Research Article

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Bioconvection transport of upper convected Maxwell nanoliquid with gyrotactic microorganism, nonlinear thermal radiation, and chemical reaction

Abstract: The microorganisms’ concept has appealed substantial consideration of modern researchers because of its utilization in commercial and industrial products, for illustration, biofuel (prepared from the waste), drug delivery, and fertilizers. Keeping such utilizations of microorganisms in mind, an analysis based on gyrotactic microorganisms featuring the mixed convective nonlinear radiative Maxwell nanoliquid stagnation point flow conjugated by permeable stretching surface is presented. Boundary layer stretching flow subjected to transpiration effects is formulated. Modeling is based on Buongiorno’s nanoliquid model. This model captures Brownian diffusion along with thermophoresis aspects. Energy expression is formulated under nonlinear version of radiative heat-flux, heat source, thermal Robin conditions, and heat sink. Mass transport analysis is presented considering solutal Robin conditions and chemical reaction. In addition, the Robin conditions for motile microorganisms are also considered. The complex mathematical expressions of Maxwell liquid are simplified utilizing the Boundary layer concept and then suitable transformations assist to obtain the mathematical problems in ordinary differential forms. The analytical approach (that is homotopy analysis methodology) is utilized for computational analysis. The outcomes obtained are presented graphically and numerically. The detailed description of emerging physical non-dimensional parameters is included. Our findings indicate that the motile density field strongly boosted with the increment in Peclet number and microorganisms Biot number; however, they are suppressed with the increase in the values of bioconvection Schmidt number and motile microorganism concentration difference parameter.

Keywords: Maxwell nanoliquid, gyrotactic microorganisms, Brownian diffusion, Robin conditions, thermophoresis, chemical reaction, transpiration, nonlinear thermal radiation

Nomenclature

\( A \) ratio of rates
\( C_f \) heated fluid concentration
\( C_m \) ambient fluid concentration
\( D_t \) thermophoretic diffusion coefficient (m\(^2\) s\(^{-1}\))
\( D_b \) Brownian diffusion coefficient (m\(^2\) s\(^{-1}\))
\( f \) dimensionless velocity
\( G_r \) thermal buoyancy number
\( G_r^* \) concentration buoyancy number
\( N \) thermophoresis parameter
\( N^* \) buoyancy ratio parameter
\( N_b \) Brownian motion parameter
\( N_t \) thermodiffusive parameter
\( N_{Nu} \) local Nusselt number
1 Introduction

Owing to their exceptionally reliable rheological characteristics, non-Newtonian liquids have acquired remarkable attraction among engineers, scientists, and researchers. Their utilization in distinct biomedical, pharmaceutical, technological, and other disciplines of science is growing every day. Different models featuring non-Newtonian characteristics are suggested. The model considered here (Maxwell fluid) is the rate type non-Newtonian model which captures viscoelastic aspects in terms of stress relaxation time factor and this model has ample usage in mechanical engineering, computers, automobiles, polymer, and medical industries. Researchers considered this model under diverse physical aspects. For illustration, Hayat et al. [1] evaluated Maxwell liquid stagnation point laminar flow configured by moving convective surface. They employed homotopy analysis methodology (HAM) for computational outcomes. They noticed temperature reduction subjected to larger Prandtl number. Analysis of mixed convective Maxwell liquid based on Buongiorno’s model was scrutinized by Nadeem et al. [2]. Their outcomes indicate increase in velocity when material parameter (Deborah number) is augmented. Megahed [3] analyzed variable properties effectiveness in magnetized Maxwell liquid incompressible flow under porous medium and Robin conditions. According to his findings, larger Eckert number yields higher temperature. Transport of hybrid Maxwell nanoliquid considering buoyancy forces and improved Fourier–Fickian relations is reported by Madkhal et al. [4]. They declared that thermal memory aspects play vital contribution in augmenting the wall heat flux. Waqas [5] explored variable properties based on temperature and concentration in flow of chemically reactive Maxwell liquid laminar flow. It is examined that Prandtl number and heat sink factor corresponds to lower temperature when compared with variable conductivity and heat source parameters.

