the nonlinear nonlocal Kirchhoff and Mindlin plate theories with von Kármán nonlinearity.

Golmakani and Rezatalab [152] performed nonlinear nonlocal bending analysis of orthotropic nanoplates resting on an elastic matrix foundation using nonlocal first-order shear deformation theory (FSDT). They found that an increase in scale coefficient results rise in linear to nonlinear deflection ratio for the nanoplate without elastic foundation and reversed effect of scale coefficient on the mentioned ratio for the nanoplate with elastic foundation. They also found sharper deference in results of local and nonlocal theories for clamped boundary condition than the simply supported, under a large load. While under small loads it is sharper in simply supported than clamped one. Far and Golmakani [153] used same tools as in [152] to study the large deflection behavior of a bilayer GS resting on polymer matrix under thermo-mechanical loads. Yan et al. [154] derived an infinite higher-order governing differential equations to formulate nanoplate models for bending analysis. They studied the small scale effect on bending behavior of the circular nanoplates under different boundary conditions and observed that the variation trend is independent of the constraints.

Wang and Wang [155] developed a FE model for static and dynamic analysis of nanoscale plates based on a mathematical model developed by Lu et al. [156] with surface effects. Zhang and Jiang [157] incorporated the surface effects and the flexoelectricity into the Kirchhoff plate model to study the size-dependent properties of a bending piezo-
electric nanoplate. Raghu et al. [158] developed nonlocal third-order shear deformation theory taking into account the surface stress effects, to study the bending and vibration of nanoplates.

5.1.3 Nonlocal strain gradient theory

Lu et al. [159] developed a size-dependent unified high-order beam model to study the structural response of simply supported nanobeams. They found that the nanobeam stiffness-softening effect and stiffness-hardening effect depends on the relative magnitude of the material length scale parameter and nonlocal parameter. Xu et al. [110] obtained the closed-form solutions for bending and buckling loads of geometrical imperfect nanobeams by using the NSGT. They found that the higher-order boundary conditions and the material length parameters have a prominent effect on the buckling loads. However static bending is not affected by the higher-order boundary conditions. Tang et al. [160] revealed that the NSGT with thickness effect, the stiffness-hardening and stiffness-softening effects are dependent on the thickness of the beam. For constant length, decreasing thickness leads to an increase in stiffness-hardening effect.

Rajasekaran and Khaniki [161] reported the same type analysis as Li et al. [162] for isotropic tapered nanobeams using the FE method. Fakher et al. [163] investigated bending behavior along with vibration behavior using three different forms of NSGT. Which are differential forms of nonlocal strain gradient, a basic form of integral nonlocal strain gradient without satisfying higher-order boundary conditions, integral nonlocal strain gradient with satisfying higher-order boundary conditions. They found a significant difference in results with different approaches. Ouakad et al [164] studied the nonlinear response of electrically actuated CNTs by modeling it as C-C Euler–Bernoulli beam, which accounts both nonlinear von-Karman strain and electric actuating force. Recently, Norouzzadeh et al. [165] studied the nonlinear bending behavior of FE modeled nanobeams.

5.1.4 Nonlocal doublet mechanics

Ebrahimian et al. [166] studied the effect of chirality on the softening or hardening behavior of a Euler-Bernoulli and Timoshenko nanobeams in bending. Incorporation of the scale parameter and chiral angle makes the nanobeam softer. However, as the chiral angle rises from 0 to 30 degrees, the nanobeam becomes stiffer. Under same loading and boundary conditions, Timoshenko nanobeam changes from a softening to a stiffening behavior with an increase in chiral angle. Recent work related to flexural analysis of nanostructures are listed briefly and explicitly in Table 2.

5.2 Buckling analysis

5.2.1 Molecular dynamics

Montazeri et al. [174] explored the effect of chemically adsorbed Hydrogen atoms on the buckling behavior of graphene nanoribbons (GNRs) at 0.01 K temperature by using Large-scale Atomic/Molecular Massively Parallel Simulator (LAMMPS) software. They found that an increase in hydrogen concentration reduces the stiffness of GNRs. Wu and Soh [175] observed that when the axial compressive force is below the critical value, the nanorod contracts in the axial direction. While rod exhibits buckling when the force reaches a critical value. Further, an increase in external loading results in post-buckling. Zhang and Shen [176] performed MDS to study buckling and post-buckling behavior of SWCNTs under combined loading (axial compressive and torsion) at 300 K, 800 K, and 1500 K. They found that the SWCNTs has higher temperature dependence under large axial load as compared to under low axial load.

MDS performed on SWCNTs with intermolecular junction (IMJ) revealed that the critical strain for SWCNTs depends on the loading velocity under high strain-rate compressions while this dependency is negligible under low strain-rate compression. Two more things were also found: (i) buckling modes of an IMJ may transfer from shell buckling to column buckling when its aspect ratio exceeds a threshold value and (ii) As compared to CNTs, IMJ has a lower critical aspect ratio at which the transition of buckling mode occurs. These results may suggest a reasonable loading velocity for MDS [177, 178].

Hao et al. [179] studied the effect of a single vacancy on the buckling behaviors of SWCNTs and double-walled carbon nanotubes (DWCNTs) under axially compressed loading. They observed that the ratio of the number of defects to the number of atoms in the tube influences the buckling behavior of SWCNTs. They reported that the location of defects has a strong effect on the buckling behavior of the DWCNTs. Akita et al. [180] examine the influence of the number of layers on the buckling behaviors of MWCNTs with the fixed outer diameter, using a nanomanipulation scanning probe microscopy (SEM) and MDS. Zhang et al. [181] compared the buckling behaviors of (5,5) CNTs, ((4,4), (10,10)) DWCNTs, ((5,5),(10,10)) DWC-
Table 2: Representation of published articles on the bending analysis of nanobeams/nano plates/nanoshells

