Research Article

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An extended model to assess Jeffery–Hamel blood flow through arteries with iron-oxide (Fe₂O₃) nanoparticles and melting effects: Entropy optimization analysis

https://doi.org/10.1515/ntrev-2023-0160
received April 19, 2023; accepted November 5, 2023

Abstract: Nano fluids are utilized in cancer therapy to boost therapeutic effectiveness and prevent adverse reactions. These nanoparticles are delivered to the cancerous tissues under the influence of radiation through the blood vessels. In the current study, the propagation of nanoparticles within the blood in a divergent/convergent vertical channel with flexible boundaries is elaborated computationally. The base fluid (Carreau fluid model) is speculated to be blood, whereas nanofluid is believed to be an iron oxide–blood mixture. Because of its shear thinning or shear thickening features, the Carreau fluid model more precisely depicts the rheological characteristics of blood. The arterial section is considered a convergent or divergent channel based on its topological configuration (non-uniform cross section). An iron oxide (Fe₂O₃) nanoparticle is injected into the blood (base fluid). To eliminate the viscous effect in the region of the artery wall, a slip boundary condition is applied. An analysis of the transport phenomena is preferred using the melting heat transfer phenomena, which can work in melting plaques or fats at the vessel walls. The effects of thermal radiation, which is advantageous in cancer therapy, biomedical imaging, hyperthermia, and tumor therapy, are incorporated in heat transport mechanisms. The governing equation for the flow model with realistic boundary conditions is numerically tickled using the RK45 mechanism. The findings reveal that the flow dynamism and thermal behavior are significantly influenced by melting effects. Higher Re can produce spots in which the track of the wall shear stress fluctuates. The melting effects can produce agitation and increase the flow through viscous head losses, causing melting of the blockage. The maximum heat transfer of 5% is achieved with We when the volume friction is kept at 1%. With higher estimation of inertial forces Re and same volume friction, the skin drag coefficient augmented to 34%. The overall temperature is greater for the divergent flow scenario.

Keywords: melting effects, artery section as a converging conduit, Carreau model as blood, entropy analysis, skin friction and heat transfer, iron oxide nanoparticles

Nomenclature

$v(r, \theta)$ radial velocity (m s⁻¹)
$V_{\text{max}}$ maximum velocity (m s⁻¹)
$(r, \theta, z)$ polar coordinates
$\nabla$ gradient operator
$\kappa_{sl}$ thermal diffusivity (m² s⁻¹)
$\kappa_{sf}$ thermal conductivity at the free stream (W m⁻¹ K⁻¹)
$(\rho c_p)_H$ heat capacitance of the fluid (J kg⁻¹ K⁻¹)
$(\rho c_p)_N$ heat capacitance of the nanoparticles (J kg⁻¹ K⁻¹)
$\sigma$ stress tensor
$\mu$ dynamic viscosity (kg m⁻¹ s⁻¹)
$\mu_1, \mu_0$ infinite and zero shear rate viscosity (kg m⁻¹ s⁻¹)
$J_1$ first kind Rivlin–Erickson tensor
$\gamma_\mu$ shear rate
$\lambda$ material constant
$\nabla p$ pressure differential

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\[ n \quad \text{power index } 1 < n < 2 \]
\[ \rho_f, \rho_p \quad \text{fluid and nanoparticle density (kg m}^{-3}\text{)} \]
\[ \mu \quad \text{nanofluid dynamic viscosity} \]
\[ N_s = \left( \frac{\sigma_{nf}}{a} \right) \quad \text{slip parameter} \]
\[ S_T \quad \text{dimensional entropy} \]
\[ N_{gen} \quad \text{dimensionless entropy} \]
\[ T, T_w \quad \text{fluid and wall temperature (K)} \]
\[ a \quad \text{channel semi-width} \]
\[ \sigma_{th} \quad \text{shear stress at the wall} \]
\[ V_w \quad \text{wall velocity (m s}^{-1}\text{)} \]
\[ \lambda \quad \text{slip parameter} \]
\[ \varphi \quad \text{volumetric friction of nanoparticles} \]
\[ \sigma^* \quad \text{Stefan–Boltzmann coefficient (W m}^{-2}\text{K}^{-4}\text{)} \]
\[ k^* \quad \text{mean absorption coefficient} \]
\[ g \quad \text{gravitational acceleration (9.8 m s}^{-2}\text{)} \]
\[ y \quad \text{air expansion coefficient} \]
\[ \delta \quad \text{latent heat capacity (J kg}^{-1}\text{)} \]
\[ C_s \quad \text{heat capacity of the solid surface} \]
\[ C_l \quad \text{skin friction coefficient} \]
\[ Nu \quad \text{Nusselt number} \]
\[ We = \left( \frac{\mu_{nf} V_w}{\rho_f} \right) \quad \text{Weissenberg number} \]
\[ Re = \frac{ar_{nf}}{V_w} \quad \text{Reynolds number} \]
\[ Pr = \frac{\mu_{nf} C_p h}{\kappa} \quad \text{Prandtl number} \]
\[ Rd = \frac{16 \sigma^* T_0^2}{3k(\rho \gamma)} \quad \text{thermal radiation parameter} \]
\[ Ec = \frac{V_w^2}{r_{nf} C_p h} \quad \text{Eckert number} \]
\[ Gr = \frac{g r_{nf} T_0^3}{V_w^2} \quad \text{Grashof number} \]
\[ Mt = \frac{C_l(T_w)}{\kappa^2 + C_l(T_w)} \quad \text{melting parameter} \]

1 Introduction

The unprecedented use of nanotechnology in every field, including the environment, medicine, therapeutics, biotechnology, and drug development, has altered the dynamics and expanded the horizons of today’s technological world. Because of the medical studies already described, it is now clear that blockage in arteries is a dangerous and fatal condition, leading researchers to attempt to use nanotechnology to solve the issue. Because nanoparticles can pass through cells, tissues, and organs, researchers are optimistic that nanotechnology may lead to significant advances in the treatment of many disorders. Nanoparticles are used in a wide range of biological processes, including medication administration, biosensing, and imaging. They can be made functional in multilayer channels to make it easier to move materials through various biological tissue layers, improving therapeutic efficacy and minimizing negative effects. Nanoparticles have also improved imaging contrast and function as biosensors to find biological substances. The development of surgical equipment, the usage of X-rays, the killing of cancerous cells, the cure of brain tumors, and other biomedical and health sciences have all benefited significantly from the usage of nanotechnology. Small energy assets have a considerable impact on engineering and industrial products, which is a well-known fact. The thermal characteristics of base liquids are markedly improved by the inclusion of nanoparticles. Choi and Eastman [1] demonstrated in a groundbreaking study on nanofluid that the interaction of nanoparticles enhances the thermal features of the base liquid. The arteries in the body are becoming thinner due to arterial stenoses, which restrict blood flow to the organ structures and can eventually cause mortality. When solid micrometer atoms are added to base fluids, the thermal characteristics and heat system quantity of the fluids are improved compared to the ordinary liquids. To investigate the rheological performance of nanoparticles, Yan et al. [2] conducted an experimental study. Elelaly et al. [3] investigated the arterial flow of a nanofluid through a cardiac valve with the distribution of temperature and slip effects. Shojaie Chahregh and Dinavand [4] investigated blood-based nanofluid transportation through an artery with the delivery of drugs and the cardiac system in acute respiratory utilization. Shahzadi and Bilal [5] studied the blood flow of nanofluids passing via diverged stenosed arteries when affected by penetration. Through such an arterial section, Abdelsalam et al. [6] provided description of the modifications in blood flow brought on by nanofluid forces using finite difference computation. Although the exact etiology of narrowing arteries is uncertain, its effects on flow characteristics are extensively researched both theoretically and empirically [7–10]. The physicochemical features of nanoparticles have enabled researchers to fill the gaps and surmount obstacles in a variety of industrial and biological applications. Nanotechnology has attracted a lot of devotion in recent years [11–16]. By offering special benefits of imaging, traceability, encapsulation, and drug conjugation, as well as by enhancing drug biodistribution and pharmacokinetics profiles, directing drug delivery to the desired tissues, and having adjuvant and antibacterial properties, nanoparticles assist in overcoming this barrier [17,18].