Nanoliquids upsurge the transportation of heat transfer utilized in intensification procedure of reactors and heat exchangers. Consideration of bioconvection in nanoliquids convalesce mass transportation, enhance stability, induce microvolume fraternization, and prevent nanoparticles clustering [6,7]. The nanoliquids together with bioconvection aspect find relevance in medical filtration, micro-fluidic devices, microbial fuel cells, modernized energy conservation mechanisms, and bio-nano coolant structures. The gyrotactic microorganisms significance in nanoliquid stagnant point flow is evaluated by Zaimi et al. [8]. Xu and Pop [9] modeled the mixed convective gyrotactic microorganisms based nanoliquid flow employing homotopic scheme. Their outcomes signify concentration augmentation subject to increasing thermophoresis variable. Magneto-hydrodynamic laminar flow based on Oldroyd-B model capturing Buongiorno’s nanoliquid and stratifications was inspected by Waqas et al. [10]. It is noticed that higher Peclet number yields diminution of microorganisms’ density. Further analyses featuring gyrotactic microorganisms in stretching flow of rheological liquids are mentioned in previous studies [11–13]. In these investigations, multi-physical effects along with distinct geometries are considered. From these studies, it is concluded that nanoliquid consideration together with gyrotactic microorganisms stabilize the adjourned nanoparticles.

Undoubtedly, numerous engineering and geological structures capturing double-diffusive convection effects encompass the phenomenon of chemical reaction. Geochemical flows involve rainfall reaction (for example a calcium feldspar and acidic brine), pollutant leaching [14], mineralogical dissolution, and brine chemo-geothermics [15]. Further examples based on industrial technologies comprise catalytic conversion, manipulation of polymer radical, materials synthesis [16], and exothermic type chemical reactions featuring porous media based reactors [17]. Rheological liquid models subjected to chemical reactions are scrutinized by various researchers considering multi-physical effects and geometries. In this direction, Reddy et al. [18] examined chemically reactive magnetized laminar flow subjected to porous medium and viscous dissipation. They considered micropolar model
for flow formulation. They inspected higher thermal field for increasing radiation factor and Eckert number. Chemically reactive Eyring–Powell liquid incompressible stretching flow confined by heated convective surface is scrutinized by Hussain et al. [19]. They computed nonlinear problems using bvp4c algorithm. They found significant impact of chemical reaction factor on mass transportation. Chu et al. [20] evaluated Maxwell nanoliquid stagnation point reactive flow considering cylindrical geometry. They declared that consideration of exponential heat source together with nonlinear radiation is significant when improvement in thermal processes is required.

The literature mentioned above witness that Darcy–Forchheimer concept of porous medium in rate type stretching liquids subjected to gyrotactic microorganisms is less considered. With this objective, our focus here is to develop a mathematical model featuring nonlinear rheological Maxwell liquid. The current study accounts the novel aspects like thermal radiation, stagnant point flow, chemical reaction, and Robin conditions. The boundary layer approach yielding parabolic expressions is used. Various computational algorithms [21–24] are available. Here the governing mathematical problems of Maxwell liquid are analytically solved through HAM [25–28]. The outcomes of significant variables against dimensionless quantities are presented and elaborated comprehensively. The considered non-Newtonian nanoliquid stagnation model encompasses coating fabricating procedures for biomimetic sensors [29] and optical fiber nano-coatings [30,31].

### 2 Flow model

Consider a laminar boundary layer stretching flow based on rate type rheological Maxwell liquid [26] confined by vertical convective permeable surface. Stagnation point incompressible flow is modeled in vertical direction considering buoyancy forces and velocity $u_0(x) = cx$. The Maxwell nanoliquid is electrically conductive and is exposed to a uniform magnetic field $B_0$. The dispersion of nanoparticles is homogeneous, which prevents agglomeration and accumulation, resulting in a dilute nanofluid suspension. In addition, the model assumes that the swimming direction and velocity of gyrotactic microorganisms are not impacted by nanoparticles presence. The density variation in buoyancy term is determined utilizing the Boussinesq approximation.

The flow configuration is illustrated in Figure 1.

The governing nonlinear expressions subjected to mixed convection, gyrotactic microorganisms, thermal radiation, Robin conditions, and heat source/sink are as follows [27]:

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
$$

$$
\frac{u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left[ u \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 v}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right] = \frac{\partial^2 u}{\partial y^2} + u \frac{\partial u}{\partial x} + \frac{1}{\rho_f} \left[ (1 - C_m) \rho_f \beta (T - T_m) - (\rho_p - \rho_f) g (C - C_m) - (\rho_m - \rho_f) g (n - n_m) \right].
$$