<table>
<thead>
<tr>
<th>Sr no.</th>
<th>Ref.</th>
<th>Theory/Method used</th>
<th>Major outcome</th>
<th>Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[167]</td>
<td>Micropolar theory, TBT, Hamilton’s principle, variational principle</td>
<td>A finite element approach is proposed for the scale dependent bending analysis of nanobeam using the micropolar theory in combination with the nonlocal elasticity.</td>
<td>Shear co-efficient is assumed</td>
</tr>
<tr>
<td>2</td>
<td>[168]</td>
<td>Euler-Bernoulli beam theory, nonlocal elasticity</td>
<td>A nonlocal nonlinear model of nanobeam with small initial curvature is developed and used for mechanical analysis of SWCT.</td>
<td>Shear effect is not considered and this represents only stiffness softening</td>
</tr>
<tr>
<td>3</td>
<td>[169]</td>
<td>Higher order shear deformation theory, nonlocal elasticity theory</td>
<td>Influences of nonlocal parameter, applied electric potential, Winkler and Pasternak’s parameters of foundation are investigated on the electro-elastic bending of piezoelectric doubly curved nanoshell.</td>
<td>Magnetic and thermal effects are not considered</td>
</tr>
<tr>
<td>4</td>
<td>[170]</td>
<td>Principle of virtual work, FSDT, nonlocal strain gradient piezo-magneto-elasticity theory</td>
<td>Influences of electric and magnetic potentials, porosity volume fraction, geometrical characteristics, and foundation parameters on the bending behavior of the sandwich nanoplate are studied.</td>
<td>Temperature effect is not considered and Poisson ratio is constant.</td>
</tr>
<tr>
<td>5</td>
<td>[171]</td>
<td>Kirchhoff’s plate theory, NSGT, Navier’s method, principle of the virtual works</td>
<td>Bending behavior of S-S isotropic, anti-symmetric orthotropic cross and angle-ply laminated nanoplates of rectangular shape subjected to uniformly and sinusoidally distributed loads are studied.</td>
<td>Only simple supported boundary conditions are used</td>
</tr>
<tr>
<td>6</td>
<td>[172]</td>
<td>Higher order refined curved nanobeam theory, Two power-law models, Hamilton’s principle, Kelvin–Voigt model</td>
<td>The influences of nonlocal parameter, structural damping, elastic foundation coefficients, and strain gradient length scale parameter on bending, buckling, and vibration response of viscoelastic FG curved nanobeam are investigated.</td>
<td>Poisson’s ratio is considered constant and temperature effect is not considered</td>
</tr>
<tr>
<td>7</td>
<td>[173]</td>
<td>EBT, power-law distribution, principle of the calculus of variation, Duhamel’s integration, concept of the neutral surface</td>
<td>Analytical solutions for static displacement, critical buckling load, free vibration frequencies and dynamic displacement are obtained. Also, emphasized on the influence of the excitation frequency and the moving load velocity on the mechanical behavior of FG nanobeam.</td>
<td>Solution is only for simply supported end condition and considered only limited type of loading conditions. Also, simple-power law model is used</td>
</tr>
</tbody>
</table>

NTs, ((6,6), (10,10)) DWCNTs, and ((7,7), (10,10)) DWCNTs by analyzing the van der Waals (vdW) forces. They used Tersoff–Brenner potential to describe the interaction of atoms in the tube, while Lennard–Jones potential to describe the van der Waals force between inner and outer tubes during the MDS. Wang [182] found that the CNTs containing polyethylene molecules buckle with the same process as CNTs, but the buckling strain is reduced by 35% as compared to empty CNTs.

Guo et al. [183] compared the buckling behaviors of gold-filled SWCNTs with gas-filled SWCNTs subjected to axial loading. They found that the gas or fullerenes filled SWCNTs collapses directly from the elastic region at the critical strain, while Au filled SWCNTs results in an elastic–inelastic transition before the critical strain. Which is
a consequence of the difference between vdW forces and metallic interactions. Ansari et al. [184] investigated the mechanical properties and buckling behavior of diethyltoluenediamines functionalized CNTs using LAMMPS. In this, they implemented the Nose–Hoover thermostat algorithm within velocity-Verlet integrator algorithm to reduce the fluctuations in temperature. They used conjugate gradient algorithm to reach sufficient minimum relative energies and minimize the energy of the simulation system. Their results show a linear increase in Young’s modulus and critical buckling force for both regular and random polymer distributions, while critical strain decreases with different modes depending on the type of polymer distribution. Jing et al. [185] performed MDS with Stillinger–Weber potential to study the effects of simulation temperature, wire length, and strain rate on buckling behavior of single-crystalline silicon NWs under the uniaxial load. They found that critical load decreases with increase in temperature, decrease in strain rate and increase of wire length.

Salmalian et al. [186] employed LAMMPS MD code with adaptive Intermolecular Reactive Empirical Bond Order (AIREBO) potential function and velocity Verlet algorithm to study the effects of side length, atomic structure and aspect ratio on the critical compressive force and critical strain of GSs. They found that for a constant aspect ratio, smaller GSs acquire larger buckling forces, but buckle at smaller strains. They also found two more things (i) critical forces of zigzag GSs greater than the armchair GSs and (ii) decrease in the critical strain as well as the critical force of GSs with an increase in the number of acetylene links. Jeong and Kim [89] performed MDS to investigate the failure behaviors of C60 fullerenes filled SWCNTs under tensile, compressive, torsional, and combined loads at 300K temperature. Inter atomic forces are modeled using modified Reactive Empirical Bond Order potential function and vdW interactions are modeled using Lennard-Jones potential.

Wong and Vijayaraghavan [90] performed MDS with Brenner’s second-generation bond order function coupled with the long-range Lennard-Jones potential to study the buckling behavior of the non-uniform CNT bundle such as P–C, P–S and C–S bundle. They found that the C–S bundle have lower critical strain and buckling load compared to the C–C bundle. Nishimura et al. [187] investigated the local buckling of defective and non-defective five-walled CNTs under compression and bending load, using MDS with AIREBO potential. In general, it has been observed that temperature, diameter, length, and chirality of nanotubes affect Young’s modulus, strain, and buckling mode of the nanotubes. Results show that Stress is very sensitive to helicity under large strain at 0 K temperature while Young’s modulus has more dependence on the diameter than on the helicity [188, 189]. Chandra et al. [190] observed that critical buckling temperature of pre-compressed boron-nitride nanotubes increases with decrease in chiral angle as well as decrease in the diameter of the nanotube.

5.2.2 Nonlocal elasticity theory

There has been extensive research on the buckling of nanostructures, from the formulation of nonlocal models of EBT, TBT, and LBT beam theories to the nonlocal classical and shear deformation beam and plate theories using von Karman nonlinear strains [123, 151, 191–194]. Few researchers studied the buckling behavior of nanobeams using nonlocal integral and two-phase nonlocal integral model of Eringen [138, 195].

