Energy and mass transmission through hybrid nanofluid flow passing over a spinning sphere with magnetic effect and heat source/sink

Abstract: Thermophoretic particle deposition (TPD) and thermal radiation have significant uses in engineering and research, such as projectiles, electrical fuel, and production of coating sheets, thermal transference, nuclear plants, renewable energy, aerospace engineering, and gas turbines. In light of the above applications, the present analysis examines the stagnation point flow of hybrid nanofluid (hnf) around a revolving sphere. The hnf is prepared with the addition of Cu and Al₂O₃ nanoparticles in the water. The flow is examined under the impact of chemical reaction, thermal radiation, TPD, and activation energy. The flow equations are reformed into a dimensionless set of ordinary differential equations and then solved through the numerical approach parametric continuation method. The graphical and numerical results are demonstrated through graphics and tables. It has been noted that the effects of acceleration and rotational parameters boost the hnf (Cu and Al₂O₃/water) velocity. Furthermore, the energy outline reduces with the effect of acceleration parameter and nanoparticle volume friction. The influence of the rotation factor and acceleration parameters boosts the rate of skin friction. The influence of thermal radiation enriches the energy transmission rate.

Keywords: rotating sphere, hybrid nanofluidics, thermophoretic particle deposition, activation energy, thermal radiation, PCM

Nomenclature

T_w  surface temperature (K)
q_r  thermal radiation
q''  irregular heat source/sink
A' & B' < 0  heat sink
A' & B' > 0  heat source
ρ  density (kg/m³)
φ_c Cu  copper nanoparticles
A  acceleration term
M  magnetic term
Rd  radiation factor
C_f  skin friction
f'(η)  axial velocity
1/T_r  reference temperature
Ω  sphere’s angular velocity
Cu  copper
C_w  surface concentration
V_T  thermophoretic velocity (m/s)
μ  viscosity (kg/m s)
K_c  thermophoretic constant
C_p  specific heat (J/kg K)
k  thermal diffusivity (W/mK)
(φ_c = φ_{Al₂O₃})  aluminum oxide nanoparticles
Sc  Schmidt number
τ  thermophoretic constraint
λ  rotation factor
Pr  Prandil number
Sh  Sherwood number
hnf  hybrid nanofluid
μ_d(x, t) = K_A  viscous dissipation
Al₂O₃  aluminum oxide
1 Introduction

Fluid dynamics has several implementations in the field of industry and engineering, i.e., the production of polymer sheet, plastic fabrication, glass fiber, transformer, and electronic circuits. In various sectors, such as power electronics manufacture of polymers and grid stations, nanofluids with improved thermal conductivity are efficiently used due to the practical implication in thermal transfer [1]. Nanofluid has received great attention from scientists over the years, leading to a deluge of discoveries and queries into its peculiar properties and potential applications. The flow of a magnetized nanoliquid in a cylindrical permeable sheet was computationally explored by Jalili et al. [2]. At higher concentrations of solid nanoparticles, the convection process declines. Nanofluid flow through a curved stretchy surface was calculated by Naveen Kumar et al. [3]. The findings reveal that the mass transmission slows down with the effect of Brownian diffusion. Khan et al. [4] inspected the blood-based nanofluids to improve thermal expansion with the radiation effect. Traciak et al. [5] assessed the extinction, refractive index, surface tension, and mass density of nanofluids with different mass fractions. The results demonstrate a linear relation between the nanoparticle fraction and the refractive index. Nanofluids were studied by Hossain et al. [6] for their potential uses in photovoltaic-thermal collectors. There are numerous applications for the hybrid nanoliquid flow (hnf) in the industry, including biotechnology, nuclear energy, crude oil, paper production, and polyethylene solution [7]. Varun Kumar et al. [8] have described the hybrid dusty nanoliquid flow in an expanding cylinder. The findings reveal that enhancing particle mass concentration lessens the velocity and energy of the fluid in the dusty phase while improving the curvilinear parameter the temperature gradient and velocity also increase. Entropy generation in magnetized hybrid nanomaterials flowing in a variable permeability region was discoursed by Hayat et al. [9]. Ramesh et al. [10] calculated the radiative electro-magneto-hydrodynamics TiO$_2$ based nanoliquid flow across an elongating surface. The heat performance of Williamson hnf flow was scrutinized by Hussian [11]. Gul et al. [12] computationally inspected the significance of the magnetic dipole of hnf flow. Due to its effective thermophysical behavior, thermal conduction is observed to be more effective in the hybrid nanoliquid than the basic nanoliquid. The variations in temperature and thermal conveyance with the ferromagnetic flow were calculated by Eid and Nafe [13]. Ullah et al. [14] estimated the radiative MHD dusty nanoliquid flow along a stretching surface with the nanotechnology applications. Salahuddin et al. [15] assessed the outcome of slip conditions with varying thicknesses on three-dimensional nanoliquid flow. Several researchers have recently presented remarkable results [16–21].