![Figure 1: Flow diagram and coordinate system.](image)
\[
\frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \left[ D_H \frac{\partial c}{\partial y} + \frac{D_T}{T_M} (\frac{\partial T}{\partial y})^2 \right]
\]

\[
+ \frac{16\sigma^*}{3k^*(\rho c)_t} \frac{\partial}{\partial y} \left(T \frac{\partial T}{\partial y}\right) + \frac{Q_0}{(\rho c)_t} (T - T_M),
\]

\[
\frac{\partial c}{\partial x} + \nu \frac{\partial c}{\partial y} = D_k \frac{\partial^2 c}{\partial y^2} + \frac{D_T}{T_M} \frac{\partial^2 T}{\partial y^2} - k_c(C - C_0),
\]

\[
\frac{\partial n}{\partial x} + \nu \frac{\partial n}{\partial y} + \frac{bW_c}{(C_w - C_n)} \left[ \frac{\partial}{\partial y} \left( n \frac{\partial c}{\partial y} \right) \right] = D_n \left( \frac{\partial^2 n}{\partial y^2} \right)
\]

\[
u = u_w(x) = cx, \quad \nu = v_w = -v_0, \quad -k_\tau \frac{\partial T}{\partial y} = h_c(T_I - T),
\]

\[
-\frac{D_H}{\theta} \frac{\partial c}{\partial y} = h_b(C_t - C), \quad -\frac{D_n}{\theta} \frac{\partial n}{\partial y} = h_b(n_t - n) \text{ at } y = 0,
\]

\[
u - u_c(x) = ex, \quad T \to T_M, \quad C \to C_n, \quad n \to n_0 \text{ when } y \to \infty,
\]

where \(\nu = \frac{\mu}{\rho_t}\) represents the kinematic viscosity, \(\rho_t\) represents the liquid density, \(\mu\) denotes the dynamic viscosity, \(Q_0\) stands for the heat absorption (sink)/generation (source) coefficient, \(y\) is the average volume of microorganisms, \(\rho_p\) and \(\rho_m\) stand for the particle microorganisms densities, \((u_w(x), v_w)\) stand for the (stretching, suction/injection) velocity, \(\lambda_1\) is the relaxation time, \(g\) is the gravitational acceleration, \(D_H\) is the Brownian diffusion coefficient, \(b\) is the chemotaxis constant, \(W_c\) is the maximum cell swimming speed, \(D_m\) is the microorganisms diffusion coefficients, \(C\) is the liquid concentration, \(u_c(x)\) is free stream velocity, \(C_t\) is the heated liquid concentration, thermal diffusivity \(\alpha = \frac{k}{(\rho c)_t}\), heat capacity ratio \(\tau = \frac{(pc)_t}{(pc)_p}\) stands for the nanoparticles effective heat capacity, \((h_t, h_g, h_b)\) stand for the heat, mass, and motile microorganisms transfer coefficients, \(\sigma^*\) is the Stefan–Boltzmann constant, \(D_T\) is the thermophoresis diffusion coefficient, \(C_o\) is the ambient liquid concentration, \(k^*\) is the mean absorption coefficient, \(n\) is the motile microorganisms concentration, \(T\) is the liquid temperature, \(T_I\) is the heated liquid temperature, \(k\) is the thermal conductivity, \(c\) is the dimensional constant, \(k_c\) is the reaction rate, \(T_M\) is ambient liquid temperature, \(n_t\) is the microorganisms transfer coefficient, \(\beta\) is the chemotaxis constant, and \(u, v\) are the components of velocity in the \((x, y)\) direction, respectively.

Implementing [27]

\[
\eta = \sqrt{\frac{c}{y}}, \quad u = cxf'(\eta), \quad v = -\sqrt{\eta} \left[f''(\eta) - f''(\infty) - f''(\eta)\right]
\]

\[
\theta(\eta) = \frac{T - T_M}{T_I - T_M}, \quad \phi(\eta) = \frac{C - C_0}{C_t - C_0}, \quad \xi(\eta) = \frac{n - n_0}{n_t - n_0},
\]

continuity equation (that is equation (1)) is fulfilled and other equations in dimensionless form become

\[
f'' + f f'' + \beta(2f' f'' + f^2 f''' - f^2 f''') - f''^2 + \lambda(\theta - N' \phi - Rb \xi) + A^2 = 0,
\]

\[
(1 + R)\theta'' + \frac{4}{3} R((\theta_0 - 1)^3(3\theta^2 \theta'^2 + \theta^3 \theta''))
\]

\[
+ 3(\theta_0 - 1)^3(2\theta(\theta^2 + \theta' \theta') + 3(\theta_0 - 1)((\theta^2 + \theta \theta')^2)
\]

\[
+ Pr f \theta'' + Pr(N \theta'^2 + Nb \phi' \phi' + S \phi'' = 0),
\]

\[
\phi'' + Scf' \phi' + \frac{Nt}{Nb} \theta'' - Scf \phi = 0,
\]

\[
\xi'' + S c f \xi' - Pe[\xi(1 - \phi)'] = 0,
\]

\[
f = h, \quad f' = 1, \quad \theta' = -\gamma_1(1 - \theta(\eta))
\]

\[
\phi' = -\gamma_1(1 - \phi(\eta)), \quad \xi' = -\gamma_1(1 - \xi(\eta)) \text{ at } \eta = 0,
\]