Roque et al. [196] used meshless method to obtain numerical solutions for bending, buckling and free vibration of nonlocal Timoshenko nanobeams. Then compared numerical solutions with the available analytical solutions. Emam [197] developed a general nonlocal nonlinear model to present analytical solutions for the critical buckling load and the nonlinear static amplitude in the postbuckling state for nanobeams. He found that increase in nonlocal parameter results in decrease in critical buckling load and increase in postbuckling amplitude. Chen et al. [198] developed an EBT based piezoelectricity and viscoelasticity coupled nonlocal beam model, to study the buckling, post-buckling, and nonlinear dynamic stability for the piezoelectric viscoelastic nanobeams under the action of vdW forces. Baghani et al. [199] studied the influence of compressive axial load, surface energy, and magnetic field on the dynamic and stability response of the rotating nanobeams.

Lim and his research group [200, 201] studied the nonlinear buckling of nanorods and shear deformable nanocolumns using the TBT and nonlocal elasticity, under the thermal load. Their results revealed that the critical buckling loads are higher in nonlocal theory as compared to classical theories with or without shear deformation. They also found that at room temperatures, increase in temperature change led to rise in buckling load of nanostructures, while increase in temperature change leads to decrease in buckling load at high temperature. Tounsi et al. [202] incorporated the nonlocal effects and von Karman nonlinearity in the higher order beam model given by Shi and Voyiadjis [203], for thermal buckling analysis of nanobeams. In recent years, buckling and post buckling
of piezoelectric nanobeams are investigated under thermo-electro-mechanical loads [204, 205].

Many researchers investigated the buckling behavior of nanostructures such as nanobeams, nanorods, and nanotubes in a temperature field. Using variational principle, Lim et al. [201] proposed a nonlocal thermoelastic model to study the effects of nonlocal parameter and temperature change on buckling behavior of nanorods. Yang and Lim studied [200] the thermal and nonlocal parameter effects on buckling of shear deformable nanocolumns by using nonlocal stress theory with von Kármán nonlinearity. Tounsi et al. [202] extended the nonlocal beam model developed by Thai and Tai [131], for the buckling behavior of nanobeams under the thermal field. Liu et al. [205] investigated the buckling and post-buckling behaviors of piezoelectric nanobeams by using the nonlocal TBT with von Kármán geometric nonlinearity. They concluded that an increase in the nonlocal parameter as well as temperature results in a decrease in both critical buckling load and post-buckling strength. They also found that positive voltage leads to decrease in the critical buckling load and post-buckling strength, while for negative voltage this effect is reversed. Jandaghiian and Rahmani [204] studied the buckling behavior of the piezoelectric nonlocal Euler-Bernoulli beam under thermo-electro-mechanical loadings. Their results were similar to the results obtained by Liu et al. [205].

Yan et al. [206] studied the effect of temperature, scale parameter and wavenumber on the critical axial buckling load of triple-walled CNTs with the initial axial stress, using nonlocal Donnell’s shell model as used in [207]. They found that the small scale effect increases gradually with the increase in wavenumber. Furthermore, the axial buckling load increases with increase in temperature at room temperature range, while at high temperature range the axial buckling load decreases with increase in temperature.

5.2.3 Nonlocal strain gradient theory

In addition to the use of nonlocal elasticity theory, lot of research has been carried out to investigate the buckling response of nanobeams by using NSGT [110, 159, 161, 162]. Khaniki and his research team emphasized on the buckling response of tapered nanobeams using the NSGT and EBT [208, 209]. Li and his research team explored the nonlinear buckling response of the isotropic and FG nanobeams with various geometric nonlinearities by using the NSGT in conjunction with the EBT. Also, they employed the size dependent effects and higher order strain gradients along with the thickness of nanobeams and studied the influence of thickness on the post buckling response of the nanobeams. Results revealed that the stiffness-softening and stiffness-hardening effects in nanobeams depend not only on the ratio of nonlocal parameter to strain-gradient parameter, but also on the thickness of nanobeams [210–212]. Sahmani and his research team emphasized on the investigation of nonlinear buckling response of supramolecular nano-tubules and lipid protein nano-tubules nanostructures using the NSGT and third-order shear deformable beam model [213, 214].

In addition to the focus on nanobeams, researchers are now investigating nanoplates using NSGT. Farajpour et al. [215] developed a nonlocal strain gradient plate model based on the higher-order NSGT, to study the thermal buckling of orthotropic nanoplates resting on a two-parameter elastic foundation. Three different kinds of scale parameters considered in the formulation for better results at nanoscale. The higher-order governing differential equation are derived by virtual work principle and solved using the differential quadrature method. Based on physical neutral surface concept Radic [216] studied the buckling behaviors of porous double-layered FG nanoplates in hygrothermal environment.

Malikan and Nguyen [217] developed a one variable first order shear deformation plate theory for the buckling analysis of piezo-magneto-electric nanoplates resting on an elastic matrix in hygrothermal environment. In this, size effect are incorporated by the higher order NSGT and considered plate is subjected to external in-plane mechanical forces along with magnetic and electric potentials between the upper and bottom faces. Moreover, the Navier’s and Galerkin’s solutions were used to solve the stability equations to get the numerical results with several boundary conditions. Results revealed that the critical buckling load is more influenced by the magnetic potential than the electric potential. An increase in magnetic potential leads to a significant rise in critical buckling loads and electric potential rise have reversed effect on it. Hygral region potential has more significant influence on the critical buckling loads than the thermal environment. Increase in the moisture percentage leads to stiffness softening of the plate which was more significant at lower temperatures than the high temperatures.

Sahmani and Fattahi [218] developed a comprehensive size-dependent shell model using the NSGT and a refined exponential shear deformation shell theory. In the continuation, authors investigated the nonlinear buckling and post buckling response of magneto-electro-elastic composite nanoshells. Governing differential equations are derived containing the coupling terms between the mechanical load, external magnetic and electrical potential. The nonlinear buckling and the large postbuckling displace-
ments are taken into account based upon the boundary layer theory of shell buckling. Results revealed that a negative magnetic potential and a positive electric potential leads to increase in influence of nonlocality and strain gradient effects on the nonlinear buckling behavior of considered nanoshells, while a positive magnetic potential and a negative electric potential play an opposite role [219]. Authors continued his study to investigate the nonlinear instability of FG multilayer GPLs reinforced composite nanoshells using the nonlocal strain gradient hyperbolic shear deformable shell model [220].