In the biological sciences, both experimental and theoretical blood flow modeling in constricted arteries is important. Even in developed countries, cardiovascular
diseases remain the leading cause of death despite notable advancements and improvements in medical sciences and medicine. Low-density compounds, protein, and cholesterol exacerbate the growth of macrophage white cells, which leads to the chronic inflammatory response identified as atherosclerosis in the arterial walls. As a result, an incision forms on the arterial blood vessels inside the walls. Blood flow decreases or may even stop because of a thrombus, which is created when a squishy plaque bursts. The behavior of blood flow in stenosis arteries and normal arteries varies dramatically. The irregularity of the blood flow and the spontaneous characteristics of the artery play a big role in the malfunctioning of the cardiovascular system. Blood flow is hampered by stenosis, which narrows the blood flow region. Blood flow disorder in the artery can lead to blockage, shear stress, and uneven volumetric flow rate. Mathematical analysis is necessary to solve blood flow problems because it provides comprehensive information on all geometrical and flow features. The binary nature of human blood viscoelasticity has been discovered through experiments. Large shear rates give blood a Newtonian character, whereas moderate shear rates or illness give blood a non-Newtonian character that predominates. Liu et al. [19] computed a blood barge pulsating blood flow. Layek et al. [20] used Cartesian coordinates to simulate the pulsating blood flow. They used blood as a model fluid for their experiment. Shear stress reduces due to the formation of contraction, and rheological fluid is used to predict blood viscoelasticity in the narrowing domain. Sankar and Hemalatha [21] employed perturbation techniques to carry out a thorough analysis of the Herschel–Buckley fluid (HBF) via a special contraction artery. Abbas et al. [22] examined the flow of HBF through a blood vessel with time-invariant interlaced stenosis. Sharma et al. [23] investigated one-directional MHD blood flow through an artery with a single gentle shrinkage. Sarwar and Hussain [24] deliberated on human blood flow performance in response to stenosis claims. Recent studies proposed broad study inquiries on blood flow utilizing diverse fluid models [25,26].

In liquid form, blood is a connective tissue (see, e.g., Sembulingam and Sembulingam [27]). In the medical field, blood flow using nanoparticles is crucial for drug delivery and cancer treatment. Koriko et al. [28] studied the Carreau nanoliquid flow along the horizontal plate of a paraboloid of rotation. They noticed that minor values of the Deborah number displayed the highest values for surface drag and the rate of heat transfer. The blood temperature increased as the volumetric percentage of carbon nanotubes increased (see Koriko et al. [28]). According to Dinarvand et al. [29], the usage of CuO and Cu hybrid nanoparticles decreased the capillary hemodynamic effect in comparison to pure copper. Recently, Ashraf et al. [30] investigated the peristaltic flow of Casson blood-based nanomaterial incorporating platelet-shaped magnetic nanoparticles using the generalized differential quadrature method.

The term “entropy production” refers to the amount of entropy created by irreversible processes such as fluid drift through flow confrontation, Joule heating, heat flow via resistance to heat, interaction among solid materials, dispersion, and fluid viscosity inside a system. According to the second law of thermodynamics, the total entropy of the entire system does not change during a reversible operation. The fundamental law of thermodynamics states that the amount of energy is always conserved, while the second law indicates that temperature gradients are always propagated. In other words, since the quantity of energy is conserved, energy gradients gradually dissipate, reducing the quantity ability to perform work. An increase in entropy measures the loss of the energy gradient. For reducing the irreversibility rate in any closed system, Bejan [31,32] focused on the significance of irreversibility in four crucial heat transport pathways. Akbar and Butt [33] studied entropy properties in porous walled arteries. In order to evaluate the entropy formation of blood flow through a stenosed artery by means of a catheter, Mekheimer et al. [34] used entropy hemodynamics. Zhang et al. [35] studied the entropy of blood flow through constricted arteries while suspending ZnO nanoparticles. Zidan et al. [36] used a multi-stenosed artery with a thrombus to conduct an entropy generation analysis. Sharma et al. [37] inspected the influence of hybrid nanoparticles on entropy formation on blood flow through a narrowing artery. Kumawat et al. [38] presented entropy creation using a curved permeable stenosed artery with a chemical reaction and heat source. Sharma et al. [39] scrutinized the entropy production for higher-order exothermic–endothermic chemical processes. Gandhi et al. [40] investigated the impacts of entropy using a temperature-dependent viscosity artery with hybrid nanoparticles.

Flows in convergent–divergent channels are extensively studied due to its importance in numerous applications in aerospace, chemical, industrial, and biomechanical engineering (arteries and veins). Such flows are well known to be unstable [41]. These flow forms describe the exit and inlet of an incompressible Newtonian fluid in a consistently expanding conduit (wedge) with a specific angle among the walls. We consider that the angle among the wedge walls is 2α; therefore, we may consider a two-dimensional (2D) sink or source flow as a good example for comprehending canal flow. The Jeffery–Hamel flow is named after Jeffery [42], who discovered it in 1915, and Hamel [43], who identified it simultaneously in 1916. Jeffery’s goal was to find analytical
solutions to the velocity equation of a Navier–Stokes fluid. After that, Hamel examined the 2D radial flow of a viscous fluid. He noticed that certain steady flows differ significantly from radial flows. Dean [41] later investigated similar solutions and studied their impact on the stability analysis of radial motion; he also discovered additional innovative solutions for particular divergent flows. Many manufacturing processes rely heavily on flows through converging–diverging channels, flows via diffusers, nozzles, and reducers. Convergent/divergent flows have also been successfully employed to replicate dilute polymer solution flows via porous media. In the polymer sector, flows over converging conduits are also used to advance the mechanical qualities of goods, such as plastic sheets and rods. Another notable potential for this type of flow is molten polymer extrusion via convergent dies. Significant advances in the analysis of nonlinear flows in convergent and divergent conduits can be found in previous studies [44–46].

The current investigations demonstrate that several investigators explored the blood flow of several rheological fluids under certain physical circumstances considering different morphological structures of blood vessels [47–50]. They considered the arteries as a stretchable surface, pipe-like structure, and contracting/expanding channels. However, the transport of blood nanofluid with iron oxide nanoparticles through the segment of the arteries or veins that are converging and divergent in nature is not deliberated so far. The finite segment of the blood vessel is convergent/divergent based on its non-uniform and cylindrical cross section throughout the body. With these objectives in mind, the current investigation analyzes the heat transport and irreversibility mechanism due to energy losses with blood flow in the arterial network, whose structure are long conduit with cylindrical cross section. The Carreau fluid model reveals that the shear thinning behavior limitations under elevated shear stress are considered blood. The heart exerts pressure on the blood to force it through the arteries. The fully developed flow of nanofluid through an arterial wall in the presence of mixed convection, viscous dissipation, radiative heat flux, and melting heat is analyzed. The iron oxide (Fe$_3$O$_4$) nanoparticle is considered because of its significant biomedical applications, such as magnetized bioseparation, magnetic-fecion medication, DNA molecule hyperthermia, identification, and targeted medication transport. The fluid viscous dissipation, in conjunction with a radiative heat flux, is used to model the entropy equation for an irreversible procedure using the second law of thermodynamics. The similarity mechanism is used to convert the controlling equations into a dimensionless framework. These modified equations are solved numerically using the fourth- and fifth-order Runge–Kutta (RK) approach together with the shooting technique. The simulated results are used to account for the effects of skin drag coefficient, heat transfer, and entropy analysis. This perspective is particularly valuable in fields such as the treatment of cancer, nano-drug transport in blood vessels, cardiovascular disease, and infusing medication, blood, and drugs into the circulatory system.