Researchers have focused on the topic of thermal transmission through nanofluids in recent years. The fundamental chemical structure of Cu and Al$_2$O$_3$ consists of aluminum nanoparticles. The advantages of these nanoparticles over other nanoparticles originate from their unique tube structure, tiny size, high surface area, chemical stability, and extreme hardness. Anuar et al. [22] considered the impact that MHD has on the two-dimensional flow that is constant of carbon nanotubes (CNTs) passing across a curvilinear surface. Gul et al. [12] and Tulu and Ibrahim [23] have calculated the hnf flow using distinct nanoparticles. The numerical calculation of the fractal non-integer order model of couple stress nanoliquid flow with cadmium telluride nano-particles was conducted by Murtaza et al. [24]. Using a water-based iron oxide and CNT hybrid nanoliquid, Bilal et al. [7] inspected the upshot of electrohydrodynamic on the nanoliquid flow across circling plates. Bilal et al. [25] documented the upshot of CNTs, Fe$_3$O$_4$, microorganisms, and water on a wavy fluctuating rotating disc. In a horizontal parallel channel, Bilal et al. [26] evaluated the flow of a magnetized hnf (Fe$_3$O$_4$-CNT/H$_2$O) and also considered the effects of heat radiation through the channel’s dilating and compressing porous walls. Lv et al. [27] investigated magnetic field, heat radiation, and a Hall effect that affected the surface of a spinning disc as an hnf flowed over it. The micro-polar liquid flow consists of CNTs with the effects of heat flux across a melting surface was estimated by Reddy and Kumar [28]. The Casson hnf flow with iron-oxide Darcy–Forchheimer was described by Khan et al. [29]. Alhussain and Tassaddiq [30] calculated the hnf flow confined of CNTs across a stretching surface. Some latest results may be found in previous studies [31–37].

Thermophoresis refers to the migration of atoms and molecules from a hotter to a colder environment. When microparticles in a non-isothermal gas migrate in the direction of lowering temperature gradients, an alarm is activated by the thermophoresis process. It is impossible to emphasize the implication of theoretical and practical insight on thermophoretic particle deposition (TPD). Air purifiers, building ventilation systems, powdered coal burners, and heat transformers are only a few applications of TPD. The effect of TPD on the laminar flow of a 2D Casson hnf through a nonlinearly elongating sheet was considered by Ramesh et al. [38]. According to the findings, a decrease in fluid velocity is noticed whenever the values for the solid nanoparticles and the permeable parametric quantities have improved. The applications of a laminar, 2D nanofluid flow with Newtonian heating mechanisms, TPD
and uniform thermal sink source were evaluated by Madhukesh et al. [39]. Bashir et al. [40] described the behavior of an electrically non-conducting two-dimensional Oldroyd-B fluid as it flowed through a stretching sheet with TPD. The outcomes demonstrate that the rate of thermophoretic deposition slows with increasing thermophoretic coefficient. In the existence of a porous media and bioconvection, Madhukesh et al. [41] considered the effect of TPD on the motion of a water-based micropolar nanoliquid sheet with the applications of TiO₂. The performance of nanoparticle aggregation through a revolving sphere with TPD was considered by Al Nuwairan et al. [42] and Shah et al. [43]. The outcomes indicate that mass distribution increases with thermophoretic parameters, and thermal transmission increases with solid volume fraction. Khan et al. [44] evaluated the effect of TPD on the energy transmission in a 2D wall jet that transports graphene oxide nanoparticles in a kerosene oil-based thermal fluid. The outcomes reveal that the concentration distribution declines as the thermophoretic factor is amplified.