\[
\text{as } \eta \to \infty,
\]

where ('') signifies the differentiation concerning \(\eta\); \(\lambda\) is the mixed convection parameter, \(Sc\) is the Schmidt number, \(Sc_n\) is the bioconvection Schmidt number, \(S < 0\) is the heat absorption variable, \(Rb\) is the bioconvection Rayleigh number, \(Gr_c\) is the concentration buoyancy number, \(A\) is the ratio of rates, \(Nt\) is the thermodiffusivity parameter, \(Pr\) is the Prandtl number, \(Gr_b\) is the thermal buoyancy number, \(Nb\) is the Brownian motion parameter, \(R\) is the radiation variable, \(S > 0\) is the heat generation variable, \(\theta_b\) is the temperature ratio parameter, \(\gamma_1\) is the thermal Biot number, \(\beta\) is the Deborah number, \(N^*\) is the buoyancy ratio parameter, \(\sigma = \frac{\gamma^2}{\tau}\) is the dimensionless reaction rate, \(\gamma_3\) is the motile microorganisms conjugate parameter, \(\sigma > 0\) is the destructive reaction variable, \(Q\) is the concentration motile microorganisms parameter, \(Pe\) is the bioconvection Peclet number, \(\sigma < 0\) is the generative reaction variable, \(\gamma_2\) is the concentration Biot number, \(h > 0\) is the suction, and \(h < 0\) is the injection. These parameters are defined as follows:
\[ \beta = \lambda_0 c, \quad \lambda = \frac{(1 - C_0)\beta}{(C_0 - C_m)}, \]
\[ \text{Re}_x = \frac{v_{hw}}{v}, \quad \text{Pr} = \frac{\nu}{\alpha}, \quad \gamma_3 = h_0 \sqrt[3]{\frac{v}{c}}, \]
\[ N^* = \frac{(\beta_0 - \beta_c)(C_0 - C_m)}{\beta_0 \beta_c (C_0 - C_m) (1 - C_m)} N_t = \frac{x D_c (T - T_m)}{T_n a}, \]
\[ \text{Rb} = \frac{r(n_t - n_m)(\rho_m - \rho_t)}{\beta_0 \beta_c (1 - C_m)(T_0 - T_m)}, \quad S = \frac{Q_0}{(\rho c)_0 c}, \]
\[ \text{Sc} = \frac{v}{D_h}, \quad \text{Sc}_n = \frac{v}{D_n}, \quad \text{Pe} = \frac{b W_c}{D_h \nu}, \quad \gamma_1 = \frac{h_0}{K} \sqrt{\frac{v}{c}}, \]
\[ Nh = \frac{x D_c (C_0 - C_m)}{a}, \quad A = \frac{e}{c}, \quad \theta_R = \frac{T_0}{T_m}, \]
\[ h = \frac{-v_0}{\sqrt{v c}}, \]
\[ Y_2 = \frac{D_h}{D_n} \sqrt{\frac{v}{c}}, \quad \sigma = \frac{k_\nu^2}{c}, \quad \Omega = \frac{n_m}{n_t - n_m}. \]

Heat-mass transportation and microorganism diffusion rates (\( \text{Nu}_x, \text{Sh}_x, \text{Nn}_x \)) in mathematical forms are defined as follows:
\[ \text{Nu}_x = \left[ -\frac{x}{(T - T_m)} \left( \frac{\partial T}{\partial y} \right) - \frac{16 \sigma x}{3 k (T_0 - T_m)} \right] \gamma_{y=0}, \]
\[ \text{Sh}_x = \frac{x q_m}{D_h (C_0 - C_m)}, \quad q_m = -D_h \left[ \frac{\partial c}{\partial y} \right] \gamma_{y=0}, \]
\[ \text{Nn}_x = \frac{x q_m}{D_n (n_t - n_m)}, \quad q_n = -D_n \left[ \frac{\partial \xi}{\partial y} \right] \gamma_{y=0}, \]
\[ \text{Nu}_x \text{Re}_x^{0.5} = -\left( 1 + \frac{4}{3} R (1 + (\theta_R - 1)\theta(0)) \right) \theta(0), \]
\[ \text{Sh}_x \text{Re}_x^{0.2} = -\phi(0), \]
\[ \text{Nn}_x \text{Re}_x^{-1.2} = -\xi(0). \]

### 3 Analytical solution procedure

In numerous engineering and industrial applications, the resultant coupled ordinary differential equations are extremely nonlinear, which is always a difficult task for engineers and researchers. Such difficult problems are frequently addressed either analytically or numerically. Beyond the analytical approaches, HAM approach is one that efficiently calculates the required convergence series solution of the system. One of the prominent features of this approach is that it does not impose the restriction of big or small parameter in the problem. The convergence domain associated with this approach can be managed more effectively as related to other methodologies. It provides a lot of flexibility to build the necessary functions to calculate the solution. Liao [25] initiated the pioneering study on this approach. Later on, several researchers used this approach to solve highly nonlinear problems [26–28]. Following initial estimations, \( f_0(\eta), \theta(\eta), \phi(\eta), \xi(\eta) \) are proposed to start the computations.