5.2.4 Nonlocal doublet mechanics

In the last two years, Gul and Aydogdu reported an extensive study on mechanical analysis of various nanostructures using the DM. They performed static deformation, buckling, vibration and wave propagation analysis of nanorods and nanobeams by using doublet mechanics in conjunction with the Classical rod theory and EBT. They found that the natural frequencies and critical buckling loads reduce with rising in the doublet distance, especially for short lengths. While bending of the beams, critical buckling loads, vibration frequencies, elongation of the rod, and wave properties predicted by the DM approach to the classical mechanics results with decreasing the doublet distance. In wave propagation of the graphite, DM results reported a better agreement with the experimental results compared to strain gradient models and classical models [221]. In continuation, the authors investigated the buckling behavior of double nanobeams system embedded in an elastic medium based on a Euler-Bernoulli beam model and scale-dependent doublet mechanics theory. They obtained governing equations based on the minimum potential energy principle and reported the exact solution for the critical buckling loads of the considered system [114]. Authors continued their study to investigate the vibration and buckling response of embedded nanotubes using the size-dependent DM. Nanotube was modeled based on the Euler–Bernoulli beam embedded in an elastic medium and reported solution for Free vibration frequencies and critical buckling loads under simply supported and clamped boundary conditions [222]. Furthermore, authors modeled embedded DWCNTs based on the Euler-Bernoulli beams within the framework of DM and reported numerical results for free vibration and buckling of considered DWCNTs system, for simply supported boundary conditions. They found that the elastic medium increases the dimensionless frequency parameter [113]. Recent work related to buckling analysis of nanostructures are listed briefly and explicitly in Table 3.

5.3 Vibration analysis

5.3.1 Nonlocal elasticity theory

Since nanostructures are extensively used as resonators in nano-electro-mechanical systems, pioneering studies on vibration response of nanostructures are carried out by many researchers in the last decade of years. The nonlocal elasticity theory in conjunction with other classical theories has been broadly adopted for analyzing the vibration response of nanostructures [1, 235–237]. Lee and Chang [238] applied the nonlocal elastic theory to investigate the surface and small-scale effects on the frequency response of a tapered nanobeam with fixed-free boundary condition. Mechab et al. [239] investigated the free vibration response of orthotropic beams by using the nonlocal higher-order shear deformation laminates theory including the Poisson effect. Apuzzo et al. [240] formulated a stress-driven integral model for Bernoulli-Euler nanobeams and computed the fundamental natural frequencies of nanobeams with various boundary conditions. Khaniki [241] emphasized on the Eringen’s two-phase nonlocal integral model to study the vibration response of double-layered nanobeam systems with various boundary conditions. This model overcomes the limitations of differential form of Eringen’s nonlocal elastic theory in the analysis of nanostructures.

Many researchers employed nonlocal elasticity theory in various local plate theories such as classical plate theory [242–245], first-order shear deformation plate theory [246, 247], two-variable refined plates [248, 249], and higher-order shear deformation plate theory [250–252] to study the linear vibration response of nanoplates.

Malekzadeh and Farajpour [253] examined the free and forced vibrations response of embedded single and double-layered circular nanoplates under the initial in-plane radial stresses. They found that the natural frequencies of considered nanoplates decreases with increase in the value of nonlocal scale parameter. Also, the higher-order natural frequencies are more sensitive to the nonlocal parameter and the plate radius. Furthermore, increasing the magnitude of the initial radial tensile stresses increases the natural frequencies. Kiani [251] studied the in-plane and out-of-plane free vibrations of a conducting nanoplate under the unidirectional in-plane steady magnetic field by incorporating the nonlocal elasticity into the Kirchhoff, Mindlin, and Reddy plate theories.
Table 3: Representation of published articles on analysis of nanobeams/nano plates/nanoshell using nonlocal elasticity.

<table>
<thead>
<tr>
<th>Sr no.</th>
<th>Ref.</th>
<th>Theory/Method used</th>
<th>Major outcome</th>
<th>Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[223]</td>
<td>Weighted residual approach, modified nonlocal theory, Euler–Bernoulli beam theory</td>
<td>The variational consistent higher order boundary conditions are formulated for the three characteristic-lengths featured size-dependent gradient-nanobeam and closed form solution for buckling load is obtained</td>
<td>Material is isotropic and shear effect is not considered</td>
</tr>
<tr>
<td>2</td>
<td>[224]</td>
<td>Fractional calculus, nonlocal fractional derivative model, TBT, Galerkin method</td>
<td>The free vibration and buckling of a nanobeam is studied based on the conformable fractional derivatives</td>
<td>Only simply supported boundary condition is considered</td>
</tr>
<tr>
<td>3</td>
<td>[225]</td>
<td>Extended Hamilton’s principle, and extended Galerkin’s approach</td>
<td>Influence of piezoelectric voltage and surface effects on the vibration and stability behavior of double nanobeam system is reported. Furthermore divergence and flutter analysis performed</td>
<td>Material is isotropic and temperature effect is not considered</td>
</tr>
<tr>
<td>4</td>
<td>[226]</td>
<td>Kelvin–Voigt model, Hamilton’s variational principle, and Bolotin’s approach</td>
<td>Effects of magnetic field, nonlocal parameter, length to thickness ratio, structural damping, static load factor, power-law index and porosity volume index, foundation type on the dynamic stability characteristics of nanobeam are investigated.</td>
<td>Temperature effect and geometric imperfection is not considered. Also, shear correction factor is assumed</td>
</tr>
<tr>
<td>5</td>
<td>[227]</td>
<td>Nonlocal elasticity, Mindlin theory, Kantorovich method, finite element method</td>
<td>A coupled finite strip–finite element formulation is developed to study the buckling and vibration response of an imperfect nanoplate</td>
<td>The obtained solutions are only approximated</td>
</tr>
<tr>
<td>6</td>
<td>[228]</td>
<td>Refined plate theory, Galerkin method, Reuss micro-mechanical scheme, Hamilton’s principle</td>
<td>Influences of material variation, temperature changes, and scale parameters on the buckling response of nanoplate are presented</td>
<td>Material properties are varying only in thickness direction and power law model is used to assume material properties</td>
</tr>
<tr>
<td>7</td>
<td>[229]</td>
<td>TBT, Variation principle, weighted residual method</td>
<td>The variationally consistent boundary conditions corresponding to the equations of motion of nonlocal strain gradient Timoshenko beams are reformulated using the weighted residual method and closed form solution for buckling load is obtained</td>
<td>Material is considered isotropic</td>
</tr>
<tr>
<td>8</td>
<td>[230]</td>
<td>NSGT, EBT, Hamilton’s principle, two-step perturbation method</td>
<td>Influences of geometrical, material, and elastic foundation parameters on the nonlinear mechanical behaviors of nanobeams are examined</td>
<td>Temperature effect is not considered</td>
</tr>
<tr>
<td>9</td>
<td>[231]</td>
<td>Refined plate theory, Hamilton’s principle, Rayleigh-Ritz method</td>
<td>Using various plate theories, results of nonlocal strain theory and nonlocal stress theory are compared for the buckling load and natural frequency</td>
<td>Mindlin’s nonlocal plate theory is not considered</td>
</tr>
<tr>
<td>10</td>
<td>[232]</td>
<td>higher-order nonlocal strain gradient theory, Liapunov method, EBT</td>
<td>A dynamic stability and instability problem of a nanobeam is investigated by considering the stochastic parametric vibrations</td>
<td>Inertia effect and shear effects are not considered</td>
</tr>
<tr>
<td>11</td>
<td>[233]</td>
<td>Nonlocal couple stress theory, higher order refined beam theory, Chebyshev–Ritz method</td>
<td>Chebyshev–Ritz method is implemented for static stability and vibration analysis of FG nanobeam</td>
<td>Material properties are graded only following power law</td>
</tr>
<tr>
<td>12</td>
<td>[234]</td>
<td>Eringen’s nonlocal elasticity theory and TBT</td>
<td>The critical loads in the nanobeams were obtained assuming a functional dependence of the nonlocal parameters on the state of stress and vibrational frequency</td>
<td>Small-scale effect in the shearing force is neglected</td>
</tr>
</tbody>
</table>