On the basis of the above literature, so far, no attempt has been made on the stagnation point flow of hnf composed of aluminum alloys (Cu and Al₂O₃) flowing around a revolving sphere. The flow is reported under the influence of thermal radiation, chemical reaction, and TPD. The flow mechanism is modeled in the form of the system of PDEs, which are first reduced to a non-dimensional system of ordinary differential equations (ODEs) and then solved through the numerical parametric continuation method (PCM).

2 Mathematical formulation

The unsteady incompressible stagnation point flow of hnf composed of nanoparticles (Cu and Al₂O₃) flowing around a revolving sphere is studied. This x-coordinate is on the surface of the sphere, while the y-axis is perpendicular to the sphere surface, as revealed in Figure 1. The sphere's angular velocity is mathematically expressed as \( \dot{\Omega}(t) = \frac{B}{t}, \quad B > 0 \). At time \( t = 0 \), the sphere has no angular velocity in the fluid. It is assumed that the temperature near the sphere surface is constant. Furthermore, viscous dissipation is defined as \( \varphi_e(x, t) = \frac{x}{t}, \quad A > 0 \), where the angular and free stream velocity are time dependent. The surface temperature and concentration is signified by \( T_w \) and \( C_w \). The modeled equations are formulated as follows [46–48]:

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} = 0, \quad (1)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{\text{hnf}} \frac{\partial^2 u}{\partial y^2} + \frac{u^2}{r} \frac{\partial u}{\partial x} + \frac{\partial u_e}{\partial x}
\]

\[
\frac{\partial w}{\partial x} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \nu_{\text{hnf}} \frac{\partial^2 w}{\partial y^2} - \frac{u}{r} \frac{\partial w}{\partial x} \quad (2)
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{-1}{(\rho C_p)_{\text{hnf}}} \frac{\partial q_r}{\partial y} + \frac{k_{\text{hnf}}}{(\rho C_p)_{\text{hnf}}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{\text{hnf}}} q^r, \quad (3)
\]

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial y} \left( V_T (C - C_w) \right) - K_c (C - C_b) \left( \frac{T}{T_w} \right)^\eta \exp \left( - \frac{E_a}{kT} \right) \quad (4)
\]

Initial conditions:

\[
v = v_w, \quad u = u_w, \quad w = w_w, \quad T = T_w, \quad C = C_w \quad \text{at} \quad t = 0. \quad (6)
\]

Boundary conditions [46–48]:

\[
\begin{align*}
u & = 0, \quad v = 0, \quad Q(t) = \frac{B}{t}, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \\
w & \rightarrow 0, \quad u \rightarrow -Ax, \quad C \rightarrow C_w, \quad T \rightarrow T_w, \quad \text{as} \quad y \rightarrow \infty. \quad (7)
\end{align*}
\]

Thermal radiation \( q_r \) and thermophoretic velocity \( V_T \) are mathematically expressed as follows [48]:

\[
q_r = -\frac{16}{3} \left( \frac{T^2 \sigma}{k^r} \right) \frac{\partial T}{\partial y}, \quad V_T = -\frac{\nu K_c}{T} \frac{\partial T}{\partial y}. \quad (8)
\]
In equation (8), \( \frac{1}{\ell} \), \( K_{\ell}^* \), and \( q_r \) are the reference temperature, thermophoretic constant, and radiation term. By setting equation (8) in equation (4):

\[
\frac{\partial T}{\partial t} + \frac{1}{\ell} \left[ \frac{\partial T}{\partial x} + \sqrt{\frac{\partial T}{\partial y}} \right] = \left[ \frac{16(\sigma T^2)}{3k^*(\rho C_p)_{\text{htf}}} + \frac{k_{\text{htf}}}{(\rho C_p)_{\text{htf}}} \right] \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{\text{htf}}} q'''.
\]

(9)

The irregular heat source/sink, \( q''' \), is stated as follows [18]:

\[
q''' = k_{\text{htf}}^* \left( A' (T_{\text{w}} - T_{\text{f}}) + B' (T - T_{\text{w}}) \right),
\]

(10)

where \( B' \) and \( A' \) are the heat source/sink and space factors. The case \( A' < 0 \), \( B' < 0 \) presents the heat sink and \( A' > 0 \), \( B' > 0 \) shows the heat source.