\[ f_0(\eta) = h + A \eta + (1 - A)(1 - e^{-\eta}), \]
\[ \theta(\eta) = \left[ \frac{y_1}{1 + y_1} \right] e^{-\eta}, \]
\[ \phi(\eta) = \left[ \frac{y_2}{1 + y_2} \right] e^{-\eta}, \]
\[ \xi(\eta) = \left[ \frac{y_3}{1 + y_3} \right] e^{-\eta}. \]

Let us assume the essential linear operators \( (L_f, L_\theta, L_\phi, L_\xi) \) as follows:
\[ L_f = f''' - f, \]
\[ L_{\theta} = \theta''' - \theta, \]
\[ L_{\phi} = \phi''' - \phi, \]
\[ L_{\xi} = \xi''' - \xi, \]

with
\[ L_f (A_1 + A_2 e^\eta + A_3 e^{-\eta}) = 0, \]
\[ L_{\theta} (A_4 e^\eta + A_5 e^{-\eta}) = 0, \]
\[ L_{\phi} (A_6 e^\eta + A_7 e^{-\eta}) = 0, \]
\[ L_{\xi} (A_8 e^\eta + A_9 e^{-\eta}) = 0, \]
where \( A_i (i = 1 - 9) \) indicate the arbitrary constants.

Now defining the problems at zeroth order, we have
\[ (1 - w^*) L_f [\tilde{f}(\eta; w^*) - f_0(\eta)] = w^* h_0 N_4 [\tilde{f}(\eta; w^*)], \]
\[ \tilde{\theta}(\eta; w^*), \quad \tilde{\phi}(\eta; w^*), \quad \tilde{\xi}(\eta; w^*), \]
\[ (1 - w^*) L_{\theta} [\tilde{\theta}(\eta; w^*) - \theta_{0}(\eta)] = w^* h_0 N_4 [\tilde{f}(\eta; w^*)], \]
\[ \tilde{\theta}(\eta; w^*), \quad \tilde{\phi}(\eta; w^*), \quad \tilde{\xi}(\eta; w^*), \]
\[ (1 - w^*) L_{\phi} [\tilde{\phi}(\eta; w^*) - \phi_{0}(\eta)] = w^* h_0 N_4 [\tilde{f}(\eta; w^*)], \]
\[ \tilde{\theta}(\eta; w^*), \quad \tilde{\phi}(\eta; w^*), \quad \tilde{\xi}(\eta; w^*), \]
\[ (1 - w^*) L_{\xi} [\tilde{\xi}(\eta; w^*) - \xi_{0}(\eta)] = w^* h_0 N_4 [\tilde{f}(\eta; w^*)], \]
\[ \tilde{\theta}(\eta; w^*), \quad \tilde{\phi}(\eta; w^*), \quad \tilde{\xi}(\eta; w^*), \]
\[ \hat{f}(0; w^*) = h, \hat{f}'(0; w^*) = 1, \hat{f}'(\infty; w^*) = A, \]
\[ \hat{\theta}(0; w^*) = -\gamma(1 - \hat{\theta}(0; w^*)), \hat{\theta}(\infty; w^*) = 0, \]
\[ \hat{\phi}(0; w^*) = -\gamma(1 - \hat{\phi}(0; w^*)), \hat{\phi}(\infty; w^*) = 0, \]
\[ \hat{\xi}(0; w^*) = -\gamma(1 - \hat{\xi}(0; w^*)), \hat{\xi}(\infty; w^*) = 0, \]

\[
\begin{align*}
N_{\eta}\{\hat{f}(q; w^*), \hat{\theta}(q; w^*), \hat{\phi}(q; w^*), \hat{\xi}(q; w^*)\} & = \frac{\partial^2 \hat{f}(q; w^*)}{\partial \eta^2} + (\hat{f}(q, w^*) \frac{\partial^2 \hat{f}(q; w^*)}{\partial \eta^2}) - \left\{ \frac{\partial \hat{f}(q; w^*)}{\partial \eta} \right\}^2 \\
& + \beta \left[ \frac{2\hat{f}(q, w^*)}{\partial \eta} \frac{\partial \hat{f}(q; w^*)}{\partial \eta} \right] \\
& - (\hat{f}(q, w^*))^2 \frac{\partial \hat{f}(q; w^*)}{\partial \eta} + 2\lambda (\hat{\theta}(q; w^*)) \\
& - N^\prime \hat{\phi}(q; w^*) - Rb \hat{\xi}(q; w^*) + \lambda^2,
\end{align*}
\]