2 Description of the physical model

Blood is considered a super-natural fluid that flows through living beings, attracting a great deal of magical and mythological attention. According to the literature, blood has been researched as both a quantitative and qualitative entity since the beginning of time, although our method of study is quantitative.

The mathematical framework contracts through the implementation of several assumptions on the blood flow in the human vascular system.  
1) The 2D Navier–Stokes equation and continuity equation for an incompressible non-Newtonian fluid in cylindrical coordinate (r, θ) were utilized to develop the mathematical modeling of arterial blood flow: \( r \) denotes the radial direction, and \( θ \) is the axial flow direction (Figure 1).
2) We evaluate an artery with an arbitrary cross-sectional area (which is comparable to a pipe), and the viscoelastic effect is ignored.
3) Iron oxide (Fe$_3$O$_4$) nanoparticles are suspended in blood.
4) In order to inspect the heat transport, the melting properties are taken into account.
5) Blood flow exerts stresses on the channel wall, and because of the strain, the channel expands, and fluid travels in a diverging section at its fastest possible rate.
6) It is presumed that the blood temperature is higher than the arterial wall temperature, \( i.e., T > T_w \).
7) We assume that the boundary conditions are established not only by stress at the arterial walls but also by constant pressure differentials at the inlet and outflow of the artery.

The mass, momentum, and energy conservation are the balanced equation for the laminar incompressible Carreau fluid in the presence of gravitational forces [51]:

\[
\rho \frac{d \mathbf{V}}{d t} = \nabla \cdot \mathbf{F},
\]

\[
\rho \left( \nabla \cdot \mathbf{V} \right) = 0,
\]

where \( \rho \) denotes the density, \( \mathbf{V} \) is the velocity vector, \( \mathbf{F} \) is the force vector, and \( \rho \) represents the fluid density.
where is the body force due to mixed convection.

\[ \frac{dT}{dr} = \kappa_0 V^2 T + (\tau : \nabla V) - \frac{\partial q_r}{\partial r}, \quad (3) \]

\( \lambda \quad \) Material characteristics of the fluid
\( \dot{\gamma} \quad \) Shear rate
\( n \quad \) Power index describing the shear thinning and shear thickening features at \( 0 \leq n < 1 \) and \( n > 1 \), respectively
\( J_1 \quad \) First Rivlin–Ericksen tensor provided in equation (6)
\( \sigma \quad \) The extra stress tensor for the Carreau liquid provided in equation (4).

The shear rate \( \dot{\gamma} \) is defined as

\[ \dot{\gamma} = \sqrt{\frac{1}{2} \sum_{i,j} \eta \gamma \gamma_{ij}} = \sqrt{\frac{1}{2} \Pi} = \sqrt{\frac{1}{2} \text{trac}(J_1)^2}, \quad (6) \]

where \( J_1 \) is the second invariant or first Rivlin–Ericksen tensor defined as

\[ J_1 = L + L^\mu, \quad L = \nabla V, \quad (7) \]

We consider \( \mu_w \ll \mu_0 \); therefore, \( \mu_w = 0 \). Thus, we attain the physical situation. Equation (5) can thus be written as

\[ \mu(\dot{\gamma}) = \mu_0 [1 + (\lambda \dot{\gamma})^n]^{\frac{n-1}{2}}. \quad (8) \]

When nanoparticles are suspended in the working liquid, the rest of the viscosity \( \mu_0 \) of the Carreau liquid is upgraded with the Carreau nanofluid viscosity \( \mu_{nf} \), and other physical features turn into physical attributes of nanofluid physical characteristics:

\[ \mu(\dot{\gamma}) = \mu_{nf} [1 + (\lambda \dot{\gamma})^n]^{\frac{n-1}{2}}(L + L^\mu). \quad (9) \]
The dimensional $L$, $J_i$, and shear rate $\dot{\gamma}$ in view of the velocity field $V = \nu(r, \theta)$ are

$$L = \begin{bmatrix} \frac{\partial \nu}{\partial r} & \frac{1}{r} \frac{\partial \nu}{\partial \theta} & 0 \\ \frac{\nu}{r} & \frac{1}{r^2} \frac{\partial \nu}{\partial \theta} & \frac{2v}{r^2} \end{bmatrix}, \quad J_i = \begin{bmatrix} \frac{2}{r} \frac{\partial v}{\partial r} + \frac{1}{r^2} \frac{\partial v}{\partial \theta} & \frac{1}{r} \frac{\partial v}{\partial \theta} \end{bmatrix},$$

$$\dot{\gamma} = \begin{bmatrix} \frac{2}{r^2} \left( \frac{\partial v}{\partial r} \right)^2 + \frac{1}{r \theta} \left( \frac{\partial v}{\partial \theta} \right)^2 + \frac{2v^2}{r^2} \end{bmatrix}.$$ (10)

Eq. (4) in the context of Eqs. (9)–(11) takes the following form:

$$\begin{pmatrix} \sigma_{rr} & \sigma_{r\theta} \\ \sigma_{\theta r} & \sigma_{\theta\theta} \end{pmatrix} = \begin{pmatrix} -p & 0 \\ 0 & -p \end{pmatrix}$$

$$+ \mu_1 \left[ 1 + \lambda \left( 2 \left( \frac{\partial v}{\partial r} \right)^2 + \frac{1}{r} \left( \frac{\partial v}{\partial \theta} \right)^2 + \frac{2v^2}{r^2} \right) \right] \frac{n-1}{2} \left( \frac{2v^2}{r^2} \right).$$ (12)

Thus, Eq. (4) in component notation is as follows:

$$\begin{cases} \sigma_{rr} = -p + \mu_1 \left[ 1 + \lambda \left( 2 \left( \frac{\partial v}{\partial r} \right)^2 + \frac{1}{r} \left( \frac{\partial v}{\partial \theta} \right)^2 + \frac{2v^2}{r^2} \right) \right] \frac{n-1}{2} \frac{2v^2}{r^2}, \\ \sigma_{\theta\theta} = -p + \mu_1 \left[ 1 + \lambda \left( 2 \left( \frac{\partial v}{\partial r} \right)^2 + \frac{1}{r} \left( \frac{\partial v}{\partial \theta} \right)^2 + \frac{2v^2}{r^2} \right) \right] \frac{n-1}{2} \frac{2v^2}{r^2} \end{cases}$$ (13)

4 Governing equations

Under the above formulation, the system of equations governing mass, momentum conservation, and energy becomes [53]

$$\rho_1 \left( \frac{\partial v}{\partial r} \right) = 0,$$ (16)
5 Boundary conditions

The initial and boundary conditions included outlet, inlet, arterial wall, and temperature and are defined as follows:

**Inlet:** At the arterial entrance, the blood circulation rate was estimated. The blood velocity at the opening and core region may regulate the blood volume. Figure 1 shows how the model intake was determined.

