Research Article

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Optimal design of the flywheel using nature inspired optimization algorithms

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Abstract: This paper presents the optimal design procedure of a flywheel using a cubic B-spline curve. The flywheel plays a vital role in storing kinetic energy in modern machines. The kinetic energy evaluates the performance of the flywheel. In order to improve the kinetic energy of a flywheel, a shape optimization model of the flywheel, with maximization of kinetic energy, is formulated using a cubic spline curve under the constraints of the mass of flywheel, and the maximum value of Von Mises stresses at all points along the radial direction. The Von Mises stresses at all points are determined using the two-point boundary value differential equation. The control points of the cubic B-spline curve are taken as design variables. Then the formulated problem is solved using particle swarm algorithm (PSO), genetic algorithm (GA), and Jaya algorithm. The proposed approach is applied to the flywheel of an agricultural thresher machine. It is found that the Jaya algorithm gives better results compared to the other algorithms. The optimized shape of the flywheel is simulated using MSC ADAMS software.

Keywords: Flywheel; Kinetic energy; Jaya algorithm; Von-mises stresses; Cubic B-spline

1 Introduction

The flywheel is an energy storage device and helps to smooth the torque fluctuations by storing and releasing energy. Generally, it is used in many applications such as internal combustion engines, punch presses, agricultural thresher machines, vehicle power station, aerospace, etc. due to its high efficiency, high cyclic lifetime, easy

maintenance, and low environmental pollution (Norton 2011). However, the performance of the flywheel can be increased by increasing its energy storing capacity. The energy storing capacity of the flywheel can be determined by its material property, rotating speed, and geometry. A material with high density can significantly increase the flywheel mass and thus increase its energy storing capacity.

Further, the high strength material can withstand more stress. As a result, the flywheel can rotate at higher speed and store more kinetic energy. Unfortunately, the number of these materials that can be chosen from for most designs is limited due to their availability. Moreover, a large mass flywheel is restricted by the available space in the machine and can also transfer large gravitational forces to the bearings. Fortunately, the geometry of a flywheel can also influence the storage capacity of kinetic energy for the same flywheel mass. Composite materials (Danfelt et al. 1977; Metwalli et al. 1983; Takahashi et al. 2001; Tzeng et al. 2006) are widely used to achieve long life and high performance in the design of the geometry of a flywheel. But, the cost of composite materials is high compared to metallic materials and the forming process is also relatively complex. Therefore, metallic materials are used to design for the various geometries of flywheels due to its low price and a simple manufacturing process.

Recently, most studies have been focused on shape optimization of the flywheel rotor cross-section. It is an efficient method to increase the kinetic energy by optimizing the rotor thickness along a radial direction (Jiang and Wu 2017). To achieve optimal thickness distribution along the radial direction, the shape optimization problem with objectives of minimizing the volume, maximizing the kinetic energy, and minimizing the stresses has been formulated under suitable design constraints using polynomial expansion. The coefficients of polynomial expansion of thickness are taken as design variables. Further, the formulated problem is solved using conventional optimization technique (Sandgren and Ragsdell 1983).

In another study, coefficients of a Fourier series and Fourier sine series are considered as design variables.

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The ratio of inertia to volume and kinetic energy are considered as objective functions with constraints of the maximum allowable stress, maximum thickness and maximum mass for optimal shape of the flywheel. These series represent the thickness as a function of the radius. However, the radial and tangential stresses of the flywheel are computed using a two-point boundary value differential equation (Ebrahimi 1988; Ghotbi and Dhingra 2012). The exact optimal shape is determined under the constraints of the geometry, which is derived from arbitrary design parameters, rotational speeds and the strength of disk, using discrete optimization (Berger and Porat 1988). An injection island genetic algorithm is used to search for the optimal shape of the flywheel with the maximum kinetic energy (Eby et al. 1999). However, the shape of the flywheel is optimized for different kinds of heterogeneous materials using conventional optimization techniques (Huang and Fadel 2000). Further, a nonlinear optimization problem is formulated with the objective to maximize the energy density (stored energy per unit mass) using the parametric geometry modeling method. Then, the downhill simplex method has been used to solve this problem (Jiang et al. 2016).

Unfortunately, the polynomial expansion, Fourier series, and Fourier sine series are limited by the number of coefficients and do not describe the degree of the resulting curve. However, Bezier curves describe the degree of the curve, but these curves are controlled globally and are limited by control points. Moreover, the optimal shape of the flywheel is determined using conventional optimization techniques and finite element analysis. But, these methods increase the computational cost and are also less efficient.