\[
N_{\eta}\{\hat{f}(q; w^*), \hat{\theta}(q; w^*), \hat{\phi}(q; w^*), \hat{\xi}(q; w^*)\} = \frac{\partial^2 \hat{\phi}(q; w^*)}{\partial \eta^2} + Sc_\phi \frac{\partial \hat{\phi}(q; w^*)}{\partial \eta} \\
+ \frac{N_{\eta}}{N_0} \frac{\partial^2 \hat{\theta}(q; w^*)}{\partial \eta^2} + Sc_\phi \frac{\partial \hat{\phi}(q; w^*)}{\partial \eta},
\]

\[ (27) \]

\[ (28) \]

\[ (29) \]

\[ (30) \]

\[ (31) \]

The embedding variable is shown by \( w^* \) while \( h_f, h_\theta, h_\phi, \) and \( h_\xi \) are non-zero auxiliary variables.

### 4 Convergence analysis of HAM

The analytical scheme known as HAM is deployed for computational outcomes. It comprises the auxiliary factors \( h_f, h_\theta, h_\phi, \) and \( h_\xi \) which assist in controlling convergence of homotopy solutions. Here the \( h \)-curves for velocity distribution \( f''(0) \), temperature distribution \( \theta'(0) \), concentration distribution \( \phi(0) \), and motile density distribution \( \xi'(0) \) are depicted for particular values of prominent parameters in Figures 2 and 3. These figures witness that \( h_f, h_\theta, h_\phi, \) and \( h_\xi \) lie in \(-1.3 \leq h_f \leq -0.4, -1.2 \leq h_\theta \leq -0.6, \)

**Figure 2:** \( h \)-curve impact on \( f''(0) \) and \( \theta'(0) \).
\[ -1.2 \leq h_{\phi} \leq -0.2, \quad \text{and} \quad -1.3 \leq h_{\xi} \leq -0.2. \] Convergence is numerically presented in Table 1.

5 Results and discussions

This segment is presented to clarify the importance of various pertinent parameters like Deborah number \( \beta \), buoyancy ratio parameter \( N^* \), Prandtl number \( \text{Pr} \), radiation parameter \( R \), thermophoresis parameter \( N_t \), heat generation (sink) absorption parameter \( S \), temperature ratio parameter \( \theta_R \), mixed convection parameter \( \lambda \), bioconvection Schmidt number \( S_{sc} \), bioconvection Rayleigh number \( R_b \), thermal Biot number \( y_1 \), Schmidt number \( S_c \), Brownian motion parameter \( N_b \), concentration Biot number \( y_2 \), motile microorganisms Biot number \( y_3 \), bioconvection Peclet number \( Pe \), motile microorganisms concentration difference parameter \( \Omega \), and chemical reaction parameter \( \gamma \) on the fluid velocity \( f''(\eta) \), temperature \( \theta'(\eta) \), concentration \( \phi'(\eta) \), and motile density field \( \xi'(\eta) \).

Table 1: Assessment of convergence series solutions for the various orders of approximations when \( A = N_b = N_t = R = R_b = S = \sigma = \text{Pe} = \Omega = 0.1 \), \( S_{sc} = S_c = \text{Pr} = 1.2, \theta_R = 1.1, h = N^* = \lambda = \beta = y_1 = y_2 = y_3 = 0.2 \).

<table>
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<th>Order of approximations</th>
<th>( f''(0) )</th>
<th>( \theta'(0) )</th>
<th>( \phi'(0) )</th>
<th>( \xi'(0) )</th>
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<td>0.1623</td>
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<td>0.1479</td>
<td>0.1611</td>
</tr>
<tr>
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<td>0.1477</td>
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Figure 3: \( h \)-curve impact on \( \phi'(0) \) and \( \xi'(0) \).

5.1 Velocity profile

Figures 4 and 5 illustrate the aspect of \( \beta, \lambda, R_b, \) and \( N^* \) on velocity \( f''(\eta) \). It is noticed from Figure 4 that the Maxwell fluid velocity \( f'(\eta) \) displays diminishing behavior with increasing values of \( \beta \), while it upsurges with increasing \( \lambda \). The reducing trend in \( f'(\eta) \) is noticed for increasing \( \beta \). Clearly Deborah number encompasses relaxation time and when this factor upsurges, Deborah number becomes greater which engenders resistance in liquid flow and as a result velocity decreases. Moreover, the ratio of acting viscous and buoyant forces in the flow regime is taken into account by the mixed convection parameter \( \lambda \). Higher values of \( \lambda \) corresponds to a lower viscous force which ultimately accelerates...
$f'(\eta)$. Figure 5 depicts the impact of $R_b$ and $N^*$ on the velocity $f'(\eta)$. This figure displays that the increasing values of $R_b$ diminish $f'(\eta)$. This diminishing trend is noticed because $R_b$ and $N^*$ are associated with the buoyant force induced by bioconvection which permits the velocity field to fall.