Farajpour and his research team explored the vibration response of coupled piezoelectric nanoplate systems with various boundary conditions. They studied the influence of various parameters such as initial stress, nonlocal parameter, external electric voltage, elastic foundation parameter, temperature change, aspect ratio, length-to-thickness ratio, and mode number on the vibration characteristics of the considered systems [254–256]. Furthermore, they developed a nonlocal continuum model for the size-dependent nonlinear free vibration of magneto-electroelastic nanobeams under the action of external electric and magnetic potentials. They modeled geometric nonlinearity based on von Kármán’s assumptions. They obtained the closed-form solutions for the nonlinear nato-
rational frequencies, critical external electric voltages and critical magnetic potentials of considered nanoplates with distinct boundary conditions. Also, they emphasized on the influence of initial edge displacement on obtained solutions [257, 258].

Together with the analysis of nanobeams and nanoplates, the vibration response of nanorods [259–261], embedded nanorods [262, 263], double nanorod systems [264, 265], and tapered nanorods [266] has been examined using the nonlocal continuum mechanics in the literature. Murmu et al. [267] presented the analytical solution for the axial vibration of embedded nanorods under the excitation of a transverse magnetic field by using the nonlocal rod theory. They revealed that the transverse magnetic field weakens the influence of nonlocal effect. Also, the alteration of frequency with the axial stiffness parameter of a nanorod in an elastic medium and magnetic field becomes more nonlinear by considering the nonlocal effect. Karlicic and his research team emphasized on the longitudinal vibration analysis of nonlocal viscoelastic coupled multi-nanorod system and the influence of transverse magnetic field on vibration response of the multi-nanorod systems [265, 268].

Li et al. [269] employed the nonlocal elasticity theory in the Bishop-Love rod theories to develop a nonlocal higher-order model for studying the axial free vibration of nanorods. The developed model consists of radial inertia and deformation effect. Furthermore, by using the nonlocal elasticity theory and Love rod Li et al. [270] studied the vibration response of a nanorod carrying a tip mass. They found that the attached mass and inertia of radial motion reduces the resonance frequencies, however, large tip mass rises the frequency shift, irrespective of considering or neglecting the nonlocal effect. Numanoglu et al. [271] investigated the longitudinal free vibration characteristics of nanorods carrying tip mass with various boundary conditions.

Recently, Karlicic et al. [272] obtained the exact closed-form solutions for the fundamental frequencies of the multiple nanorods system installed on an elastic medium with various boundary conditions. The governing equations of motion of the system are determined by using the nonlocal elasticity theory, Bishop’s rod theory, and Hamilton’s principle. They concluded that the increase of elastic medium stiffness coefficients leads to an increase of natural frequencies, while the rise in the number of nanorods has reversed effect on the natural frequency of the system.

5.3.2 Nonlocal strain gradient theory

NSGT based models have been also reported for the vibration response of nanoscale beams [161–164, 273]. Lu et al. [274] developed a size-dependent sinusoidal shear deformation beam model based on the NSGT. Hamilton’s principle was employed to derive the governing equations and boundary conditions. Moreover, analytical solutions for natural frequencies of S-S nanobeams obtained by Navier’s method. They found higher natural frequencies by the NSGT than those obtained by nonlocal theory and lower than those predicted by strain gradient theory. Furthermore, they observed the higher influence of shear deformation for nanobeams with lower values of slenderness ratios and at higher modes. Apuzzo et al. [275] obtained an exact solution for axial and flexural free vibrations of the cantilever and fully-clamped Euler-Bernoulli nanobeams using the modified nonlocal strain gradient elasticity model developed in [276]. Zhen et al. [277] explored the free vibration response of viscoelastic nanotubes under the action of a longitudinal magnetic field by utilizing the Timoshenko beam model and Kelvin-Voigt model in conjunction with the NSGT. Wang et al. [278] investigated the transverse free vibration response of axially moving Euler-Bernoulli nanobeams based on NSGT with the consideration of geometrical nonlinearity. They revealed that the increase in material characteristic parameter leads to a rise in critical velocity and nonlocal parameter has reversed effect on it. Also, the first few natural frequencies can be raised by reducing the axial speed in the subcritical range or raising the axial speed in the supercritical range. But considered nanobeam loses stability with an increase in axial speed. In continuation, the author explored the dynamic vibration behavior of an axially moving viscoelastic nanobeams [279]. Also, the exact solution was presented by Khaniki and Hashemi [280] for dynamic transverse vibration behavior of tapered nanobeams by utilizing the generalized DQM. Moreover, Guo et al. [281] promoted the dynamic transverse vibration characteristics of axially moving and rotating nanobeams using Hamilton’s principle and Galerkin approach.

Nonlocal strain gradient rod models have been also proposed for the vibration analysis of nanorods. Li et al. [282] used Hamilton’s principle to formulate the equations of motion and boundary conditions for the vibration analysis of nanorods. They determined the analytical solutions for the natural frequencies and mode shapes of the nanorods. Also, a FE method is developed to solve the vibration problems, which accounts both classical and nonclassical boundary conditions. They extended their study to solve the dynamic vibration problem of a nanorod [283].
The first time, Simsek [284] investigated the axial vibration response of an embedded nanorod using NSGT. Xu et al. [285] reformulated the variational consistent boundary conditions for the nonlocal strain gradient nanorods with the help of the weighted residual method. They identify and solve the boundary value problems to determine the frequency of the nanorods. Furthermore, they studied the asymptotic dynamic behaviors of nanorods.