In this paper, the shape optimization model of the flywheel is formulated using a cubic B-spline curve. The B-Spline curves have local control and are not limited by control points. Then, the optimization problem is solved by GA, PSO, and Jaya algorithm. The effectiveness of the approach is illustrated by the flywheel design of an agricultural thresher machine. It is found that the Jaya algorithm gives better results than the other algorithms and stores more energy compared to that of original flywheel.

2 Material and Methods

In this section, the profile of the flywheel thickness is represented by the cubic B-spline curve as shown in Figure 1. R_1 and R_2 are the inner and outer radius of flywheel respectively. The flywheel is axial symmetric in

X and Y directions and thickness distribution along the radial direction is represented by cubic B-splines curve. The control points describe the vertices of the polygon of the B-spline curve. A set of control points as $p_1, p_2, \dots, p_n$ approximates or interpolates the curve as defined in Eq. (1) (Mortenson 2006; Zeid 1991). X and Y coordinates of control points are taken as radius and thickness of flywheel, respectively.

Figure 1: 2D symmetric profile of flywheel

Moreover, a composite sequence of curve segments ($S=n-k+1$) defines the cubic B-spline curve that connected with C^2 continuity. Further, it blends two curve segments with the same curvature as shown in Figure 2.

Figure 2: B-spline curve segments

$$p(u) = \sum_{i=1}^n N_{i,k}(u) p_i, \quad 0 \leq u \leq S \quad (1)$$

Where parameter k controls the degree of curve and the value of k for a cubic B-spline curve is 4. $N_{i,k}(u)$'s are B-spline blending functions, which are defined by the following expression;

$$N_{i,k} = (u - u_i) \frac{N_{i,k-1}(u)}{u_{i+k-1} - u_i} + (u_{i+k} - u) \frac{N_{i+1,k-1}(u)}{u_{i+k} - u_{i+1}} \quad (2)$$

Where

$$N_{i,1} = \begin{cases} 1, & u_i \leq u \leq u_{i+1} \\ 0, & \text{elsewhere} \end{cases} \quad (3)$$

Where, $N_{i,j}$ is a unit step function and are called the parametric knots. The values of u_i depend on whether the B-spline curve is an open (non-periodic) or periodic curve. The periodic B-spline curve does not pass through any of control points, while a non-periodic B-spline curve passes through the two end points. The knot values for periodic and non-periodic B-spline curves are given by:

$$\text{For periodic B-spline curve } u_i = i - 1, i - 1 \leq S \quad (4)$$

For non-periodic B-spline

$$u_i = \begin{cases} 0, & i < k \\ i - k + 1, & k \leq i < n \\ n - k + 1, & i > n \end{cases} \quad (5)$$

Where $1 \leq i \leq n + k$

The coordinates of any point on the i^{th} segment of the curve for periodic B-spline curves are determined using Eqs.(1-4) and are given as:

$$r^i(u) = \frac{a_1 r_i + a_2 r_{i+1} + a_3 r_{i+2} + a_4 r_{i+3}}{6} \quad (6)$$

$$t^i(u) = \frac{a_1 t_i + a_2 t_{i+1} + a_3 t_{i+2} + a_4 t_{i+3}}{6} \quad (7)$$

Where

$$\left. \begin{aligned} a_1 &= -u^3 + 3u^2i - 3ui^2 + u^3 \\ a_2 &= 3u^3 + u^2(3 - 9i) + u(9i^2 - 6i - 3) - 3i^3 + 3i^2 + 3i + 1 \\ a_3 &= -3u^3 + u^2(9i - 6) + u(-9i^2 + 12i) + 3i^3 - 6i^2 + 4 \\ a_4 &= u^3 + u^2(-3i + 3) + u(3i^2 - 6i + 3) - i^3 + 3i^2 - 3i + 4 \end{aligned} \right\} i - 1 \leq u \leq i \quad (8)$$

The coordinates of any point on the i^{th} segments of the curve for non-periodic B-spline curves are determined using Eqs. 1-3 and 5 and are given as:

$$r^i(u) = a_1^i r_i + a_2^i r_{i+1} + a_3^i r_{i+2} + a_4^i r_{i+3} \quad (9)$$

$$t^i(u) = a_1^i t_i + a_2^i t_{i+1} + a_3^i t_{i+2} + a_4^i t_{i+3} \quad (10)$$

Where, a_1^i, a_2^i, a_3^i are a_i^i obtained by solving Eqs. (2), (3) and (5) for each segment in MAT LAB.

