7 Appendix

This section contains the proofs.

Proof. (Proposition 1) The existence of the minimum acceptable offer and the resulting value function (Equation 3) are already presented in the text. Here, we will derive Equation 4. For this, use Equation 3 to rewrite Equation 2 as

$$\frac{\omega^*}{1-\beta} = c + \beta\left(\int_0^{\omega^*} \frac{\omega^*}{1-\beta} dF(\omega) + \int_{\omega^*}^M \frac{\omega}{1-\beta} dF(\omega)\right).$$

Extending the left hand side,

$$\begin{aligned}\int_0^{\omega^*} \frac{\omega^*}{1-\beta} dF(\omega) + \int_{\omega^*}^M \frac{\omega^*}{1-\beta} dF(\omega) &= c + \beta\left(\int_0^{\omega^*} \frac{\omega^*}{1-\beta} dF(\omega) + \int_{\omega^*}^M \frac{\omega}{1-\beta} dF(\omega)\right) \\ \omega^* \int_0^{\omega^*} dF(\omega) - c &= \frac{1}{1-\beta}\int_{\omega^*}^M (\beta\omega - \omega^*)\, dF(\omega).\end{aligned}$$

Adding $\omega^* \int_{\omega^*}^M dF(\omega)$ to both sides gives

$$\omega^* - c = \frac{\beta}{1-\beta}\int_{\omega^*}^M (\omega - \omega^*)\, dF(\omega).$$

To obtain the expected payoff expression, Equation 5, note that by definition

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*} \omega^* dF(\omega) + \int_{\omega^*}^M \omega dF(\omega)\right).$$

It can be rewritten as

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*} (\omega^* - \omega)\, dF(\omega) + \int_0^M \omega dF(\omega)\right).$$

Integrating by parts,

$$\int_0^{\omega^*} (\omega^* - \omega)\, dF(\omega) = (\omega^* - \omega)\, F(\omega)\mid_0^{\omega^*} - \int_0^{\omega^*} F(\omega)\, d(\omega^* - \omega) = \int_0^{\omega^*} F(\omega)\, d(\omega).$$

Using this equality, we obtain

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega)\right),$$

the desired expression.

To obtain the expression for the average duration of the conflict, let $\xi = F(\omega^*)$ be the probability that a peace offer is rejected. Then the conflict lasts for t periods with probability

$prob\,(L = t) = (1-\xi)\,\xi^{t-1}$ (since the first $t-1$ offers are rejected with probability ξ^{t-1} and the last one is accepted with probability ξ). That means, the duration of the conflict is geometrically distributed. Then, the average duration of the conflict is

$$\sum_{t=1}^{\infty} t\,(1-\xi)\,\xi^{t-1} = \frac{1}{1-\xi},$$

the desired expression. ■

Proof. (Proposition 2) An increase in c decreases the left hand side in Equation 4. Alternatively, an increase in β increases the right hand side. In either case, however, the left hand side is smaller than the right hand side.

Now, the right hand side of Equation 4 is continuously decreasing in ω^* and the left hand side is increasing in ω^*. Thus, to equate the two sides again, ω^* must increase, thereby decreasing the right hand side and increasing the left hand side continuously.

The increase in ω^* also increases $F\,(\omega^*)$, thus increasing the average duration of the conflict due to expression 6.

It follows from Equation 5 that an increase in c, by increasing ω^*, leads to an increase in Ev. ■

Proof. (Proposition 3) Suppose F' first-order stochastically dominates F. Suppose ω^* is the minimum acceptable offer under F. Then ω^* satisfies Equation 4:

$$\omega^* - c = \frac{\beta}{1-\beta}\int_{\omega^*}^{M} (\omega - \omega^*)\,dF\,(\omega)\,.$$

By definition of first-order stochastic dominance, this equation implies

$$\omega^* - c < \frac{\beta}{1-\beta}\int_{\omega^*}^{M} (\omega - \omega^*)\,dF'\,(\omega)\,.$$

To compensate, ω^* must increase, decreasing the right hand side and increasing the left hand side, until the two are equal again. Thus, the reservation wage under F', $\omega^{*\prime}$ must satisfy $\omega^{*\prime} > \omega^*$.