5.2 Temperature profile

Figures 6–9 display the effect of $R$, $Pr$, $y_1$, $N_b$, $S$, and $N_b$ on temperature $\theta(\eta)$. The decline in temperature field $\theta(\eta)$ is detected for increasing $Pr$ (Figure 6). $Pr$ is a non-dimensional parameter and it is described as the ratio of...
diffusivities (i.e., momentum and thermal diffusivity). Larger Pr yields lower thermal diffusion due to which $\theta(\eta)$ decays. On the other hand, a rise in $R$ produces $\theta(\eta)$ enhancement. Physically, the transportation of heat with the assistance of liquid elements transpires because of radiation aspect. For that reason, certain heat amount is augmented in flow field. It is evident from Figure 7 that larger values of $N_b$ and $N_t$ cause an enhancement of temperature field $\theta(\eta)$. The thermophoresis process comprises of relocated fluid particles from hotter to a cooler surface area. In thermophoresis phenomenon, the heated fluid particles shift in the boundary domain from a lower to higher temperature area, causing the temperature to rise.

On the other hand, the Brownian motion diffusion $N_b$ depicts the chaotic fluid particles motion in the regime of flow. With higher values of $N_b$, the stochastic fluid particles movement upsurges which outcomes an improved temperature field $\theta(\eta)$. Figure 8 is sketched to elucidate on the effects of heat generation ($S > 0$)/absorption ($S < 0$) against $\theta(\eta)$. Here $\theta(\eta)$ is enhanced when $S > 0$ whereas it reduces when $S < 0$. It is investigated that consideration of heat source/sink tends to hasten the liquid movement. Note that $S > 0$ signifies the generation of heat, while $S < 0$ means absorption of heat. Physically the heat source

Figure 6: Variation in $\theta(\eta)$ via $Pr$ and $R$.

Figure 7: Variation in $\theta(\eta)$ via $N_b$ and $N_t$. 
indicates heat generation which upsurges \(\theta(\eta)\). Therefore, as heat source variable upsurges, \(\theta(\eta)\) increases. On the other hand, \(\theta(\eta)\) decays for \(S < 0\). Figure 9 illustrates the consequences of \(\theta_R\) and \(\gamma_1\) on temperature \(\theta(\eta)\). As expected, larger \(\theta_R\) and \(\gamma_1\) corresponds to \(\theta(\eta)\) enhancement. Higher \(\theta_R\) implies a boost in convective liquid temperature, which produces deeper thermal penetration depth into the boundary layer. Heat transfer into flow towards wall is consequently promoted. Furthermore, when \(T_f\) surpasses \(T_w\), a larger thermal differential is generated across the boundary layer, which strengthens thermal diffusion from wall towards free-stream and results in a rise in \(\theta(\eta)\). On the other hand, it is investigated that the temperature field \(\theta(\eta)\) escalates with a rise in \(\gamma_1\). In fact, increasing \(\gamma_1\) increases the strength of the coefficient of heat transmission which contributes to accelerating the heat transfer rate.

Figure 8: Variation in \(\theta(\eta)\) via \(S\).

Figure 9: Variation in \(\theta(\eta)\) via \(\theta_R\) and \(\gamma_1\).
5.3 Concentration profile

Figures 10–12 show the impacts of \( \text{Sc}, \, \sigma, \, N_t, \, y_2, \) and \( N_b \) on \( \phi(\eta) \). Figure 10 illustrates that an intensification in \( \text{Sc} \) leads to a decline in volumetric concentration \( \phi(\eta) \) of nanoparticles. In fact, \( \text{Sc} \) has an inverse relationship with Brownian diffusion. Higher values of \( \text{Sc} \) corresponds to a lower Brownian diffusion which causes a depletion in the nanoparticle’s concentration profile \( \phi(\eta) \). From this Figure, it is also found that the concentration of nanoparticles \( \phi(\eta) \) increases when \( y_2 \) is augmented. The mass transfer quantity increases for higher values of \( y_2 \) which accelerates the liquid concentration of nanoparticles. Figure 11 portrays the concentration distribution \( \phi(\eta) \) for distinct values of \( N_t \) and \( N_b \). A diminishing trend is observed when \( N_b \) is enlarged. The fluid particle collision accelerates for greater values of \( N_b \). Hence, the concentration of Maxwell nanofluid reduces. On the other hand, contradictory behavior is noticed for \( N_t \) on the concentration profile \( \phi(\eta) \). Physically, the thermophoretic body force elevates which causes the nanoparticles transportation from upper to lower region when \( N_t \) is increased. Therefore, \( \phi(\eta) \) reduces. Figure 12 demonstrates that with higher destructive chemical reaction parameter \( \sigma > 0 \), concentration \( \phi(\eta) \) of nanoparticles