**Outlet:** In order to provide a realistic portrayal of the model, we depicted the flow model velocity at the outflow. The outlet, shown in Figure 1, was located on the opposite side of the entry and served as the point at which blood departed to the capillaries.

The topological conditions on the channel symmetry line (i.e., on \( \theta \rightarrow 0 \)) are [54–56]

\[
v = V_m, \frac{\partial v}{\partial \theta} = 0, \text{ and } \frac{\partial T}{\partial \theta} = \rho_c[\delta + C_\text{C}(T_w)].
\]

(23)

When the artery wall is treated rigidly, the velocity boundary conditions are the typical velocity slip conditions provided by

\[
v(r, \theta) = V_w - \frac{\mu_{nf}}{\rho_{nf}} \left[ 1 + \lambda \left( \frac{\partial v}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial v}{\partial \theta} \right)^2 + \frac{2v^2}{r^2} \right] \frac{\partial v}{\partial \theta} \bigg|_{r=a},
\]

(24)

\[
T = T_w, \text{ as } \theta \rightarrow a.
\]

The flow is considered to be entirely developed and laminar at the inlet; hence, the inlet velocity conditions are thought to have a parabolic velocity profile similar to a long circular cross-sectional channel of arbitrary radius.

And

\[
\rho_{nf} = (1 - \phi) \rho_{bf} + \phi \rho_{np}, \text{ and } \mu_{nf} = \frac{\mu_{bf}}{(1 - \phi)^{2/5}}, \text{ and } \rho_c = \rho_{np}.
\]

(25)

The constitutive relations for fluid density \( \rho_{nf} \), viscosity, \( \mu_{nf} \), heat capacitance \( \rho_c \), thermal conductivity of the base liquid and nanoparticles, as displayed in equation (25), respectively. In the above relations, \( \mu_{bf} \) and \( \mu_{np} \) are the density of the base fluid and nanoparticles, \( \rho_c \) and \( \rho_{np} \) are the heat capacitances, \( \phi \) is the volumetric friction quantity, \( \delta \) is the latent heat capacity, \( C_\text{C} \) denotes the heat capacity of the solid surface, and \( \gamma \) is the air expansion coefficient. Furthermore, \( \sigma_\text{St} \) is the Stefan–Boltzmann coefficient, \( k* \) is the coefficient of mean absorption, and \( g \) is the gravitational acceleration (9.8 m s\(^{-2}\)). Furthermore, \( V_m \) is the maximum velocity at the central track, and \( \theta \rightarrow 0 \) and \( \theta \rightarrow a \) indicate the central region of the channel and wall surface, respectively.

To admit the similarity solution of the governing equation, the following variables are introduced [54,57]:

\[
f(\eta) = \frac{F(\theta)}{f_{\text{max}}}, \eta = \frac{\theta}{\alpha}, \beta = \frac{T}{T_0}, f_{\text{max}} = rv_m.
\]

(26)

\[
(f'''' + 4a^2f''')(1 + We^2(4a^2f^2 + f'^2))^{n-1/2}
+ 2n Re \left[ 1 - \phi + \phi \frac{\rho_{np}}{\rho_{bf}} \right] (1 - \phi)^{2n} f'''
+ \left( 1 - \phi + \phi \frac{\rho_{np}}{\rho_{bf}} \right) a^2 Gr \theta
+ (n - 1) We^2(1 + We^2(4a^2f^2 + f'^2))^{n-3/2}(3f''f'^2)
+ 32a^2ff'''' + f'^2 f'' + 64a^4f^2 f'^2 + (n - 1) (n - 3)(We^2)^2(f''f'^2)
+ 16a^2ff''''f'' + 32a^4f^2f'' + 16a^4f^2f'^2 + 64a^4f'^2 + 4a^2ff'''' = 0,
\]

(27)

\[
\left[ \frac{k_{nf}}{k_t} + Rd \right] \beta'' + \left( 1 - \phi \right) \left( \frac{\rho C_{np}}{\rho C_{bf}} \right) \left( \frac{\rho C_{np}}{\rho C_{bf}} \right)
= \frac{Pr Ec}{(1 - \phi)^{2/5}} \left( 1 + We^2(4a^2f^2 + f'^2) \right)^{n-1/2} (4a^2f'^2 + f''')
\]

(28)

\[
f(0) = 1, f'(0) = 0, f(0)
+ \frac{1}{a Pr} \frac{k_{nf}}{k_t} \frac{M_{t}'(0)}{\phi} = 0, \eta \rightarrow 0,
\]

(29)

\[
f(1) = \frac{N_1}{(1 - \phi)^{2/5}} \left( 1 + We^2(4a^2f^2 + f'^2) \right)^{n-1/2}
= 0.
\]

(30)

\[
f'(1), \beta(1) = 1, \eta \rightarrow a.
\]

5.1 Physical quantities

It is important to measure both the rate of heat transfer from the wall to the fluid and the surface drag (shear stress at the artery wall) of the fluid flow.

The shear stress and heat flux at the walls for the flow problem are defined as

\[
\sigma_{\theta} = \mu_{nf} \left[ 1 + \lambda \left( \frac{\partial v}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial v}{\partial \theta} \right)^2 + \frac{2v^2}{r^2} \left( \frac{1}{r} \frac{\partial v}{\partial \theta} \right) \right] \bigg|_{\theta=a},
\]

(31)
Table 1: Important derivatives

<table>
<thead>
<tr>
<th>Partial derivatives</th>
<th>Dimensional form; using continuity equation ( u = \frac{\nabla \rho}{r} )</th>
<th>Dimensionless form; using similarity variables ( f(\eta) = \frac{f_{\eta \max}}{r_{\max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial v}{\partial r} )</td>
<td>( \frac{F}{r^2} )</td>
<td>( -V_{\max} f )</td>
</tr>
<tr>
<td>( \frac{\partial v}{\partial \theta} )</td>
<td>( \frac{F}{r} )</td>
<td>( V_{\max} f )</td>
</tr>
<tr>
<td>( \frac{\partial^2 v}{\partial r \partial \theta} )</td>
<td>( -\frac{2F}{r^3} )</td>
<td>( V_{\max} f' )</td>
</tr>
<tr>
<td>( \frac{\partial^2 v}{\partial \theta^2} )</td>
<td>( \frac{F}{r} )</td>
<td>( V_{\max} f'' )</td>
</tr>
<tr>
<td>( \frac{\partial^2 v}{\partial r^2} )</td>
<td>( -\frac{F}{r} )</td>
<td>( V_{\max} f''' )</td>
</tr>
</tbody>
</table>

\[ q_{\alpha} = -k_{nf}\left[ \frac{1}{r} \frac{\partial T}{\partial \theta} \right]_{r=\alpha}, \quad (32) \]

The dimensional notation of the skin drag coefficient and heat transport rate can be settled as

\[ C_l = \frac{\sigma_{nf}}{\rho_{nf} V_f^2}, \quad \text{Nu} = -\frac{r q_{\alpha}}{\kappa_{nf} T_{nf}}, \quad (33) \]

The normalized form of equation (33) is

\[ C_l = \frac{1}{(1 - \beta)^{2.5} \text{Re}} \times \left[(1 + We^2(4\alpha_0 r^2(1 + f^2(1))))\frac{n-1}{2} - f(1)\right], \quad (34) \]

\[ \text{Nu} = -\frac{1}{\alpha} \left[ \frac{\kappa_{nf}}{\kappa_l} + Rd \right] f'(1). \]