To show the welfare effect, note that Equation 5 can be rewritten as

$$\begin{aligned}
Ev &= \frac{1}{1-\beta}\left(\int_0^{\omega^*} \omega^* dF\,(\omega) + \int_{\omega^*}^{M} \omega dF\,(\omega)\right)\\
Ev &= \frac{1}{1-\beta}\left(\int_0^{\omega^*} \omega^* dF\,(\omega) + \int_{\omega^*}^{M} \omega dF\,(\omega) + \int_{\omega^*}^{M} \omega^* dF\,(\omega) - \int_{\omega^*}^{M} \omega^* dF\,(\omega)\right)\\
Ev &= \frac{1}{1-\beta}\left(\omega^* + \int_{\omega^*}^{M} (\omega - \omega^*)\,dF\,(\omega)\right)
\end{aligned}$$

Integrating by parts,

$$\int_{\omega^*}^{M} (\omega - \omega^*) \, dF(\omega) = (\omega - \omega^*) F(\omega) \mid_{\omega^*}^{M} - \int_{\omega^*}^{M} F(\omega) \, d(\omega - \omega^*)$$
$$= M - \omega^* - \int_{\omega^*}^{M} F(\omega) \, d(\omega).$$

Using this equality, we obtain

$$Ev = \frac{1}{1-\beta} \left(M - \int_{\omega^*}^{M} F(\omega) \, d(\omega) \right).$$

By first-order stochastic dominance and the fact that $\omega^{*\prime} > \omega^*$

$$\int_{\omega^{*\prime}}^{M} F'(\omega) \, d(\omega) < \int_{\omega^*}^{M} F(\omega) \, d(\omega)$$

establishing

$$\frac{1}{1-\beta} \left(M - \int_{\omega^{*\prime}}^{M} F'(\omega) \, d(\omega) \right) > \frac{1}{1-\beta} \left(M - \int_{\omega^*}^{M} F(\omega) \, d(\omega) \right),$$

that is, Ev increases from F to F'. ■

Proof. (Proposition 4) Suppose F' is a mean-preserving spread of F. Suppose ω^* is the minimum acceptable offer under F. We will first rewrite Equation 4 as follows:

$$\omega^* - c = \frac{\beta}{1-\beta} \int_{\omega^*}^{M} (\omega - \omega^*) \, dF(\omega)$$
$$\omega^* - c = \frac{\beta}{1-\beta} \left(\int_{\omega^*}^{M} (\omega - \omega^*) \, dF(\omega) + \int_{0}^{\omega^*} (\omega - \omega^*) \, dF(\omega) - \int_{0}^{\omega^*} (\omega - \omega^*) \, dF(\omega) \right)$$
$$\omega^* - c = \frac{\beta}{1-\beta} \left(E\omega - \omega^* - \int_{0}^{\omega^*} (\omega - \omega^*) \, dF(\omega) \right)$$
$$\omega^* - (1-\beta) c = \beta E\omega - \beta \int_{0}^{\omega^*} (\omega - \omega^*) \, dF(\omega)$$

and integrating by parts gives

$$\omega^* - c = \beta (E\omega - c) + \beta \int_{0}^{\omega^*} F(\omega) \, d\omega.$$

Now, by definition of a mean-preserving spread, $\int_0^{\omega^*} (F'(\omega) - F(\omega)) \, d\omega \geq 0$. Thus,

$$\omega^* - c \leq \beta (E\omega - c) + \beta \int_{0}^{\omega^*} F'(\omega) \, d\omega.$$

Both sides are increasing in ω^*, but the right hand at a smaller rate. Thus, increasing ω^* increases the left hand side more than it does increase the right hand side and equates them. Thus,

$$\omega^{*\prime} - c = \beta (E\omega - c) + \beta \int_0^{\omega^{*\prime}} F'(\omega)\, d\omega$$

implies $\omega^{*\prime} \geq \omega^*$, with strict inequality if $\int_0^{\omega^*} (F'(\omega) - F(\omega))\, d\omega > 0$ (that is, if the mean-preserving spread is strict at ω^*).