![Figure 10: Variation in \( \phi(\eta) \) via \( \text{Sc} \) and \( y_2 \).](image)

![Figure 11: Variation in \( \phi(\eta) \) via \( N_b \) and \( N_t \).](image)
is suppressed since a larger amount of the nanoparticles are transformed to another species in the domain. The inverse trend is reported with greater generative chemical reaction parameter $\sigma < 0$ for which nanoparticle concentrations are elevated and thickness of solutal boundary layer is also improved.

5.4 Motile microorganism profile

Figures 13 and 14 show the impacts of $Sc_n$, $Pe$, $\gamma_3$, and $\psi$ on $\xi(\eta)$. From Figure 13, it is examined that an enhancement in $Sc_n$ causes a decay in the diffusivity of microbes in polymeric liquid, which results in the depletion of the
microorganism field. Moreover, Figure 13 displays the aspect of Pe on the microorganism profile $\xi(\eta)$. In fact, Pe is the key parameter to evaluate the microorganisms swimming in the liquid regime. Pe is defined as the ratio of high cell swimming velocity to microorganism diffusion rate. Diffusion is the mechanism in which a molecule transfers from a region of high concentration to a region of low concentration. It proves the movement of the fluid.

Table 2: Numerical values of $\mathrm{Nu}_b\,\mathrm{Re}_T^1$, $\mathrm{Sh}_b\,\mathrm{Re}_T^1$, and $\mathrm{Nn}_b\,\mathrm{Re}_T^{1/2}$ for different flow parameters

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particles in the boundary regime. It is noted that microbe’s diffusivity is suppressed in the case of an enhancement in Pe. Hence, the microorganism profile $\xi(\eta)$ decreases. Figure 14 describes the influence of $\Omega$ on the microorganism field $\xi(\eta)$. It is detected that by enhancing the values of $\Omega$, the concentration of motile microorganisms in ambient liquid is reduced. Also, Figure 14 shows the performance of $\gamma_3$ on motile microorganism profile $\xi(\eta)$. It is found that the microorganism distribution $\xi(\eta)$ enlarges when $\gamma_3$ is accelerated.

### 5.5 Physical quantities

The variation in the rates of heat transfer, mass transfer, and microorganism diffusion for flow parameters are presented through Table 2. Here it is found that a diminishing trend in these physical quantities for Rayleigh factor, buoyancy ratio, and thermophoresis parameters is observed, while increasing behavior is observed for the temperature ratio parameter. Furthermore, increasing $N_b$ and $y_1$ cause a decrease in heat transfer rate; however increasing behavior is observed for $y_2$. An enhancement is observed in the rate of mass transfer with the growing values of $N_b$ and $y_2$. Increasing $y_1, y_2$, and $y_3$ cause an elevation in microorganism diffusion rate.

### 6 Conclusion

The present study captures the thermophoresis body force and Brownian motion aspects in the flow of upper convected Maxwell nanofluid subjected to gyrotactic motile microorganisms. The flow is considered over a porous vertical surface. Nonlinear radiative stagnated flow is formulated considering chemical reaction. The convergence series solutions are obtained through HAM. The key findings are elaborated below:

1) The fluid velocity is an increasing function of the mixed convection parameter but opposite outcomes are obtained for the Deborah number.
2) The temperature ratio factor yields higher temperature and heat transportation rate.
3) Brownian motion and radiation parameters upsurge the nanofluid temperature.
4) Concentration profile decays for larger Brownian motion parameter whereas it elevates for larger thermophoresis parameter.
5) Larger solutal Biot number and Brownian motion parameter result in the augmentation of the magnitude of local Sherwood number.

6) Microorganisms profile declines for larger bioconvection Schmidt number and motile microorganisms difference parameter whereas it improves with stronger Pelet number.

7) Magnitude of microorganism diffusion rate boosts directly for larger $y_1, y_2$, and $y_3$.

The present communication provides a suitable platform to researchers regarding nanoliquid consideration in higher temperature based nano-polymeric flows subjected to coating processes. However, this research work has overlooked certain effects like activation energy, magnetohydrodynamics, Cattaneo–Christov based dual diffusive flow, entropy generation, and bio-convective flows featuring gyrotactic microorganisms. Such effects will be reported imminently.

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### References