2.1 Parameters of Flywheel

The volume, mass, and kinetic energy are the parameters of the flywheel, which also describe the performance of flywheel. These parameters, for each segment, are calculated using the expression of $r^i(u)$ and $t^i(u)$ as

$$v^i = 2\pi\rho \int_{u_{i-1}}^{u_i} t^i(u) r^i(u) \frac{dr^i(u)}{du} du \quad (11)$$

$$m^i = 2\pi\rho \int_{u_{i-1}}^{u_i} t^i(u) r^i(u) \frac{dr^i(u)}{du} du \quad (12)$$

$$e_K^i = \pi\rho\omega^2 \int_{u_{i-1}}^{u_i} t^i(u) (r^i(u))^3 \frac{dr^i(u)}{du} du \quad (13)$$

Where ρ is the density of the material and ω represents the angular velocity of the flywheel.

Further, the whole volume, mass, and kinetic energy of flywheel are calculated by the summing of volume, mass, and kinetic energy of each segment, respectively as

$$V = \sum_{i=1}^{n-k+1} v^i \quad (14)$$

$$M = \sum_{i=1}^{n-k+1} m^i \quad (15)$$

$$E_K = \sum_{i=1}^{n-k+1} e_K^i \quad (16)$$

2.2 Stress analysis of flywheel

A nonperiodic uniform B-spline cubic curve is used for stress analysis of the flywheel. It passes through the end points. Stresses at the inner and outer radii are calculated using this curve. While a periodic uniform B-spline cubic curve does not pass through the endpoints, it does not give information about stresses at an inner and outer radius of the flywheel.

However, tangential and radial stresses occur due to centrifugal forces during the operation of the flywheel. The following relationship between tangential and radial stresses for each segment has been obtained, based on the force balance on each small element of the flywheel (Timoshenko and Goodier 1970).

$$\frac{d}{dr^i(u)} (t^i(u) r^i(u) \sigma_r^i) - t^i(u) \sigma_\theta^i + \rho (r^i(u))^2 t^i(u) \omega^2 = 0 \quad (17)$$

$$(\sigma_\theta^i - \sigma_r^i)(1 + \nu) + r \frac{d\sigma_\theta^i}{dr^i(u)} - r^i(u) \nu \frac{d\sigma_r^i}{dr^i(u)} = 0 \quad (18)$$

Where σ_r^i and σ_θ^i are tangential and radial stresses for each segment respectively. $t^i(u)$ and $r^i(u)$ are thickness and

radius of the flywheel F^i for each segment given in Eqs. (9) and (10).

Let us define a stress function for each segment as

$$F^i = t^i(u)r^i(u)\sigma_r^i \quad (19)$$

$$\sigma_r^i = \frac{F^i}{t^i(u)r^i(u)} \quad (20)$$

Substituting Eq. (20) in Eq. (17) and solving for σ_θ as

$$\sigma_\theta^i = \frac{1}{t^i(u)} \left(\frac{dF^i}{dr^i(u)} + \rho(r^i(u))^2 \omega^2 t^i(u) \right) \quad (21)$$

A second ordinary differential equation is obtained by solving the Eqs. (18), (20) and (21) as:

$$r^2 \frac{d^2 F}{dr^2} + r \frac{dF}{dr} - F + (3+v)\rho r^3 \omega^2 t - \frac{r}{t} \frac{dt}{dr} \left(r \frac{dF}{dr} - vF \right) = 0 \quad (22)$$

Eqs. (21) and (22) are written in parametric form for each segment by converting the independent variable r into u by using chain rule of differentiation as

$$\sigma_\theta^i = \frac{1}{t^i(u)} \left(\frac{dF^i}{du} + \rho(r^i(u))^2 \omega^2 t^i(u) \right) \quad (23)$$

$$\begin{aligned} & (r^i(u))^2 \frac{dr^i(u)}{du} \frac{d^2 F^i}{du^2} \\ & + \left\{ r^i(u) \left(\frac{dr^i(u)}{du} \right)^2 - (r^i(u))^2 \frac{d^2 r^i(u)}{du^2} - \frac{(r^i(u))^2}{t^i(u)} \frac{dr^i(u)}{du} \frac{dt^i(u)}{du} \right\} \frac{dF^i}{du} \\ & + \left\{ v \frac{r^i(u)}{t^i(u)} \frac{dt^i(u)}{du} \left(\frac{dr^i(u)}{du} \right)^2 - \left(\frac{dr^i(u)}{du} \right)^3 \right\} F^i \\ & + (3+v)\rho \omega^2 t^i(u) (r^i(u))^3 \left(\frac{dr^i(u)}{du} \right)^3 = 0 \end{aligned} \quad (24)$$