The welfare effect follows from Equation 5:

$$Ev = \frac{1}{1-\beta}\left(\int_0^{\omega^*} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega)\right).$$

By definition, a mean-preserving spread does not effect $\int_0^M \omega dF(\omega)$ and increases $\int_0^{\omega^*} F(\omega)\, d(\omega)$. Combining this with the fact that $\omega^{*\prime} \geq \omega^*$ we obtain

$$\frac{1}{1-\beta}\left(\int_0^{\omega^*} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega)\right) < \frac{1}{1-\beta}\left(\int_0^{\omega^{*\prime}} F'(\omega)\, d(\omega) + \int_0^M \omega dF'(\omega)\right),$$

that is, Ev increases from F to F'. ■

Proof. (Proposition 5) In Equation 7, the second expression is independent of ω and the first expression is increasing in ω. Therefore, there is a unique minimum acceptable offer $\omega^*_{\tau\rho}$ such that$v(\omega) = \max\left\{\frac{\omega}{1-\beta}, c + \beta\tau\rho\frac{M}{1-\beta} + \beta(1-\tau)\, Ev\right\}$

$$\frac{\omega^*_{\tau\rho}}{1-\beta} = c + \frac{\beta\tau\rho M}{1-\beta} + \beta(1-\tau)\, Ev.$$

Note that

$$Ev = \int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF(\omega) + \int_{\omega^*_{\tau\rho}}^M \frac{\omega}{1-\beta} dF(\omega).$$

Thus we have

$$\frac{\omega^*_{\tau\rho}}{1-\beta} = c + \frac{\beta\tau\rho M}{1-\beta} + \beta(1-\tau)\left(\int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF(\omega) + \int_{\omega^*_{\tau\rho}}^M \frac{\omega}{1-\beta} dF(\omega)\right).$$

Extending the left hand side,

$$\begin{aligned}
\int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF(\omega) + \int_{\omega^*_{\tau\rho}}^M \frac{\omega^*_{\tau\rho}}{1-\beta} dF(\omega) &= c + \frac{\beta\tau\rho M}{1-\beta} + \beta(1-\tau)\begin{pmatrix} \int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF(\omega) \\ + \int_{\omega^*_{\tau\rho}}^M \frac{\omega}{1-\beta} dF(\omega) \end{pmatrix} \\
(1-\beta(1-\tau)) \int_0^{\omega^*_{\tau\rho}} \frac{\omega^*_{\tau\rho}}{1-\beta} dF(\omega) &= c + \frac{\beta\tau\rho M}{1-\beta} + \int_{\omega^*_{\tau\rho}}^M \frac{\beta(1-\tau)\omega - \omega^*_{\tau\rho}}{1-\beta} dF(\omega) \\
(1-\beta(1-\tau)) \int_0^{\omega^*_{\tau\rho}} \omega^*_{\tau\rho} dF(\omega) &= (1-\beta)\, c + \beta\tau\rho M + \int_{\omega^*_{\tau\rho}}^M \left(\beta(1-\tau)\omega - \omega^*_{\tau\rho}\right) dF(\omega)
\end{aligned}$$

Adding $(1-\beta(1-\tau))\int_{\omega^*_{\tau\rho}}^{M}\omega^*_{\tau\rho}dF(\omega)$ to both sides gives

$$\omega^*_{\tau\rho}-c=\frac{\beta}{(1-\beta(1-\tau))}\left(-\tau c+\tau\rho M+(1-\tau)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF(\omega)\right)$$

The derivation of the expected payoff expression, Equation 9, is identical to that in the proof of Proposition 1.