Here, the similarity variables are [47]

\[
r = x', \ \eta = (v(t)^{-1} y'), \ \frac{\partial \eta}{\partial x} = 1, \ \frac{\partial \eta}{\partial t} = \frac{B_{\ell} x}{t} g(\eta),
\]

(11)

\[
\Theta(\eta) = \frac{T - T_{\text{w}}}{T_{\text{w}} - T_{\text{f}}}, \ \psi = A x (v(t)^{-1} f(\eta)), \ \Phi(\eta) = \frac{C - C_{\text{w}}}{C_{\text{w}} - C_{\text{f}}}
\]

The mathematical model for the water-based hnf (\( \phi_1 = \phi_{\text{Cu}} \), \( \phi_2 = \phi_{\text{Al}_2\text{O}_3} \)) is expressed as follows [50]:

**Viscosity:**

\[
\mu_{\text{htf}} = \frac{\mu_1}{(1 - \phi_1)^{2/5}(1 - \phi_2)^{2/5}}.
\]

**Density:**

\[
\rho_{\text{htf}} = [(1 - \phi_2)(1 - \phi_1)\rho_1 + \phi_1 \rho_2], \ \Phi_{\text{htf}} = [\phi_1 \rho_2].
\]

**Thermal capacity:**

\[(\rho C_p)_{\text{htf}} = \phi_2 (\rho C_p)_2 + [(1 - \phi_2)(\rho C_p)_1 + \phi_1 (\rho C_p)_2](1 - \phi_2).
\]

**Thermal conductivity:**

\[
k_{\text{htf}} = \frac{k_2 + k_1(n - 1) - \phi_2(k_2 - k_1)(n - 1)}{k_2 + k_1(n - 1) - \phi_2(k_2 - k_1)(n - 1)} , \ \text{where} \ \frac{k_1}{k_2} = \frac{k_2 + k_1(n - 1) - \phi_2(k_2 - k_1)(n - 1)}{k_2 + k_1(n - 1) - \phi_2(k_2 - k_1)(n - 1)}.
\]

**Electrical conductivity:**

\[
\frac{\sigma_{\text{htf}}}{\sigma_2} = \frac{\sigma_2 + 2\sigma_1 - 2\phi_2(\sigma_2 - \sigma_1)\phi_2}{\sigma_2 + 2\sigma_1 - 2\phi_2(\sigma_2 - \sigma_1)}, \ \text{where} \ \frac{\sigma_1}{\sigma_2} = \frac{\sigma_1 + 2\sigma_1 - 2\phi_2(\sigma_1 - \sigma_1)\phi_2}{\sigma_1 + 2\sigma_1 - 2\phi_2(\sigma_1 - \sigma_1)\phi_2}.
\]

Table 1 shows the numerical values used for the validation during simulation and model for the hnf flow problem.

Using equation (11), we obtain

\[
f''' + \sigma_1 \sigma_2 \left( f' - M f''^2 + f'' \left( \frac{n}{2} \right) \right) + \sigma (\lambda g^2 + 1) - 1 = 0,
\]

(12)

\[-M f' = 0,\]

(13)

\[
g''' + \sigma_1 \sigma_2 \left[ g(1 - 2\sigma f') + g' \left( \frac{n}{2} + f\sigma \right) \right] = 0,
\]

(14)

\[
\frac{1}{\sigma_3} \left( k_{\text{htf}} + \frac{4}{3} R_d \right) \phi''' + \rho \left( \frac{n}{2} + f\sigma \right) + \left( \lambda_1 f' + \lambda_2 \phi' \right) = 0,
\]

(15)

The boundary conditions for the system of ODEs are

\[
f(\eta) = 0, \ f'(\eta) = 0, \ g(\eta) = 1, \ \Phi(\eta) = 1, \ \Theta(\eta) = 1, \ \text{at} \ \eta = 0,
\]

(16)

\[
f'(\eta) = 1, \ g'(\eta) = 0, \ \Phi(\eta) = 0, \ \Theta(\eta) = 0 \ \text{as} \ \eta \to \infty.
\]