Here, \( \beta \) is the slip parameter and \( \text{We} = \frac{16\sigma_{\alpha_0} V_{\alpha_0}^2}{k_{nf} T_{nf}} \) is the Carreau fluid number known as the Weissenberg number. The flow of non-Newtonian liquid can be captured when \( \text{We} \gg 0 \), and Newtonian when \( \text{We} = 0 \). \( \text{Re} = \frac{\sigma_{\alpha_0} V_{\alpha_0}}{\nu_{nf}} \) is the inertial term, known as the Reynolds number. \( \text{Pr} = \left(\frac{\mu_{nf} V_{\alpha_0}}{k_l}\right) \) is the Prandtl number \( \text{Pr} \) that defines the relationship between the momentum and thermal diffusivity in a fluid, and when \( \text{Pr} \gg 1 \), the dimension of the temperature boundary layer is much greater than the thickness of the momentum boundary layer, and the opposite is true when \( \text{Pr} \ll 1 \). \( \text{Rd} = \frac{16\sigma_{\alpha_0} r^2}{k_{nf} (\rho_{nf})_{\alpha_0}} \) is the thermal radiation parameter, which may be a laser source or other medical radiation equipment. \( \text{Ec} = \frac{V_{\alpha_0}}{F_{\alpha_0} (G_{\alpha_0})_{\alpha_0}} \) is the Eckert number \( \text{Ec} \) and contributes to quantifying heat dissipation. It is an indicator of the flow kinetic energy in relation to the enthalpy variation at the thermal boundary layer. \( \text{Gr} = \frac{\theta V_{\alpha_0}^2}{k c_{\alpha_0} T_{\alpha_0}} \) is the ratio of the strength of a buoyant force to a viscous force acting on a fluid in the velocity boundary layer and is referred to as the Grashof number. Just like Reynolds number in forced convection, it plays a similar role in natural convection. Higher estimation of Grashof number leads to turbulent flow within the boundary layer, while for lower Gr within \( 10^3 < \text{Gr} < 10^6 \), the flow is laminar within the boundary layers. \( M_s = \frac{C_{\alpha_0}(T_{\alpha_0})}{\delta + C_{\alpha_0}(T_{\alpha_0})} \) is the process of turning a solid into a liquid while absorbing heat is known as melting. Higher convective flow towards a cooling surface is caused by the dominant melting parameter.

6 Entropy assessment

The subsequent equations are utilized to compute the overall entropy generation, and the volume integral symbolizes entropy generation across the entire fluid domain. The following mathematical equations can be used to determine the thermal and frictional entropy production [58]:

\[ \dot{S}_T = \dot{S}_{\text{therm}} + \dot{S}_{\text{fric}}, \quad (35) \]

where

\[ \dot{S}_{\text{therm}} = \oint S_{\text{therm}} dV, \quad \dot{S}_{\text{fric}} = \oint S_{\text{fric}} dV. \quad (36) \]

The total entropy can be optimized by computing the volume integral over the whole fluid domain:

\[ S_{\text{therm}} = \iint S_{\text{therm}} dV + \iint S_{\text{fric}} dV. \quad (37) \]

Thus,

\[ S_T = \iint S_{\text{therm}} dV + \iint S_{\text{fric}} dV. \quad (38) \]

In this context, the total entropy generation for the nanofluid can be computed as

\[ S_T = \frac{k_l}{k_{nf}} \left[ \kappa_{nf} + \frac{16\sigma_{\alpha_0} T_{\alpha_0}^3}{3k_l (\rho_{nf})_{\alpha_0}} \left[ \left( \frac{\partial T}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial T}{\partial \theta} \right)^2 \right] \right] \]

\[ + \left[ \frac{\mu_{nf}}{T_{nf}} + \frac{16\sigma_{\alpha_0} T_{\alpha_0}}{3k_l (\rho_{nf})_{\alpha_0}} \left[ \left( \frac{\partial T}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial T}{\partial \theta} \right)^2 \right] \right]^n \]

\[ \times \left[ 1 + \lambda^2 \left( \frac{\partial T}{\partial r} \right)^2 + \frac{1}{r^2} \left( \frac{\partial T}{\partial \theta} \right)^2 + \frac{2\nu}{r^2} \right]. \quad (39) \]
In the context of dimensionless variables (equation (26)),
\[
N_{\text{gen}} = \frac{r^2 \alpha^2 \kappa_{\text{T}}}{\kappa_l} = \left( \frac{K_{\text{nf}}}{\kappa_l} + \text{Rd} \right) \beta^2 + \frac{\text{PrEc}}{\left( 1 - \varphi \right)^{2.5}} \times \left[ 1 + \text{We}^2 \left( 4 \alpha^2 \frac{t^2}{2} + f^2 \right) \right]^{\frac{n-1}{2}} \left( 4 \alpha^2 t^2 + f^2 \right) \right]
\]
(40)

7 Computational algorithm

The nonlinear governing differential equations (27) and (28) are decoded using the RK Fehlberg [59] integration method with boundary constraints (29) and (30) numerically. The RK45 methodology uses the shooting technique and is composed of a number of repeating procedures for estimating the solutions. While computationally integrating ordinary differential equations, the RK approach is adopted in this computational mechanism [60].

Step I: The prerequisite initial conditions
\[
t_1 = f, \; t_2 = f', \; t_3 = f'', \; t_4 = \beta, \; t_5 = \beta', \quad (41)
\]

\[
\begin{align*}
t_1' &= t(2) \\
t_2' &= \frac{X_5}{X_4} - \frac{C_3 C_6 X_6}{X_4} - \frac{X_5}{X_4} + \frac{C_2 X_6}{X_4} \\
t_3' &= t(4) \\
t_4' &= -\frac{C_3 \text{PrEc} X_6 (t^2 + 4 \alpha^2 t^2)^{\frac{n-1}{2}} \left( t^2 + 4 \alpha^2 t^2 \right)}{A_t (C_4 + \text{Rd})}
\end{align*}
\]
(42)

where
\[
C_1 = \left( 1 - \varphi \right)^{2.5}, \quad C_2 = 1 - \varphi + \varphi \frac{\rho_{\text{np}}}{\rho_{\text{bf}}},
\]
(43)

\[
C_3 = \left( 1 - \varphi \right) + \frac{\left( \rho_{\text{Pnp}} \right)_{\text{np}}}{\left( \rho_{\text{Pbf}} \right)_{\text{bf}}}, \quad C_4 = \frac{K_{\text{nf}}}{\kappa_l},
\]

\[
X_1 = \left[ 1 + \text{We}^2 (t^2 + 4 \alpha^2 t^2) \right]^{\frac{n-1}{2}} \left( t^2 + 4 \alpha^2 t^2 \right)
\]
(44)

\[
X_2 = \left[ 1 + \text{We}^2 (t^2 + 4 \alpha^2 t^2) \right]^{\frac{n-3}{2}}, \quad X_3 = \left[ 1 + \text{We}^2 (t^2 + 4 \alpha^2 t^2) \right]^{\frac{n-5}{2}}, \quad X_4 = \left[ X_1 + (n - 1) \text{We}^2 X_6 t^2 \right]
\]
\[
X_5 = -4 \alpha^2 t^2, \quad X_6 = 2 \alpha \text{Re} t^2
\]
\[
X_7 = (n - 1) \text{We}^2 \left[ 1 + \text{We}^2 (t^2 + 4 \alpha^2 t^2) \right]^{\frac{(n-3)}{2}} \left[ 3 t^2 t^2 \right] + 32 \alpha^2 t^2 t^2 + 64 \alpha^4 t^2 t^2
\]
\[
X_8 = \left[ t_2^{2} t^2 + 4 \alpha^2 t^2 t^2 \right] + 32 \alpha^2 t^2 t^2 t^2 + 64 \alpha^4 t^2 t^2 t^2
\]
\[
X_9 = -\alpha^2 Gr t^4
\]

Step II: The computing algorithm uses a testing step in the middle of an interval to eliminate lower-order error factors. A series of first-order differential equations are created by combining the boundary restrictions and the guided equations. The following steps are adopted in this computational mechanism [60].