To obtain the expression for the average duration of the conflict, let $\xi=F\left(\omega^*_{\tau\rho}\right)$ be the probability that a peace offer is rejected. Let L be the duration of time until the conflict ends either with a successful peace settlement or with victory of one side.

The conflict lasts for one period with probability $prob(L=1)=\tau+(1-\tau)(1-\xi)$, that is, either the conflict terminates with violence (with probability τ) at the end of the first period, or the first peace offer is accepted (with probability $(1-\tau)(1-\xi)$). Similarly, the conflict lasts for two periods with probability $prob(L=2)=(1-\tau)\xi(\tau+(1-\tau)(1-\xi))$ where $(1-\tau)\xi$ covers the probability that in the first period the conflict does not terminate with violence and the first offer is rejected. The second part of the expression is as explained in the previous case. Generalizing, the conflict lasts for t periods with probability $prob(L=t)=(1-\tau)^{t-1}\xi^{t-1}(\tau+(1-\tau)(1-\xi))$. Then, the average duration of the conflict is

$$\begin{aligned}
\sum_{t=1}^{\infty}t(1-\tau)^{t-1}\xi^{t-1}(\tau+(1-\tau)(1-\xi)) &= \\
(\tau+(1-\tau)(1-\xi))\sum_{t=1}^{\infty}t(1-\tau)^{t-1}\xi^{t-1} &= \\
(\tau+(1-\tau)(1-\xi))\sum_{t=1}^{\infty}\sum_{k=1}^{t}(1-\tau)^{t-1}\xi^{t-1} &= \\
(\tau+(1-\tau)(1-\xi))\sum_{k=0}^{\infty}\sum_{t=1}^{\infty}(1-\tau)^{t-1+k}\xi^{t-1+k} &= \\
(\tau+(1-\tau)(1-\xi))\sum_{k=0}^{\infty}\frac{(1-\tau)^k\xi^k}{1-(1-\tau)\xi} &= \\
\frac{(\tau+(1-\tau)(1-\xi))}{1-(1-\tau)\xi}\sum_{k=0}^{\infty}(1-\tau)^k\xi^k &= \frac{1}{1-\xi(1-\tau)}
\end{aligned}$$

which is the desired expression. ■

Proof. (Proposition 6)

Effect of ρ: An increase in ρ increases the right hand side of Equation 8. To compensate, $\omega^*_{\tau\rho}$ must increase, increasing the left hand side and decreasing the right hand side of Equation

8. By Equation 9, this increases the decision-maker's expected payoff. By Equation 10, it also increases the average duration of the conflict.

Effect of τ : The derivative of the right hand side of Equation 8 with respect to τ is

$$\frac{\partial\left(\frac{\beta\left(-\tau c+\tau\rho M+(1-\tau)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF(\omega)\right)}{(1-\beta(1-\tau))}\right)}{\partial\tau}=-\frac{\beta\left(c\left(1-\beta\right)-M\rho\left(1-\beta\right)+\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF\left(\omega\right)\right)}{\left(\beta\tau-\beta+1\right)^2}$$

If $\rho M > c+\int_{\omega^*_{\tau\rho}}^{M}\frac{\left(\omega-\omega^*_{\tau\rho}\right)}{1-\beta}dF\left(\omega\right)$, the sign of this derivative is positive, that is, an increase in τ increases the right hand side of Equation 8. To compensate, $\omega^*_{\tau\rho}$ must increase, increasing the left hand side and decreasing the right hand side of Equation 8. By Equation 9, this increases the decision-maker's expected payoff.

Alternatively if $\rho M < c+\int_{\omega^*_{\tau\rho}}^{M}\frac{\left(\omega-\omega^*_{\tau\rho}\right)}{1-\beta}dF\left(\omega\right)$, the sign of this derivative is negative, that is, an increase in τ decreases the right hand side of Equation 8. To compensate, $\omega^*_{\tau\rho}$ must decrease, decreasing the left hand side and increasing the right hand side of Equation 8. By Equation 9, this decreases the decision-maker's expected payoff. By Equation 10, it also decreases the average duration of the conflict.