Eq. (24) is a second order differential equation with the independent variable 'u' and dependent variable 'F'. In the case of flywheel design, two boundary conditions like radial stresses being zero at the inner and outer radius are usually known. Then, Equation (24) becomes a two-point boundary value problem with a second order differential equation. It can be solve using the finite difference method and the shooting method (Ahsan and Farrukh 2013; Kharab and Guenther 2012). Furthermore, a MATLAB inbuilt function as ode45 is directly used to solve this equation.

Once radial and tangential stresses are known at each point of each segment, the Von Mises stresses at each point are calculated using the application of distortion energy theory as

$$\sigma_t = (\sigma_r^2 + \sigma_\theta^2 - \sigma_r \sigma_\theta)^{1/2} \quad (25)$$

2.3 Formulation of the optimization problem

A nonlinear constrained optimization problem for optimal distribution of the thickness, along the radial direction, is formulated using the cubic B-spline. The design parameters of the flywheel are shown in Figure 1. OXY represents the global coordinate system. The coordinates of control points $r_1, r_2, \dots, r_n$ in X and $t_1, t_2, \dots, t_n$ in Y direction are radius and the thickness of flywheel, respectively.

2.4 Design variables

The radius of flywheel between R_1 to R_2 is equally divided. Hence, the X- coordinates of control points are fixed and given as $r_i = r_1 + \frac{(r_2 - r_1)(i-1)}{n-1}$ where $i = 1, \dots, n$

Furthermore, the Y-coordinates as thicknesses are taken as the design variables and are expressed in the vector form as:

$$\mathbf{x} = [t_1 \ t_2 \ t_3 \ \dots \ t_n]^T \quad (26)$$

Where n represents the control points or design variables between R_1 to R_2 .

2.5 Objective function and constraints

In general, the flywheel should store as much kinetic energy as possible for its better performance. However, the kinetic energy stored in the flywheel is determined by the shape of the flywheel. Hence, an optimal shape of the flywheel improves its energy storing capacity. Therefore, the optimization problem with maximization of the kinetic energy is formulated under the constraints of the mass of the flywheel and the maximum value of Von Mises stresses at all points of each segment. Although, the kinetic energy of the flywheel is determined by using Eq. (16).

Moreover, the mass of the flywheel should be less than the maximum mass (M_{max}) of the flywheel. Mass of flywheel is calculated using Eq. (15). The mass constraint is define as

$$g_1(\mathbf{x}) = M \leq M_{max} \quad (27)$$

Furthermore, the maximum value of Von Mises stress at all points of each segment in the radial direction should be less than the allowable stress (σ_a). The maximum value of Von Mises stress at all points of each segment in the radial direction is calculated using Eq. (25). The stress constraint is defines as

$$g_2(\mathbf{x}) = \max(\sigma_t) \leq \sigma_a \quad (28)$$

Finally, the optimization problem is posed as

$$\begin{aligned} \text{Minimize } f(\mathbf{x}) &= -E_K \\ \text{Subjected to } g_1(\mathbf{x}) &= M - M_{max} \leq 0 \\ g_2(\mathbf{x}) &= \max(\sigma_t) - \sigma_a \leq 0 \\ LB_i &\leq x_i \leq UB_i \quad i = 1, \dots, n \end{aligned} \quad (29)$$

The negative sign in the expression for the objective function implies that the function E_k is being maximized. LB_i and UB_i are the lower and upper bounds on the i^{th} design variable, and n represents the number of design variables.