Here,

\[
\sigma_1 = (1 - \phi_{\text{Cu}})^{2/5}(1 - \phi_{\text{Al}_2\text{O}_3})^{2/5} + \rho_{\text{Cu}} \rho_{\text{Al}_2\text{O}_3}, \ \sigma_2 = (1 - \phi_{\text{Al}_2\text{O}_3}) + \phi_{\text{Cu}} \rho_{\text{Al}_2\text{O}_3},
\]

(17)

\[
\sigma_3 = (1 - \phi_{\text{Al}_2\text{O}_3}) + \phi_{\text{Cu}} \rho_{\text{Cu}} \rho_{\text{Al}_2\text{O}_3}.
\]

\[
\lambda_1 = \frac{2}{\ell_1} \sigma_1, \ \lambda_2 = \frac{2}{\ell_2} \sigma_2, \ \tau = \frac{K_{\ell}(T_{\text{w}} - T_{\text{f}})}{r}, \ \text{Rd} = \frac{4\sigma_1 g^2}{K_{\ell} r}.
\]

\[
\text{Pr} = \left( \frac{\rho_{\text{Cu}}}{\ell_1} \right), \ \text{is} \ \text{the} \ \text{Prandtl} \ \text{number}.
\]

The physical quantities are

\[
Cl = \frac{\mu_{\text{htf}}}{u_0} \left( \frac{\partial u}{\partial y} \right) \bigg|_{y=0} = \left( \tfrac{\Re}{\ell_1} \right) Cl = \frac{f''(0)}{\sqrt{\ell_1}}, \ \text{(17)}
\]

\[
Cl = \frac{\mu_{\text{htf}}}{u_0} \left( \frac{\partial u}{\partial y} \right) \bigg|_{y=0} = \left( \tfrac{\Re}{\ell_1} \right) Cl = \frac{-g(0) \lambda_1^2}{\sqrt{\ell_1}}.
\]
Table 2: Relative assessment for the validation of the present results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Malvandi [46]</th>
<th>Ramesh et al. [54]</th>
<th>Present work</th>
<th>Malvandi [46]</th>
<th>Ramesh et al. [54]</th>
<th>Present work</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>$f''(0)$</td>
<td>$f''(0)$</td>
<td>$f''(0)$</td>
<td>$-\theta'(0)$</td>
<td>$-\theta'(0)$</td>
<td>$-\theta'(0)$</td>
</tr>
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</tr>
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<td>1.91845</td>
<td>1.918476</td>
<td>0.779526</td>
<td>0.779538</td>
<td>0.7795399</td>
</tr>
</tbody>
</table>

$$\begin{align*}
\text{Nu} &= \frac{-x\left(k_{\text{hf}} + \frac{16 \sigma^2 \tau^2}{3 \lambda^2}ight)\left(\frac{\partial f}{\partial y}\right)_{y=0}}{k_c(T_w - T_n)} \to (\text{Re})^{-\frac{1}{2}} \\
\text{Sh} &= \frac{-xD_c}{D_f(C_w - C_m)} \left(\frac{C}{\partial T}\right)_{y=0} \to (\text{Re})^{-\frac{1}{2}} \sqrt{\alpha} \Phi(0),
\end{align*}$$

(18)

3 Numerical solution and problem validation

Researchers frequently encounter challenging strong nonlinear boundary value problems (BVPs) in the real-world challenges of engineering. Other forms of numerical procedures, like RK4, the Newton-Raphson method, and bvp4c, are sensitive to initial assumptions. This method’s goal is to

Figure 2: (a) Acceleration parameter ($A$), (b) aluminum alloy nanoparticles ($\phi_1 = \phi_2$), (c) rotational parameter ($\lambda$), and (d) acceleration factor ($A$) versus the velocity curve ($f'(\eta)$, $g(\eta)$). By taking $\lambda = 0.5$, $Rd = 0.1$, $\phi_1 = \phi_2 = 0.01$, $Sc = 0.2$, $Kc = 0.5$, and $Ea = 1.0$. 