Finally, if $\rho M = c+\int_{\omega^*_{\tau\rho}}^{M}\frac{\left(\omega-\omega^*_{\tau\rho}\right)}{1-\beta}dF\left(\omega\right)$, the above derivative is zero. Thus, a change in τ has no effect on equations 8 and 9. In Equation 10, it does not affect $F\left(\omega^*_{\tau\rho}\right)$ but decreases the overall expression due to its direct effect. ■

Proof. (Proposition 7) An increase in c or β makes the left hand side smaller than the right hand side in Equation 8. To equate the two sides, one has to increase $\omega^*_{\tau\rho}$, which increases the left hand side and decreases the right hand side continuously. The new value of $\omega^*_{\tau\rho}$ is thus greater as a result of an increase in c or β.

The increase in $\omega^*_{\tau\rho}$ also increases $F\left(\omega^*_{\tau\rho}\right)$, thus increasing the average duration of the conflict due to expression 10.

It follows from Equation 9 that an increase in c or β, by increasing $\omega^*_{\tau\rho}$, leads to an increase in Ev. ■

Proof. (Proposition 8) Suppose F' first-order stochastically dominates F. Suppose $\omega^*_{\tau\rho}$ is the minimum acceptable offer under F. Then $\omega^*_{\tau\rho}$ satisfies Equation 8:

$$\omega^*_{\tau\rho}-c=\frac{\beta}{\left(1-\beta\left(1-\tau\right)\right)}\left(-\tau c+\tau\rho M+\left(1-\tau\right)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF\left(\omega\right)\right).$$

By definition of first-order stochastic dominance, this equation implies

$$\omega^*_{\tau\rho}-c<\frac{\beta}{\left(1-\beta\left(1-\tau\right)\right)}\left(-\tau c+\tau\rho M+\left(1-\tau\right)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega-\omega^*_{\tau\rho}\right)dF'\left(\omega\right)\right).$$

To compensate, $\omega^*_{\tau\rho}$ must increase, decreasing the right hand side and increasing the left hand side, until the two are equal again. Thus, the reservation wage under F', $\omega^{*\prime}_{\tau\rho}$ must satisfy $\omega^{*\prime}_{\tau\rho} > \omega^*_{\tau\rho}$.

For the second claim, suppose F' is a mean-preserving spread of F. Suppose $\omega^*_{\tau\rho}$ is the minimum acceptable offer under F. We will first rewrite Equation 5 as follows:

$$\omega^*_{\tau\rho} - c = \frac{\beta}{(1-\beta(1-\tau))}\left(-\tau c + \tau\rho M + (1-\tau)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega - \omega^*_{\tau\rho}\right)dF(\omega)\right)$$

$$\omega^*_{\tau\rho} - c = \frac{\beta}{(1-\beta(1-\tau))}\begin{pmatrix} -\tau c + \tau\rho M + (1-\tau)\int_{\omega^*_{\tau\rho}}^{M}\left(\omega - \omega^*_{\tau\rho}\right)dF(\omega) \\ +(1-\tau)\int_0^{\omega^*_{\tau\rho}}\left(\omega - \omega^*_{\tau\rho}\right)dF(\omega) - (1-\tau)\int_0^{\omega^*_{\tau\rho}}\left(\omega - \omega^*_{\tau\rho}\right)dF(\omega)\end{pmatrix}$$

$$\omega^*_{\tau\rho} - c = \frac{\beta}{(1-\beta(1-\tau))}\left(-\tau c + \tau\rho M + (1-\tau)\left(E\omega - \omega^*_{\tau\rho}\right) - (1-\tau)\int_0^{\omega^*_{\tau\rho}}\left(\omega - \omega^*_{\tau\rho}\right)dF(\omega)\right)$$

and integrating by parts gives

$$\omega^*_{\tau\rho} - c = \beta\left(\tau\rho M + (1-\tau)E\omega - c + (1-\tau)\int_0^{\omega^*_{\tau\rho}}F(\omega)\,d(\omega)\right).$$