To obtain an optimum solution, the constrained problem, as defined in Eq. (29), is converted into an unconstrained problem by using a penalty function (Singh et al. 2017). A significant penalty value is added to the objective function for each constraint violation. As a result, the objective function proceeds toward an infeasible solution. Hence, the global optimum solution is obtained by satisfying all the constraints using a suitable optimization algorithm. The original constrained optimization problem is then stated as an unconstrained optimization problem in which the first and second term describes the objective function and the penalty function respectively. Finally, the shape optimization problem of the flywheel is formulated as:

$$\text{Minimize } f(\mathbf{x}) = -E_K + \sum_{B=1}^2 C_B (CP)^B \quad (30)$$

upper and lower limit of variables

$$LB_i \leq x_i \leq UB_i \quad i=1, \dots, n \quad (31)$$

Where CP is the penalty value of the order of 10^6 which assign to objective function if the constraints are not satisfied and C_B is the defined as (Singh et al. 2017)

$$C_B = \begin{cases} 1 & \text{if } g_B(\mathbf{x}) \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

2.6 Optimization algorithm

Notations used in the Jaya algorithm are defined as follows:

p = population size

LB_i , UB_i = Lower and upper bounds for the i^{th} design variables

x_i = i^{th} design variable

x_{ij} = i^{th} design variable for j^{th} population

f_j = An objective function value of the j^{th} population (Eq. (30))

rand = Any random number in the range of 0 and 1

After formulating the optimization problem, it can be solved either by classical methods or nature inspired optimization algorithms. The traditional methods involve the computational complexity in calculating the gradient of the objective function correspond to the design variables. Moreover, these methods give only a local optimum solution (Arora 2004).

Nature inspired optimization algorithms have less computational complexity compared to classical methods. There are some nature-inspired optimization algorithms, such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Ant Colony Optimization (ACO). Moreover, these algorithms converge to a global minimum, but there is no guarantee of an optimal global solution. Some of the algorithms such as GA, PSO, and ACO have algorithmic control parameters, which affect the performance of algorithms (Rao and Savsani 2012). Furthermore, the Jaya algorithm is a parameter-less algorithm that has only one phase, unlike the two phases of the TLBO algorithm (Chaudhary and Chaudhary 2015; Rao 2015). It gives the optimal solution rapidly and updates the worst solution (Rao 2016; Venkata Rao and Waghmare 2016). A flow chart of Jaya algorithm for the shape of the flywheel is shown in Figure 3. Here, the stopping criterion for Jaya algorithm is considered as the number of iterations and number of function evaluations. The number of function evaluations depends on population size and the number of iterations. The number of function evaluations is equal to the product of the iteration numbers and population size. Therefore, design variables do not affect the function evaluations but may affect the computational time of the algorithm. The detailed procedure of GA and PSO applied in this study is explained by (Chaudhary and Chaudhary 2014) and (Pathak et al. 2017), respectively. However, the first time it is implemented for the optimal design of a flywheel is in this study.

Ethical approval: The conducted research is not related to either human or animal use.

3 Results and Discussions

The proposed optimization procedure is applied for the flywheel of an agricultural thresher machine. The thresher is an essential piece of agricultural machinery, used by farmers. It detaches the grains from harvested crops

Figure 3: A flow chart of Jaya algorithm for the shape of the flywheel

by a combination of impact and a rubbing action with minimum effort and time. It has many functional parts as shown in Figure 4., but the flywheel is an essential element of the thresher machine (Ghaly 1985; K. Madan Lal 2012, An Improved Multi Crop Thresher, Patent No. 253863). The function of the flywheel in a thresher machine is to minimize the variations in the speed of the PTO shaft due to torque fluctuations of the threshing drum by storing or releasing kinetic energy.

The material properties and design parameters of the flywheel are given in Tables 1 and 2 respectively (Varshney 2004). Design variables are equal to the control points that are divided between the inner and outer radius. The eight control points are taken for the shape of the flywheel in this study. Design variables are explained in Eq. (26). The upper and lower limits of the design variables are taken as

$$0.010 \leq x_i \leq 0.060 \quad \text{where } i = 1 \text{ to } 8$$

Von Mises stresses at each point, as expressed in Eq.25, are solved by taking a number of segments ($S=5$) and a step size (h) =0.1. A total of 51 points are taken between the inner radius and outer radius for this study.

The Jaya algorithm was coded in MATLAB. In order to compare the efficiency of the algorithm, GA, and PSO are also applied to the same flywheel design problem. All three algorithms were run for 100 iterations to find the best objective function value and corresponding design variables. GA, PSO, and Jaya give the best of the objective function as -28950, -32896.25, and -41624.8, respectively. The convergence plots of the best objective function values are shown in Figure 5. The optimum results, obtained from these algorithms, are compared with the original design (Varshney 2004) as shown in Table 3, and the best value of the objective function is shown in boldface. It was found that Jaya gives the better results compared to the results obtained from GA and PSO.