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solve high-order nonlinear BVPs with little computational effort [51]. The basic methodology of PCM approach is as follows [52,53]:

**Step 1:**

\[
\begin{align*}
\begin{cases}
    f(\eta) = \partial_1(\eta), & f'(\eta) = \partial_2(\eta), & f''(\eta) = \partial_3(\eta), \\
    g(\eta) = \partial_4(\eta), & g'(\eta) = \partial_5(\eta), \\
    \theta(\eta) = \partial_6(\eta), & \Theta'(\eta) = \partial_7(\eta), \\
    \Phi(\eta) = \partial_8(\eta), & \Phi'(\eta) = \partial_9(\eta)
\end{cases}
\end{align*}
\]

(19)

By employing equation (19) in equations (12)–(15), we obtain

\[
\begin{align*}
\partial_3'(\eta) &= \tau_1 \tau_2 \left[ \partial_2(\eta) - \nabla(\partial_2(\eta))^2 + \left( \frac{\eta}{2} \right) \partial_3(\eta) ight] \\
&\quad + \tau(\lambda(\partial_3(\eta))^2 + 1) - 1 = 0, \\
\partial_5'(\eta) &= \tau_1 \tau_2 \left[ \partial_4(\eta)(1 - 2\tau\partial_2(\eta)) ight] \\
&\quad + \partial_3(\eta) \left[ \nabla^2 + \nabla \partial_1(\eta) \right] = 0,
\end{align*}
\]

(20)

(21)

with the corresponding boundary conditions

\[
\begin{align*}
\partial_1(\eta) &= 0, \quad \partial_2(\eta) = 0, \quad \partial_3(\eta) = \partial_4(\eta) = \partial_5(\eta) = 1, \\
\text{at } \eta &= 0, \\
\partial_6(\eta) &= 1, \quad \partial_7(\eta) = \partial_8(\eta) = \partial_9(\eta) = 0, \quad \text{as } \eta \to \infty
\end{align*}
\]

(24)

**Step 2: Familiarizing parameter p in equations (20)–(23):**

\[
\begin{align*}
\partial_3'(\eta) &= \tau_1 \tau_2 \left[ \partial_2(\eta) - \nabla(\partial_2(\eta))^2 + \left( \frac{\eta}{2} \right) \left( \partial_3(\eta) - 1 \right) p \\
&\quad + 1 + \tau(\lambda(\partial_3(\eta))^2 + 1) - 1 \right] = 0,
\end{align*}
\]

(25)

Figure 3: (a) Acceleration factor \(A\), (b) thermal radiation \(Rd\), and (c) aluminum alloy nanoparticles \(\phi_1 = \phi_2\) versus the energy curve \(\theta(\eta)\). By taking \(\lambda = 0.5, \phi_1 = \phi_2 = 0.01, \text{Rd} = 0.1, \text{Sc} = 0.2, \text{Kc} = 0.5\), and \(\text{Ea} = 1.0\).
\[ \partial_t \eta + \nabla \cdot (\eta \nabla \eta) = 0, \]

where \( \partial_t \) is the material derivative, \( \eta \) is the velocity, and \( \nabla \) is the gradient operator. This equation is subject to the boundary conditions:

1. \( \eta = 0 \) at the wall,
2. \( \partial \eta / \partial n = 0 \) at the sphere.

4 Results and discussion

An incompressible stagnation point flow of hnf composed of nanoparticles (Cu and Al₂O₃) flowing around a revolving sphere is studied.

Figure 2(a)-(d) show the nature of the velocity curve against the acceleration factor (A), aluminum alloys nanoparticles (\( \phi_1 = \phi_2 \)), and rotational factor (\( \lambda \)), respectively. It can be detected that the effects of acceleration parameter and rotational factor boost the hybrid nanoliquid (Cu–Al₂O₃/water) as well as Al₂O₃/water-based nanoliquid velocity. The effects of both constraints (acceleration parameter and rotational parameter) hasten the fluid particles adjusting to the sphere surface, which causes the fast rotation of the fluid velocity \( f'(\eta) \). as shown in Figure 2(a) and (c). Figure 2(b) reports that the velocity curve \( f'(\eta) \) falls with the varying numbers of Cu and Al₂O₃ NPs. The density and thermal diffusivity of Cu and Al₂O₃ nanoparticulates are greater as compared to water. Hence, the dispersion of Cu and Al₂O₃...
nanoparticles to water declines the fluid velocity $f'(\eta)$. Figure 2(d) illustrates the application of acceleration factor on the secondary velocity $g(\eta)$. It can be perceived that the velocity $g(\eta)$ field drops with the upshot of $A$.