Now, by definition of a mean-preserving spread, $\int_0^{\omega^*_{\tau\rho}}\left(F'(\omega) - F(\omega)\right)d\omega \geq 0$. Thus,

$$\omega^*_{\tau\rho} - c \leq \beta\left(\tau\rho M + (1-\tau)E\omega - c + (1-\tau)\int_0^{\omega^*_{\tau\rho}}F'(\omega)\,d(\omega)\right)$$

Both sides are increasing in $\omega^*_{\tau\rho}$, but the right hand at a smaller rate. Thus, increasing $\omega^*_{\tau\rho}$ increases the left hand side more than it does increase the right hand side and equates them. Thus,

$$\omega^{*\prime}_{\tau\rho} - c = \beta\left(\tau\rho M + (1-\tau)E\omega - c + (1-\tau)\int_0^{\omega^{*\prime}_{\tau\rho}}F'(\omega)\,d(\omega)\right)$$

implies $\omega^{*\prime}_{\tau\rho} \geq \omega^*_{\tau\rho}$, with strict inequality if $\int_0^{\omega^*_{\tau\rho}}\left(F'(\omega) - F(\omega)\right)d\omega > 0$ (that is, if the mean-preserving spread is strict at $\omega^*_{\tau\rho}$).

The proof of the welfare effect is identical to that in the proofs of propositions 3 and 4.

■

Proof. (Proposition 9) In Equation 11, the second term is constant in ω. Postulating that $v(\omega)$ is increasing in ω, we obtain the following optimal action in the minimum acceptable offer form. Let ω^* be the minimum acceptable offer. For $\omega \leq \omega^*$ then, $v(\omega) = c + \beta Ev$. For $\omega \geq \omega^*$ on the other hand, we have

$$v(\omega) = \omega + \beta(1-\alpha)v(\omega) + \beta\alpha[c + \beta Ev]$$

which simplifies to

$$v(\omega) = \frac{\omega + \beta\alpha\left[c + \beta Ev\right]}{1 - \beta(1 - \alpha)}.$$

Note that this expression is increasing in ω, which is consistent with our postulate. Thus, the resulting value function is:

$$v(\omega) = \begin{cases} c + \beta Ev & if \quad \omega \leq \omega_\alpha^* \\ \frac{\omega + \beta\alpha[c + \beta Ev]}{1 - \beta(1 - \alpha)} & if \quad \omega \geq \omega_\alpha^* \end{cases} \tag{15}$$

where ω_α^* equates the two expressions:

$$\frac{\omega_\alpha^* + \beta\alpha\left[c + \beta Ev\right]}{1 - \beta(1 - \alpha)} = c + \beta Ev.$$

Solving this equality, we obtain

$$\omega_\alpha^* = (1 - \beta)(c + \beta Ev).$$

By definition,

$$\begin{aligned} Ev &= \int_0^{\omega_\alpha^*} v(\omega)\, dF(\omega) + \int_{\omega_\alpha^*}^M v(\omega)\, dF(\omega) \\ Ev &= \int_0^{\omega_\alpha^*} \frac{\omega_\alpha^* + \beta\alpha\left[c + \beta Ev\right]}{1 - \beta(1 - \alpha)} dF(\omega) + \int_{\omega_\alpha^*}^M \frac{\omega + \beta\alpha\left[c + \beta Ev\right]}{1 - \beta(1 - \alpha)} dF(\omega). \end{aligned}$$

Solving it for Ev, we obtain

$$Ev = \frac{1}{(1 - \beta)(\alpha\beta + 1)} \left(\int_0^{\omega_\alpha^*} \omega_\alpha^* dF(\omega) + \int_{\omega_\alpha^*}^M \omega dF(\omega) + \beta\alpha c \right).$$