Figure 4: A cad model of flywheel

Table 1: Material properties of flywheel

Material	Density (kg/m ³)	Elastic modulus (Gpa)	Poisson's ratio
grey cast iron	7250	210	0.3

Table 2: Design parameters of the flywheel

Control points	Inner radius R_1 (m)	Outer radius R_2 (m)	Angular velocity ω (rad/sec)	M_{max} (kg)	σ_a (N/mm ²)
8	0.060	0.500	65.45	115	6.4

The optimized shape of the flywheel stores 36.55% more energy compared to the existing flywheel designed, produced by other researchers (Ghaly 1985; Madan Lal 2012). As a result of this, less torque and fuel consumption will be required to run the thresher machine.

Comparison of the optimized shape of the flywheel from the Jaya algorithm with the original shape of thresher machine flywheel is shown in Figure 6. The profile of the optimized flywheel is divided into three sections; it is thin in the middle section and thicker at inner and outer section, while the original flywheel has a constant thickness profile over the entire section. Further, the optimal values of the thicknesses are also effectively converted into physically possible shapes of the flywheel using PTC Creo 3.0 as shown in Figure 7. Hence, the optimized shape of flywheel, as obtained by Jaya algorithm, stores the maximum energy compared with the existing flywheel of thresher machine.

The stress distribution of radial, tangential, and Von Mises stresses along the radial direction in original and optimized flywheel in all sections is shown in Figure 8. Fewer stresses are developed in the optimized shape of the flywheel compared with those of the original shape of the flywheel as shown in Figure 8. Thus, the material of the optimized flywheel does not fail during its operation.

3.1 Simulation of the optimized shape of the flywheel of thresher machine

The original and optimized configurations of the flywheel of thresher machine was simulated using MSC ADAMS software (ADAMS 2014). The Input torque variations for the original and optimized flywheel are shown in Figure 9. The input torque is generated by the PTO of a tractor to run the thresher machine as shown in Figure 4. So, the input torque must be minimum for better performance of the thresher machine. It was seen that the optimized shape of the flywheel reduced the maximum and minimum values of input torque up to 15.65%, and 16.58% compared with the original values.

4 Conclusion

The optimal design procedure of the flywheel, using a cubic B-spline, is described in this paper. The optimization problem, with maximization of the kinetic energy, is formulated by considering the constraints of the mass of flywheel and the maximum value of the Von Mises stresses at all points along the radial direction using a cubic spline curve. The proposed approach is applied

Figure 5: Convergence of the best objective function value in PSO, GA, and Jaya.

Table 3: The optimum value for the shape of the flywheel

Optimum Values	Original (Varshney 2004)	GA	PSO	Jaya
$t_1 (m)$	0.020	0.0267	0.01	0.0189
$t_2 (m)$	0.020	0.0122	0.01	0.01
$t_3 (m)$	0.020	0.0228	0.06	0.01
$t_4 (m)$	0.020	0.0292	0.01	0.01
$t_5 (m)$	0.020	0.0248	0.01	0.01
$t_6 (m)$	0.020	0.0182	0.01	0.01
$t_7 (m)$	0.020	0.013	0.01	0.0246
$t_8 (m)$	0.020	0.0211	0.0489	0.06
$M (kg)$	115	115	115	115
$E_k (J)$	30483.66	28950	32896.25	41624.88 (36.55%)
$f(x)$	-	-28950	-32896.25	-41624.88

Figure 6: Original shape and optimized shape of the flywheel

(c) CAD model of the optimally designed flywheel

Figure 7: A Cad model of the Optimized flywheel **Figure 8:** Stress distribution along the radial direction in (a) original flywheel (b) optimized flywheel

to the flywheel of agricultural thresher machine. It was found that the Jaya algorithm gives the better results, compared with the results obtained from GA and PSO. The optimized shape of the flywheel, obtained by using by Jaya algorithm, stores 36.55% more energy compared with that of the original flywheel, without potential failure of the material during its operation. The optimal value of thicknesses is also effectively converted into physically possible shapes of the flywheel using any CAD software. The MSC ADAMS software was also used for simulation of the optimized shape of the flywheel. It was found that the maximum and minimum values of torque are reduced up to 15.65%, and 16.58% compared with the original values. Thus, farmers can use the optimized shape of the flywheel

Figure 9: Torque variation in the original and optimized flywheel

in place of the existing flywheel. As a result of this, dynamic performance (like vibrations, torque fluctuation) of the existing thresher machine will improve, and also accidents involving humans will be reduced.

Conflict of interest: Authors declare no conflict of interest.

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