Adeli et al. [286] emphasized on the torsional vibration response of nano-cone in the framework of NSGT, Hamilton’s principle and the generalized DQM. Borgi et al. [287] modeled a viscoelastic nanorod embedded in an elastic medium in the framework of NSGT and velocity gradient theory. This model consists of three length-scale parameters, which are nonlocal, strain gradient and velocity gradient parameter. These parameters help to account both softening and stiffening of the nanorods. Also, Kelvin–Voigt viscoelastic damping model is used to model the viscoelastic behavior of the nanorod. They obtained both exact analytical and numerical solutions for frequencies and damping ratios by utilizing a Locally adaptive DQM.

In addition to nanorods, vibration responses of nanotubes are also investigated. Shafiei and She [288] predicted the thermal vibration behavior of two-dimensional FG nanotubes using the NSGT and higher-order beam model presented in [289] for tubes. The temperature considered to be uniform across the radius. However, material properties of the nanotubes vary both in the length and radial direction. They utilized Hamilton’s principle to derive the size-dependent governing equations and solved these equations using the generalized differential quadrature method. Authors continued their study to determine the vibration characteristics of porous FG nanotubes in a thermal environment using a refined beam model [290]. They found softening and stiffening of the considered nanotube depending on the values of nonlocal and the strain gradient parameters. Also, the rise in volume fraction index and temperature leads to reduced natural frequencies. However, the effect of porosity on the natural frequency is dependent on the value of volume fraction index.

Nanoplates such as silver nanoplates [291], GSs [292], and metallic carbon nanosheets [293] have a remarkable potential applications in various fields of nanotechnology. Hence, number of researcher reported the vibration analysis of nanoplates in last few years [294–296].

Lu et al. [2] determined the exact analytical solution for the natural frequencies of S-S nanoplates using the surface elasticity theory, Kirchhoff and Mindlin plate model. It was reported that the influence of surface effects on the vibration response of nanoplates is dependent on the size of nanoplates. Moreover, the influence of nonlocal stress and strain gradient on the vibration behavior is more prominent for nanoplate with lower length-to-thickness ratio and higher aspect ratio. Besides rectangular FG nanoplates, the vibration response of circular FG nanoplates has also been reported in the literature.

In addition to vibration analysis of nanoplates and GSs using NSGT, many researchers have been also reported the NSGT based vibration analysis of nanoshells in literature [297, 298]. Barati [299] modeled porous FG nanoshells in an elastic medium based on the FSDT in conjunction with the NSGT and reported the numerical solution for the natural frequency of nanoshells based on Galerkin’s method. The parametric study revealed that the rise in porosity volume fraction leads to smaller natural frequencies. But, uneven porosity distribution gave larger frequencies compared with even porosity distribution. Also, increasing the radius-to-thickness and length-to-thickness ratios led to smaller frequencies, while increasing the foundation coefficients has reversed effect on frequencies.

5.3.3 Nonlocal doublet mechanics

In addition to bending and buckling analysis of nanostructures using DM, many researchers used it for vibration analysis too. Vajari and Imam [300] presented a DM with a length scale parameter based exact solution for axial vibration of SWCNTs. A fourth-order partial differential governing equation of motion was derived and solved for natural frequency of SWCNTs. Results reported that the length scale parameter decreases the natural frequency compared to results predicted by the classical continuum mechanics models. Furthermore, they studied the effect of scale parameter radius on CNTs and chirality on the torsional vibration of SWCNTs using scale-dependent DM [301]. In the next paper [302], they applied size-dependent DM to investigate the radial breathing like mode vibration of SWCNTs. A second-order partial differential governing equation of motion was derived and solved for natural frequency of SWCNTs. With the scale parameter, they found the less natural frequency of vibration than the frequency predicted by the classical continuum mechanics models. However, with an increase in the radius of SWCNTs, the influence of the scale parameter on the natural frequency decreases. Vajari [303] continued their study to investigate the radial breathing like mode vibration of DWCNTs based on the size-dependent DM. He assumed DWCNTs as two concentric cylindrical shells restrained together with Len-Jones potential modeled vdW forces. He derived two coupled second-order partial differential governing equations.
Table 4: Representation of published articles on vibration analysis of nanobeams/nano plates/nanoshells

<table>
<thead>
<tr>
<th>Sr no.</th>
<th>Ref.</th>
<th>Theory/Method used</th>
<th>Major outcome</th>
<th>Limitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[304]</td>
<td>Von Karman nonlinear relations, Kelvin–Voigt model, Hamilton’s principle, Galerkin Technique, multiple scale method</td>
<td>Nonlinear frequency response of piezoelectric sandwich nanoplates under the influence of electric voltage exerted on piezoelectric layer and shock force pulse is reported</td>
<td>Material is isotropic and temperature effect is not considered</td>
</tr>
<tr>
<td>2</td>
<td>[305]</td>
<td>Extended Melnikov method, perturbation analyses</td>
<td>The homoclinic behavior and chaotic motions of the buckled double layered nanoplates are studied</td>
<td>Limited boundary conditions are considered</td>
</tr>
<tr>
<td>3</td>
<td>[306]</td>
<td>FEM, principle of total potential energy, nonlocal integral elasticity theory, classical plate theory</td>
<td>A finite element formulation is reported for free vibration analyses of nanoplates</td>
<td>Temperature effect is not considered and material is isotropic</td>
</tr>
<tr>
<td>4</td>
<td>[307]</td>
<td>Laplace transformation techniques</td>
<td>The scale-dependent thermo-electro-mechanical responses of multi-layered piezoelectric nanoplates are examined for the vibration control</td>
<td>Material is isotropic. Also, initial imperfection and humidity effect are not considered.</td>
</tr>
<tr>
<td>5</td>
<td>[308]</td>
<td>Classical thermo-elasticity</td>
<td>Thermo-elastic damping of in-plane vibration of a FG nanoplate is examined</td>
<td>Material properties are graded only following power law</td>
</tr>
<tr>
<td>6</td>
<td>[309]</td>
<td>Hamilton’s principle, Galerkin method</td>
<td>Dynamical instability of an axially-moving nanoplate embedded on a viscoelastic foundation is explored</td>
<td>Material is isotropic and temperature effect is not considered</td>
</tr>
<tr>
<td>7</td>
<td>[310]</td>
<td>Hamilton principle, polynomial based DQM, Sinc DQM, and Discrete singular convolution DQM</td>
<td>Sinc DQM and Discrete singular convolution DQM are examined to study vibration characteristics of elastically supported piezoelectric nanobeams embedded on nonlinear Winkler–Pasternak foundation</td>
<td>Accuracy of the results depends on the type of shape functions used and material is isotropic</td>
</tr>
<tr>
<td>8</td>
<td>[311]</td>
<td>Nonlocal elasticity, refined beam theory, Finite element method</td>
<td>Finite element formulation is presented for vibration analysis of porous metal foam nanobeams resting on an elastic medium</td>
<td>Poison ratio is considered constant and shear correction factor is not used. Only axial force is considered in dynamic response analysis and temperature effect is not considered</td>
</tr>
<tr>
<td>9</td>
<td>[312]</td>
<td>FSDT, Hamilton’s principle, method of multiple scales, Kelvin–Voigt structural damping, and perturbation method</td>
<td>The free vibrations and dynamic response of orthotropic laminated beam under the action of transient and harmonic loads are investigated</td>
<td>Crack is assumed as a torsional spring and obtained solutions are only approximated</td>
</tr>
<tr>
<td>10</td>
<td>[313]</td>
<td>FEM, Hamilton’s principal, Galerkin method</td>
<td>Influences of the crack position, crack length, temperature gradient, boundary conditions and the foundation stiffness, on the vibration response of the cracked nanobeams resting on an elastic foundations is discussed by including thermal effects</td>
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</table>