Step I:
\[
t_0 = f, \; t_1 = f', \; t_2 = f'', \; t_3 = \beta, \; t_4 = \beta', \quad (41)
\]

\[
\begin{align*}
t_1' &= t(2) \\
t_2' &= \frac{X_5}{X_4} - \frac{C_3 C_6 X_6}{X_4} - \frac{X_5}{X_4} + \frac{C_2 X_6}{X_4} \\
t_3' &= t(4) \\
t_4' &= -\frac{C_3 \text{PrEc} X_6 (t^2 + 4 \alpha^2 t^2)^{\frac{n-1}{2}} \left( t^2 + 4 \alpha^2 t^2 \right)}{A_t (C_4 + \text{Rd})}
\end{align*}
\]
(42)

where
\[
C_1 = \left( 1 - \varphi \right)^{2.5}, \quad C_2 = 1 - \varphi + \varphi \frac{\rho_{\text{np}}}{\rho_{\text{bf}}},
\]
(43)

\[
C_3 = \left( 1 - \varphi \right) + \frac{\left( \rho_{\text{Pnp}} \right)_{\text{np}}}{\left( \rho_{\text{Pbf}} \right)_{\text{bf}}}, \quad C_4 = \frac{K_{\text{nf}}}{\kappa_l},
\]

\[
X_1 = \left[ 1 + \text{We}^2 (t^2 + 4 \alpha^2 t^2) \right]^{\frac{n-1}{2}} \left( t^2 + 4 \alpha^2 t^2 \right)
\]
(44)

\[
X_2 = \left[ 1 + \text{We}^2 (t^2 + 4 \alpha^2 t^2) \right]^{\frac{n-3}{2}}, \quad X_3 = \left[ 1 + \text{We}^2 (t^2 + 4 \alpha^2 t^2) \right]^{\frac{n-5}{2}}, \quad X_4 = \left[ X_1 + (n - 1) \text{We}^2 X_6 t^2 \right]
\]
\[
X_5 = -4 \alpha^2 t^2, \quad X_6 = 2 \alpha \text{Re} t^2
\]
\[
X_7 = (n - 1) \text{We}^2 \left[ 1 + \text{We}^2 (t^2 + 4 \alpha^2 t^2) \right]^{\frac{(n-3)}{2}} \left[ 3 t^2 t^2 \right] + 32 \alpha^2 t^2 t^2 + 64 \alpha^4 t^2 t^2
\]
\[
X_8 = \left[ t_2^{2} t^2 + 4 \alpha^2 t^2 t^2 \right] + 32 \alpha^2 t^2 t^2 t^2 + 64 \alpha^4 t^2 t^2 t^2
\]
\[
X_9 = -\alpha^2 Gr t^4
\]

Step II: The prerequisite initial conditions
\[
t_0(0) = 1, \; t_1(0) = 0, \; t_2(0) + \frac{1}{a \text{Pr} a \text{A}} A_1 M t_0(0) = 0, \quad (46)
\]

\[
t_0(1) = t_0(1) - \frac{a}{a \text{Pr} a \text{A}} A_1 \left[ 1 + \text{We}^2 (t^2 + 4 \alpha^2 t^2) \right]^{\frac{n-1}{2}} t_0(1), \; t_0(1) = 1.
\]
(47)

Step III: The accuracy of approximate missing initial conditions can be guaranteed by estimating the dependent factor at the boundaries; if it is not convergent, a different value must be chosen as a guess, and this procedure is repeated before the required degree of accuracy between the provided and measured missing initial conditions is accomplished.

Step IV: Finally, the RK method is used to solve the first-order IVP system with given and computed missing initial conditions.

Table 2 shows the exact solution by Abbasbandy and Shivanian [61] for skin friction vs numerical findings of the current findings in the limiting case when \( \varphi = 0 \) and \( n = 1 \). When compared to previously published results in the literature, the calculated values are found to be in good agreement up to four decimal places. As a result, because it converges significantly, the shooting approach is particularly useful in discovering the solution to complex fluid flow issues. Table 3 depicts the thermochemical properties of nanoparticles and blood.

8 Results and discussion

The primary objective of this segment is to investigate the graphical and physical performances of several key elements, containing those in the velocity and temperature...
number may result in a smooth pattern in the center of the conduit and significant gradients toward the walls. As a result, the boundary layer thickness decreases. It is clear that under convergent flow conditions, backflow is entirely prohibited. On the other hand, there is disagreement over how Reynolds number affects divergent flow. In channels with smaller gradients approaching the walls, the diverging flow intensifies the volume flux there. According to the findings, only divergent channels greatly favor flow reversal (according to Garimella et al. [66]). Reynolds number has a growing effect on temperature fields \( \beta(\eta) \) for both converging and diverging walls. Blood viscosity is a predictor of resistance to vascular flow and is believed to contribute to blood pressure due to boundary slip. Furthermore, the cardiovascular risk factor, arterial hypertension, contributes to cardiac morbidity. The temperature of the liquid decreases in the convergent channel for larger Reynolds number estimates, while the divergent channel shows a considerable increase. The calculations also show that velocity is a general growing function of the inertial parameter. The temperature, on the other hand, increases with Re, albeit altering the value of Re does not really have a major impact on the blood temperature inside the vessel (divergent portion). It is also easy to see that the blood temperature is lower closer to the tissues and gradually increases near the channel midline. Figure 3(a and b) shows the effects of buoyancy Gr on velocity fields. The inclusion of buoyancy effects makes the problem more difficult because the flow equation is correlated with temperature distribution. Physically, the buoyancy assisting forces within the channel is represented by the positive values of the Grashof number \( Gr \geq 0 \). The Grashof number, which measures the ratio of buoyant to viscous forces, has a negative connection to viscosity. High blood pressure results from the human cardiovascular risk factor, arterial hypertension, for both converging and diverging walls.
velocity increases with low viscosity. Therefore, greater amounts of Grashof number are needed to slow the flow of blood. We examine changes in the blood flow distribution by taking the Grashof number values from lower to higher. The buoyant forces were strengthened by the acceleration in the Grashof number, while the viscous forces were attenuated, improving the velocity profile. The momentum equation shows a slight viscous effect, and the converging channel records an increase in the Grashof number, which indicates greater velocity profiles. The upward pressures put forth by fluids in opposition to an object's weight are lodged in a divergent channel. For both narrowly opening channels, natural convection has very little effect on the velocity field. As the Grashof number increases, the maximum value of the dimensionless temperature decreases since the two quantities are inversely related. The sharp temperature gradient adjacent to the channel creates a strong buoyancy-driven force that is mostly accountable for this. This force causes the thermal boundary layer to thin as it moves farther away from the channel wall. Similar patterns are observed when there are diverging channels (Figure 3b). Physically, as the buoyancy