Inserting Ev into $\omega_\alpha^* = (1 - \beta)(c + \beta Ev)$, we then obtain

$$(1 + \alpha\beta)\, \omega_\alpha^* = (1 + \alpha\beta - \beta)\, c + \beta \int_0^{\omega_\alpha^*} \omega_\alpha^* dF(\omega) + \beta \int_{\omega_\alpha^*}^M \omega dF(\omega).$$

Adding and subtracting $\beta \int_{\omega_\alpha^*}^M \omega_\alpha^* dF(\omega)$ from the right hand side, the expression simplifies into

$$\omega_\alpha^* - c = \frac{\beta}{(1 + \alpha\beta - \beta)} \int_{\omega_\alpha^*}^M (\omega - \omega_\alpha^*)\, dF(\omega),$$

the desired expression for ω_α^*.

To obtain the expected payoff expression, note that

$$Ev = \frac{1}{(1 - \beta)(\alpha\beta + 1)} \left(\int_0^{\omega_\alpha^*} \omega_\alpha^* dF(\omega) + \int_{\omega_\alpha^*}^M \omega dF(\omega) + \beta\alpha c \right)$$

can be rewritten as

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\int_0^{\omega_\alpha^*} (\omega_\alpha^* - \omega)\, dF(\omega) + \int_0^M \omega dF(\omega) + \beta\alpha c \right).$$

Integrating by parts,

$$\int_0^{\omega_\alpha^*} (\omega_\alpha^* - \omega)\, dF(\omega) = (\omega_\alpha^* - \omega)\, F(\omega) \mid_0^{\omega_\alpha^*} - \int_0^{\omega_\alpha^*} F(\omega)\, d(\omega_\alpha^* - \omega) = \int_0^{\omega_\alpha^*} F(\omega)\, d(\omega).$$

Using this equality, we obtain

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\int_0^{\omega_\alpha^*} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega) + \beta\alpha c \right),$$

the desired expected payoff expression.

The proof for the average duration of the conflict is identical to the proof of Proposition 1. ■

Proof. **(Proposition 10)** Let $\alpha_1 < \alpha_2$ and let $\omega_{\alpha_1}^*$ and $\omega_{\alpha_2}^*$ be the resulting minimum acceptable offers. By Equation 12,

$$\omega_{\alpha_1}^* - c = \frac{\beta}{(1+\alpha_1\beta-\beta)} \int_{\omega_{\alpha_1}^*}^M \left(\omega - \omega_{\alpha_1}^*\right) dF(\omega)$$

and by $\alpha_1 < \alpha_2$,

$$\omega_{\alpha_1}^* - c > \frac{\beta}{(1+\alpha_2\beta-\beta)} \int_{\omega_{\alpha_1}^*}^M \left(\omega - \omega_{\alpha_1}^*\right) dF(\omega).$$

The left hand side is increasing and the right hand side is decreasing in ω_α^*. By definition, $\omega_{\alpha_2}^*$ solves

$$\omega_{\alpha_2}^* - c = \frac{\beta}{(1+\alpha_2\beta-\beta)} \int_{\omega_{\alpha_2}^*}^M \left(\omega - \omega_{\alpha_2}^*\right) dF(\omega).$$

Therefore, we have $\omega_{\alpha_1}^* > \omega_{\alpha_2}^*$.

Now $\frac{1}{1-F(\omega_\alpha^*)}$ is increasing in ω_α^*. Thus, an increase in α, by decreasing ω_α^*, also decreases the average duration of a conflict regime.