<table>
<thead>
<tr>
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<tbody>
<tr>
<td>11</td>
<td>[314]</td>
<td>EBT, nonlinear von Karman theory, and Galerkin method</td>
<td>Emphasized on the influence of parametric excitation, by considering the instability regions and bifurcation points during the nonlinear vibration behavior and dynamic instability investigation of a nanobeam under thermo-magneto-mechanical loads</td>
<td>Material is isotropic and shear effect is not considered</td>
</tr>
<tr>
<td>12</td>
<td>[315]</td>
<td>EBT, Galerkin approximation method, Runge–Kutta numerical method</td>
<td>The primary resonance and chaotic vibration of a curved SWCNT resting on a viscoelastic foundation exposed to axial thermomagnetic and transverse harmonic forces are studied.</td>
<td>The obtained solutions are approximated</td>
</tr>
<tr>
<td>13</td>
<td>[316]</td>
<td>Zener model, Laplace transform, Bessel functions theory and the binominal series</td>
<td>The fractional Zener model is employed for dynamic analysis of a viscoelastic nanobeam</td>
<td>Only simple supported boundary condition and uniform distributed load are considered.</td>
</tr>
<tr>
<td>14</td>
<td>[317]</td>
<td>Hamilton’s concept, Love's shell theory, Fourier decomposition and DQM</td>
<td>The vibration response of rotating CNT with various boundary conditions is examined</td>
<td>Material is isotropic, and temperature effect is not considered</td>
</tr>
<tr>
<td>15</td>
<td>[318]</td>
<td>Galerkin method, FEM, Monte Carlo simulations, Newmark method, Bochner-Khinchin theorem</td>
<td>Transverse vibration behavior of a SWCNT under random loading condition is investigated.</td>
<td>Only clamped-clamped boundary condition is used.</td>
</tr>
<tr>
<td>16</td>
<td>[319]</td>
<td>Hamilton’s Principle, Non-local elasticity theory</td>
<td>Torsional vibration analysis of a bio-molecular motor is carried out by modeling it as a DWCNT system embedded in a viscoelastic medium.</td>
<td>Material is considered continuum and isotropic</td>
</tr>
<tr>
<td>17</td>
<td>[320]</td>
<td>Parabolic shear deformation theory</td>
<td>The effects of Poisson’s ratio and nonlocal parameter in thickness direction are studied</td>
<td>Considered only hinged-hinged boundary conditions</td>
</tr>
<tr>
<td>18</td>
<td>[321]</td>
<td>Inverse Laplace transform approach, refined beam theory, Galerkin’s method</td>
<td>Transient vibration analyses of porosity-dependent FG nanobeam under various impulsive loadings are studied.</td>
<td>Only even and uneven distribution of porosity are considered.</td>
</tr>
<tr>
<td>19</td>
<td>[322]</td>
<td>EBT, TBT, Galerkin’s method, Laplace transform method, surface elasticity</td>
<td>The forced vibration behaviors of nano beams with surface effects subjected to a moving harmonic load travelling with a constant velocity are examined and obtained closed-form solution for the critical velocities</td>
<td>Only S-S boundary condition is considered and variation in velocity of moving harmonic load is not considered</td>
</tr>
<tr>
<td>20</td>
<td>[323]</td>
<td>EBT, DQM, first order nonlocal strain gradient model, Winkler elastic foundation model</td>
<td>Dynamical behavior of nanobeam resting on constant, linear, parabolic, and sinusoidal types of Winkler elastic foundations are investigated. Also, influence of non-uniform parameter and Winkler modulus parameter on the frequency parameters is studied</td>
<td>Temperature effect is not taken into account</td>
</tr>
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Table 4: ...continued