**Figure 2:** (a and b) Blood velocity and thermal profile under the consequences of Re.

**Figure 3:** (a and b) Blood velocity and thermal profile under the consequences of Gr.
parameter increases, the temperature drops within the channel, causing the volumetric expansion. The peculiarities of the volumetric fraction parameter for the nanofluid on velocity and temperature outline are highlighted in Figure 4(a and b). It is evident that the velocity is an increasing function of $\varphi$. Physically, nanoparticles increase the rate of heat transmission, which improves the convective flow of nanofluid, and thus velocity increases. This shows that more iron nanoparticles injected led to a higher rate of recovery for the patient with iron oxide deficiency. We observe that the velocity profile increases in the middle of the channel and diminishes towards its walls. It was concluded that stronger flow resistance is provided by higher nanoparticle volume fractions close to the channel walls. An increase in the volumetric fraction of nanoparticles indicates an increase in the fluid temperature. This demonstrates unequivocally how the heat produced by the heat source is swiftly transmitted to the walls (due to the addition of additional nanoparticles) and subsequently results in an increase in fluid temperature. Since the temperature is dispersed equally throughout the fluid, the intermolecular collision of the liquid molecules is enhanced as a consequence of an increase in the volumetric fraction of nanoparticles from 0.05 to 0.2. The consequent increase in thermal conductivity makes the upward variation prevails for the status of nano-molecules. Figure 4b clearly illustrates that the temperature distribution is wider for iron oxide nanoparticles. This is because iron particles have a high atomic number, which raises temperature gradients and aids in the treatment of cancer. Figure 5(a and b) shows how the Weissenberg number (We) affects the temperature and blood flow rate in the conduit region. It is noted that as we increase We from 0.5 to 1.5, the flow contour for shear-thickening blood drops while that for shear-thinning blood increases. The Weissenberg number articulates the comparative importance of elastic to viscous forces for non-Newtonian liquids. Elastic forces, which are dominant when $We > 1$, cause dilatant blood to slow down. In the pseudoplastic (shear-thinning) situation, the observed acceleration is caused by a reduction in viscous resistance to the flowing blood, which is implied by the reduction of viscous forces compared to elastic forces ($We < 1$). The effects are opposite in diverse sections. The variation in the We on $\beta(\eta)$ is seen in Figure 5(b). The temperature increase and the corresponding increase in boundary layer thickness help as We increase. We actually have an extraordinary significance to amplify the temperature profile. Additionally, compared to a conventional liquid, a nanoliquid’s thermal layer thickness is superior. Figure 6(a, b) depicts the influence of $\text{Mt}$ on the blood–iron oxide nanofluid velocity field. The boundary layer $f(\eta)$ and associated thickness are improved with an increase in the melting effect, $\text{Mt}$. Physically, a sophisticated convective flow is produced toward a (vein or artery) cooling surface by the dominant melting parameter. The velocity field expands as a result. It is helpful to remove blockages like plaque or fat from the internal walls of arteries. This is because an increase in $\text{Mt}$ enhances the melting intensity, and the wall gradually transitions to a fluid, causing the velocity gradients to expand rapidly, reducing the heat gradient and its related boundary layer thickness. Figure 6b depicts the effect of the

![Figure 4: (a and b) Blood velocity and thermal profile under the consequences of $\varphi$.](image-url)
melting parameter $M_t$ on the temperature for blood-based magnesium nano fluid. The temperature field changes from an increasing function to a declining function as the melting parameter increases. As the melting factor is upgraded, the thickness of the temperature within the boundary layer grows. Physically, a higher melt parameter enables heat to transfer faster from the heated fluid to the insulated wall. This, in turn, causes an advanced rate of heat transfer to the surroundings, which ultimately leads to a low temperature. This behavior occurs since raising the melting parameter causes a spike in the temperature variations between the fluid and melting surfaces, and so the temperature differential diminishes with increasing melting parameter. Additionally, increased heat transfer to the vein surface causes the melting of lipids or cholesterol on the interior walls of veins. Higher melting can remove the blood flow barrier. With the involved parameters, a significant increase in the velocity slip and bulk acceleration leads to a significant increase in the main bloodstream and volumetric flow rate.

Figure 7 shows the effect of $Ec$ on the nanofluid blood iron oxide. The temperature distribution steadily gets better
as the Eckert number increases. Physically, this behavior results from drag forces among fluid particles, which convert mechanical energy into thermal energy. Consequently, the temperature field magnifies. The temperature of the liquid itself largely determines the viscosity of the plasma. Although nanoparticles implanted throughout blood increase its thermal conductivity, one of these characteristics seems to predominate when nanoparticles are present. Figure 8 illustrates the effects of the radiation factor $R_d$ on the thermal field. It is obvious that the thermal field expands as the $R_d$ estimates are higher. When $R_d$ is increased, the consequences of nanoliquid conduction physically increase.

Larger $R_d$ values actually imply a more heated surface, which increases the rate of heat transfer within the boundary-layer region. Additionally, the thermal layer increases with increasing $R_d$ estimations.

The numerical characterization of the volumetric friction parameter, the melting effect on skin friction, and the Nusselt number are discussed in Figures 9(a, b) and 10(a, b). A larger melting parameter allows for quick heat transfer. Stronger $M_t$ influences the skin friction coefficient in both scenarios. When there is a melting effect, the rate of heat transfer increases quickly, while it decreases when there is a convergent channel. Physically, an increased melting index results in faster heat transfer from the heated blood to the isothermal wall, which causes more heat transfer to the surroundings and, ultimately, lower heat transfer in the convergent channel. This phenomenon occurs because raising the melting parameter causes an increase in the temperature differential between the surrounding and melting surface, and so the gradient of temperature reduces as the melting parameter is increased. Raising the melting parameter $M_t$ obviously improves the thickness of the thermal boundary layer. As a result, the melting process behaves like a blowing boundary condition at the artery wall, tending to thicken the thermal barrier and reduce heat transfer via the interface between the solid and the liquid. Additionally, increased heat transfer to the surface of the artery causes the inner-wall lipids or plaque to dissolve. Higher melting, hence lowering the obstruction to blood flow. According to the findings of this study, the temperature difference is greater for divergent flow arrangements than for convergent flow arrangements (Figures 9b and 10b), indicating an increase in the heat transfer rate between the hot nanofluid and cool base fluid in converging flow arrangements.

In recent decades, several physiological processes have been the subject of investigations by a number of scientists. They act in an erratic and unpredictable manner. While a person's blood flow improves during specific physical activities, blood circulation is, however, unsteady under these circumstances. Evaporation transfers heat from the skin when the temperature of the surrounding air changes or when the body loses heat through conduction and radiation mechanisms. When evaluating such systems, entropy is quite important. In Figure 11, the effect of $E_c$ on entropy $N_{gen}$ formation is examined. The Eckert number $E_c$ controls how quickly heat is released through molecular conduction during viscous heating. The heat emitted by viscous processes is less pronounced than heat transmission through molecular conduction. Entropy increases because of a significant quantity of released heat between the layers of fluid particles. This shows that heat transmission effects are
subordinate to viscous effects. In Figure 12, the melting factor $Mt$ features for $N_{\text{gen}}$ are highlighted. Entropy increases with the growth of the melting parameter before beginning to decline after a certain threshold. Physically, as $Mt$ increases, the entropy decreases because of the heat being transferred from the heated nanofluid to the cold wall, which causes the disorderness to decrease. Physically, for a greater melting parameter, heat transfers from the heated fluid to the cold wall more quickly, leading to higher heat transmission to the surroundings and, ultimately, low temperature. The inner walls of the artery or vein fats or plaque melt because of increased heat transfer to the artery or vein surface. As the volumetric fraction $\phi$ of nanoparticles increases, fluid particle collisions occur more frequently. The fluid temperature, therefore, increases. As depicted in Figure 13, the increase in $\phi$ is directly proportional to the increase in entropy generation. This is because the formation of entropy grows as the temperature increases. This could be because of the irreversibility of heat and mass transmission being overshadowed by other notions, such as fluid friction.