Figure 3(a)–(c) shows the nature of the velocity curve against the acceleration parameter ($A$), thermal radiation ($R_d$), and aluminum alloy nanoparticles ($\phi_1 = \phi_2$). Figure 3(a) and (c) shows that the energy outline reduces with the effect of acceleration term and nanoparticle volume friction. Physically, the density and thermal diffusivity of Cu and Al$_2$O$_3$ nanoparticles are higher than water. Hence, the dispersion of Cu and Al$_2$O$_3$ nanoparticles to water absorbs heat and drops the temperature $\theta(\eta)$. This property of hybrid nanocomposites using Cu and Al$_2$O$_3$ nanoparticles has remarkable applications in modern technologies, as discussed in Section 1. Figure 3(b) shows that the effect of $R_d$ enriches the temperature field $\theta(\eta)$. Thermal radiation is a sort of radiation that a material emits as a result of its heat. Therefore, the effect of radiation improves the temperature curve of the hybrid nanoliquid.

Figure 4(a)–(c) illustrates the nature of concentration curve $\phi(\eta)$ against the varying values of ($Sc$, $K_c$), and activation energy ($E_a$), respectively. It can be noticed that the mass transmission profile drops with the effect of $Sc$ and $K_c$. Actually, the mass dispersion rate is contrariwise related to the $Sc$, while the kinetic viscosity is directly proportional to the $Sc$; hence, the variation of $Sc$ declines the mass diffusion ratio as shown in Figure 4(a). Chemical conversion occurs as a result of $K_c$. Here, the influence of $K_c$ (chemical reaction) also reduces the mass...
dissemination rate as shown in Figure 4(b). The significances of Arrhenius energy have an opposite effect on the mass curve. The Arrhenius equation, which was presented by Svante Arrhenius in 1889, has important and vast applications in determining the rate of the energy of activation and chemical reactions. Physically, $E_a$ activates the fluid molecules and accelerates the mass transition ratio as shown in Figure 4(c). Figure 5 presents the error analysis (AE) for the proposed method and bvp4c package. Figure 5(a)–(c) demonstrates that the average AE is about $10^{-10}$–$10^{-2}$. Figure 6 exposes the streamlines around a freely spinning sphere.

Table 3 illustrates the numerical outcomes for $(C_p$, $C_p)$, Nusselt number (Nu), and Sherwood number (Sh). It can be noticed that the influence of the rotation constraint and acceleration parameters enhances the rate of skin friction, whereas the upshot of thermal radiation boosts the energy transmission rate.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Physical interest quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>$C_{pK}$</td>
</tr>
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</tr>
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Figure 6: Streamlines around a freely spinning sphere.
5 Conclusions

We have analyzed the implications of TPD and thermal radiation on the stagnation point flow of hnf flowing around a gyrating sphere. The hnf is prepared with the addition of Cu and Al₂O₃ NPs in the water. The flow is scrutinized under the effect of chemical reaction, TPD, and thermal radiation. The main conclusions are as follows:

- The effects of acceleration parameter and rotational factor boost the hybrid nanoliquid (Cu and Al₂O₃/water) and Al₂O₃/water-based nanoliquid velocity.
- The velocity curve $f(\eta)$ falls with the varying numbers of nanoparticles (Cu and Al₂O₃) in the water.
- The effect of the acceleration factor shrinks the secondary velocity $g(\eta)$.
- The energy outline reduces with the effect of acceleration term and nanoparticles’ volume friction while enhances with the upshot of thermal radiation.
- The mass transfer profile drops with the variation of Sc and Kc. In contrast, the significances of Arrhenius energy have an opposite effect on the mass curve.
- The influence of the rotation factor and acceleration parameters boosts the rate of skin friction, whereas the effect of thermal radiation boosts the energy transmission rate.
- The present model may be modified by considering different physical effects and different boundary conditions.

Acknowledgments: The authors acknowledge the support and funding by the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University (IMSIU) (grant number IMSIU-RP23059).

Funding information: This work was supported and funded by the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University (IMSIU) (grant number IMSIU-RP23059).

Author contributions: All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

Conflict of interest: The authors state no conflict of interest.

Data availability statement: All data generated or analyzed during this study are included in this published article.

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