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<tbody>
<tr>
<td>21</td>
<td>[324]</td>
<td>NSGT, Kelvin-Voigt model, Hamilton principle</td>
<td>Effect of viscoelasticity and initial imperfection on the nonlinear vibration response of CNT with large deformation is reported</td>
<td>Only C-C boundary condition is considered and temperature effect is not taken into account</td>
</tr>
<tr>
<td>22</td>
<td>[325]</td>
<td>Reddy’s third-order shear deformation plate theory, improved perturbation technique, von Karman nonlinearity, Galerkin procedure and the Hamilton's principle</td>
<td>The nonlinear vibrational behavior of a VdW bonded double-layered nanoplates subjected to thermal load is studied</td>
<td>Only S-S and C-C boundary conditions are considered</td>
</tr>
<tr>
<td>23</td>
<td>[326]</td>
<td>Refined plate model, NSGT, Hamilton's principle, Galerkin's method</td>
<td>The effects of nonlocal parameter, strain gradient, porosity distributions and porosity coefficient, foundation parameters on vibration characteristics of metal foam coupled nanoplates are examined</td>
<td>Poison ratio is considered constant and influence of temperature change is not considered</td>
</tr>
<tr>
<td>24</td>
<td>[327]</td>
<td>Refined exponential shear deformation theory, closed-cell Gaussian random field scheme, Halpin–Tsai micro-mechanical modeling, variational approach, improved perturbation technique, Galerkin method</td>
<td>A vibrational response of axially loaded FG porous nanoplate reinforced with GPLs is investigated within prebuckling and postbuckling domain</td>
<td>Poison ratio and temperature is considered constant, and only axial loading is taken into account.</td>
</tr>
<tr>
<td>25</td>
<td>[328]</td>
<td>FSDT, Hamilton's principle, Gurtin-Murdoch surface elasticity model, and generalized DQM</td>
<td>Influences of surface elastic properties, residual surface stress, and surface mass density on the vibration response of cylindrical shell are investigated at different values of size-dependent parameters</td>
<td>Temperature effect is not considered</td>
</tr>
<tr>
<td>26</td>
<td>[329]</td>
<td>EBT, Kelvin-Voigt approach, Hamilton principle, Galerkin method</td>
<td>A scale-dependent coupled longitudinal-transverse nonlinear formulation is presented for the mechanical behavior analysis of viscoelastic CNTs</td>
<td>Shear effects are not considered</td>
</tr>
<tr>
<td>27</td>
<td>[330]</td>
<td>HSDT, modified power law rule, Hamiltonian principle, Navier method</td>
<td>Influences of small scale parameters, geometry conditions, material compositions, porosities and thermal environment on the free vibration analysis of doubly-curved nanoshells are examined</td>
<td>Only S-S edges boundary condition is considered.</td>
</tr>
<tr>
<td>28</td>
<td>[331]</td>
<td>Modified NSGT</td>
<td>A nanorod model is developed based on the modified NSGT to study the extensional behavior of nanorods and CNTs</td>
<td>Only extensional behavior is investigated</td>
</tr>
</tbody>
</table>
and solved to obtain the vibration of DWCNTs. He found higher frequencies of the Zigzag DWCNT than that of the Armchair. Also, the in-phase mode of DWCNTs has the lowest frequency while the anti-phase mode has the highest frequency of vibration.

Gul et al. [112] reported the axial vibration analysis of CNTs embedded in an elastic medium using scale-dependent DM. They derived governing equations and corresponding boundary conditions based on the variational principle and obtained the natural frequencies. Detailed parametric analyses were conducted to examine the influences of elastic medium stiffness, length and doublet separation distance on the axial vibration. They reported that the dimensionless frequency parameter predicted by the DM approach decreases with increase in doublet distance specifically for higher modes.

In addition to CNTs, Vajari and Imam [334] investigated the lateral vibration of single-layered GSs based on DM with scale parameter. They derived a sixth-order partial differential governing equation and solved for natural frequency of GSs. They found that the length scale parameter decreases the natural frequency. However, the influence of scale parameter decreases with increase in the length of the GSs. They also found that the frequency ratio for Zigzag GSs is slightly higher than the Armchair one.

Vajari and Azimzadeh [335] investigated the nonlinear axial vibration of SWCNTs based on Homotopy perturbation method and DM. They derived a second-order partial differential governing equation and obtained the nonlinear natural frequency in axial vibration mode using Homotopy perturbation method. They reported a parametric study to investigate the influences of boundary conditions, radius, length, amplitude of vibration, and changes in vibration modes on the nonlinear axial vibration characteristics of SWCNTs. They found that the rise of maximum vibration amplitude leads to decrease in natural frequency. However, at large tube length, the effect of the amplitude on the natural frequency is negligible. Also, the amount and change of nonlinear natural frequency are more prominent in the higher mode of vibration and clamped-clamped boundary conditions. Recent work related to vibration analysis of nanostructures are listed briefly and explicitly in Table 4.

### 6 Conclusion

A comprehensive review is given in the present paper, in which different type of fabrication techniques, specific applications of nanostructures, different methods adopted for static and dynamic response of nanostructures has been discussed. Also, critical review of various investigations on the static, buckling and vibration analyses of nanostructures including nanorods, nanobeams, nanoplates and nanoshells has been carried out. The general observations from the present literature review are listed as.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>29</td>
<td>[332]</td>
<td>NSGT, refined higher order shear deformation beam model, Hamilton’s principle, two steps perturbation method, power-law</td>
<td>Analytical solution is obtained to carry out a nonlinear vibration analysis of nanotube and influences of inner temperature, nonlocal parameter, strain gradient parameter, scale parameter ratio, slenderness ratio, radius, volume indexes, different beam models on the linear and nonlinear frequencies are studied</td>
<td>Only power law of material gradation is used.</td>
</tr>
<tr>
<td>30</td>
<td>[333]</td>
<td>TBT, Hamilton’s principle, Modified Fourier series method, weighted residual method</td>
<td>An analytical solution for dynamic analysis of the hetero-junction carbon nanotubes based mass nanosensors is proposed</td>
<td>Damping and temperature effects are not considered</td>
</tr>
</tbody>
</table>
1. Among the various methods like MDs, nonlocal DM, nonlocal elasticity theory, and NSGT employed for the analysis of nanostructures, nonlocal elasticity theory is extensively used. Some modification has also been incorporated in nonlocal elasticity theory by the researchers time to time, to increase the accuracy of results.

2. TBT in conjunction with nonlocal elasticity theory and NSGT is extensively applied for the structural analysis of nanobeams and CNTs.

3. Nonlocal parameter has stiffness softening effect, while strain gradient parameter has stiffness hardening effect on the buckling, bending and vibration analysis of all type of nanostructures.

4. Effect of nonlocal parameter is more prominent on the buckling and vibration as compared to the bending of nanostructures. Increasing nonlocal parameter leads to decline in buckling load and natural frequency, while increment in the deflection.

5. Generally, the influence of small scale parameter, aspect ratio, porosity index, porosity distribution, external voltage, magnetic field, hygro-thermal environment, boundary conditions, surface energy, initial curvature and geometric nonlinearity on the structure response of isotropic, orthotropic and FG nanostructures are analysed.

6. According to NSGT, increase or decrease of stiffness depends on the relative magnitude of the material length scale and nonlocal parameters. So critical buckling load, deflection and natural frequency may rise or fall depending on the relative magnitude of two parameters.

7. Computational cost of NSGT is high as compared to the nonlocal elasticity theory.

8. Bending analysis of nanoplates using NSGT is not explored to much.

9. Finding the structural response of nanostructures with internal junction using the nonlocal elasticity theory and NSGT is not reported yet.

10. Accuracy of upgraded continuum theories depends on the calibration of parameters.

11. Very few papers dealing with structural responses of sandwich nanostructures, cracked nanostructures and reinforced nanocomposites are reported. In addition, compared to free and forced vibration analysis, the damped vibration analysis of nanostructures has not been studied comprehensively.

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