Tables 4 and 5 show the influence of the nanoparticle volume fraction on skin friction and average Nusselt number for converging channel flow $a = -10^{6}$ at the arterial wall. The increase in the maximum drag coefficient on the solid wall indicates an increase in the frictional coefficient and heat transfer rate from the hot nanofluid to the wall is increased by increasing the volume fraction.
of the nanoparticles $\varphi = 0.0$ to $\varphi = 0.01$. The variance of wall temperature is computed by heat flux at the wall. Table 4 depicts the comparison of the skin friction coefficient $C_f$ for pure and blood nanofluid. According to Table 4, the highest level of skin friction 34% is achieved, when $Re = 150$ and the volume fraction of $\varphi = 1\%$. At low Reynolds numbers, volume fraction augmentation has less effect on the skin friction enhancement. In other words, adding nanoparticles at higher Reynolds numbers has a greater effect, whereas employing nanofluid at lower Reynolds numbers is impractical. The skin friction caused by the fluid flow is proportional to the pressure gradient and inversely proportional to the flow speed. The density of nanofluids decreases as the volume percentage of nanofluids increases. This may cause more skin friction. Skin friction is widely recognized to increase the heat transmission rate in the domain. The skin friction coefficient is a non-dimensional quantity that represents the ratio of shear stress to dynamic pressure. This effect, however, becomes stronger as the Reynolds number increases. The frictional coefficient of a nanofluid is affected by several elements, including thermal conductivity, heat capacity of the base fluid and nanoparticles, flow pattern, nanofluid concentration, particle volume fraction, particle size and shape, and flow structure. The maximum average Nusselt number of 5% is achieved when $\varphi = 1\%$ and $We = 1.0$. In addition, for lower Reynolds numbers, $Re = 50 - 150$, an increase in the nanoparticle volume fraction has a less influence on the Nusselt number (2%) augmentation. The Brownian
motion of particles in the base fluid increases the amount of energy exchange and momentum diffusion in the fluid in a laminar flow. Momentum diffusion from hot to cold parts raises the velocity gradients between the fluid and the wall, increasing the frictional forces between the fluid and the wall. As a result, the mechanism of heat transfer enhancement by nanofluid may be defined by considering two factors: first, the presence and growth of particles in the base fluid improves thermal conductivity and increases the particle’s effective surface. Second, the Brownian motion of microscopic particles speeds up thermal diffusion and, as a result, the energy transfer process in the fluid.

9 Conclusions

The primary objective of this study is to emphasize the impact of a more feasible boundary condition, referred to as the melting phenomena, on the heat transmission properties. This study investigates the effects of nanoparticles on blood as it travels through an artery having a narrower or divergent portion. The blood is modeled as a non-Newtonian Carreau fluid, while the arteries and capillary segments were considered as convergent or divergent channels. The nanofluid model seeks to improve medicine delivery via the nanoparticles in a human blood circulatory system. Steady coupled partial differential equations have been computationally resolved using the RK-45 technique. The dimensionless radial blood velocity, temperature profile, and arterial wall shear stress in terms of skin friction, heat transfer coefficient, and entropy generation have all been calculated for a wide range of physical parameters. Several significant results are listed as follows:

1) The blood velocities increase in a convergent section with respect to the Reynolds number, melting parameter, and Carreau fluid parameters. On the other hand, the blood flow rate decreases with increasing Grashof number. The study of arterial therapy will be abetted by these discoveries.

2) The production of entropy increases as the Eckert number increases. As the thermal radiation parameter increases, the fluid temperature increases as a result.

3) The rate of heat transfer and the coefficient of skin friction both increase as the volumetric friction of iron oxide nanoparticles increases from 0.05 to 0.2. If we focus the medication on a specific area suffering from iron insufficiency, the increased volumetric nanoparticles will be beneficial.

4) With the improvement of the melting parameter, the velocity outline and temperature profile quickened. This demonstrates that depending on the patient's health, the recovery rate of people with iron deficiency may be improved by medication containing enriched Fe$_3$O$_4$ nanoparticles.

5) Disorderliness (entropy) can be minimized with the aid of melting parameters.

6) The addition of nanoparticles increases the temperature and, consequently, the irreversibility.

7) The expedited melting process increases the generation of entropy due to heat transmission.

8) Wall stress and heat transport at the blood–wall interface are significant physiological elements that influence biological responses.

Table 5: Comparison of the heat transfer coefficient $\text{Nu}$ for pure and blood nanofluid

<table>
<thead>
<tr>
<th>We</th>
<th>Re</th>
<th>$n$</th>
<th>Ec</th>
<th>Rd</th>
<th>$\varphi = 0.00\text{Nu}$</th>
<th>$\varphi = 0.1\text{Nu}$</th>
<th>% increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>1.2</td>
<td>0.1</td>
<td>0.2</td>
<td>$-0.16173459$</td>
<td>$-0.12983201$</td>
<td>3</td>
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<tr>
<td>1.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$-0.17849032$</td>
<td>$-0.13039493$</td>
<td>4</td>
</tr>
<tr>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$-0.18514902$</td>
<td>$-0.14992083$</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>1.2</td>
<td>0.1</td>
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<td>$-0.15564879$</td>
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</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>$-0.158434894$</td>
<td>$-0.13767854$</td>
<td>2</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>$-0.13789302$</td>
<td>$-0.11334589$</td>
<td>2</td>
</tr>
<tr>
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<td>0.1</td>
<td>0.2</td>
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</tr>
<tr>
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<td></td>
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<td>1</td>
</tr>
<tr>
<td>1.6</td>
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<td></td>
<td></td>
<td></td>
<td>$-0.178937193$</td>
<td>$-0.16989278$</td>
<td>1</td>
</tr>
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9) With increased volume friction \( \varphi = 0.01 \) of nanoparticles, the skin friction and heat transfer rate increased to 34 and 5%, respectively.

A thorough understanding of the relationship between acceleration and slip velocity and the elastic characteristics of flowing blood could help in the medication of coronary artery disease. A simultaneous evaluation of body acceleration and slip velocity can also help in the identification and therapy of specific health problems, such as vision loss, joint discomfort, and vascular concerns. The slip situation is crucial in the spurt, hysteresis, and shear skin effects. Fluids with boundary slips are useful in both technology and medicine.

10 Future recommendation

A suitable magnetic field can be used to control blood pressure and is also beneficial for illnesses such as poor circulation (necrosis), migraines and headaches, traveling sickness, muscular soreness, and so on. Furthermore, the problem may be more realistic by including a radiation source like a laser.

Funding information: The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University for funding this work through a large group research project under Grant No. RGP2/290/44.

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Data availability statement: All data generated or analyzed during this study are included in this published article.

References


[6] Abdelsalam SI, Mekheimer KS, Zaher AZ. Alterations in blood pressure and is also beneficial for illnesses such as poor circulation (necrosis), migraines and headaches, traveling sickness, muscular soreness, and so on. Furthermore, the problem may be more realistic by including a radiation source like a laser.