Regarding the decision-maker's expected payoff, Equation 13 can be rewritten as

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)} \left(\omega_\alpha^* + \int_{\omega_\alpha^*}^M (\omega - \omega_\alpha^*)\, dF(\omega) + \beta\alpha c \right).$$

Integrating by parts

$$\int_{\omega_\alpha^*}^M (\omega - \omega_\alpha^*)\, dF(\omega) = M - \omega_\alpha^* - \int_{\omega_\alpha^*}^M F(\omega)\, d(\omega).$$

Using this equality,

$$\begin{aligned} Ev &= \frac{1}{(1-\beta)(\alpha\beta+1)}\left(M - \int_{\omega_\alpha^*}^{M} F(\omega)\, d(\omega) + \beta\alpha c\right) \\ Ev &= \frac{M - \int_{\omega_\alpha^*}^{M} F(\omega)\, d(\omega)}{(1-\beta)(\alpha\beta+1)} + \frac{\beta\alpha c}{(1-\beta)(\alpha\beta+1)} \end{aligned}$$

The first part of this expression is decreasing in α. However, the second part is increasing in α. The overall effect of α on Ev thus depends on the relative sizes of these opposite effects. ■

Proof. (Proposition 11) The proof that β and c both increase ω_α^* is identical to the proof of Proposition 2. Since an increase in ω_α^* also increases, $\frac{1}{1-F(\omega_\alpha^*)}$, we also have the effect on the average duration.

It is straightforward to see from Equation 13 that an increase in c increases the decision-maker's expected payoff. The effect of β is less obvious. To see it, note that

$$\frac{\partial\left(\frac{\beta\alpha c}{(1-\beta)(\alpha\beta+1)}\right)}{\partial\beta} = \frac{\alpha c\left(\alpha\beta^2+1\right)}{(\beta-1)^2(\alpha\beta+1)^2} > 0.$$

Thus, an increase in β increases the third term in Equation 13. It also increases the first term since, ω_α^* is increasing in β. Finally,

$$\frac{\partial\left(\frac{1}{(1-\beta)(\alpha\beta+1)}\right)}{\partial\beta} = \frac{(2\alpha\beta-\alpha+1)}{(\beta-1)^2(\alpha\beta+1)^2} > 0.$$

Therefore, an increase in β increases Ev. ■

Proof. (Proposition 12) The proof that first-order stochastic dominance increases ω_α^* is similar to that of Proposition 3. To see that it increases expected payoff, note that Equation 13 can be rewritten as

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)}\left(\omega_\alpha^* + \int_{\omega_\alpha^*}^{M}(\omega-\omega_\alpha^*)\, dF(\omega) + \beta\alpha c\right).$$

Integrating by parts,

$$\int_{\omega_\alpha^*}^{M}(\omega-\omega_\alpha^*)\, dF(\omega) = M - \omega_\alpha^* - \int_{\omega_\alpha^*}^{M} F(\omega)\, d(\omega)$$

and inserting in the previous equality

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)}\left(M - \int_{\omega_\alpha^*}^{M} F(\omega)\, d(\omega) + \beta\alpha c\right).$$

By first-order stochastic dominance and the fact that ω_α^* is increasing, the expression $\int_{\omega_\alpha^*}^M F(\omega)\, d(\omega)$ is decreasing. As a result, the whole expression for Ev is increasing.

Suppose F' is a mean-preserving spread of F. Suppose ω_α^* and $\omega_\alpha^{*\prime}$ are the minimum acceptable offers under F and F', respectively. We will first rewrite Equation 12 as follows:

$$\begin{aligned}
\omega_\alpha^* - c &= \frac{\beta}{1+\alpha\beta-\beta}\int_{\omega_\alpha^*}^M (\omega-\omega_\alpha^*)\, dF(\omega) \\
\omega_\alpha^* - c &= \frac{\beta}{1+\alpha\beta-\beta}\begin{pmatrix} \int_{\omega_\alpha^*}^M (\omega-\omega_\alpha^*)\, dF(\omega) \\ +\int_0^{\omega_\alpha^*} (\omega-\omega_\alpha^*)\, dF(\omega) - \int_0^{\omega_\alpha^*} (\omega-\omega_\alpha^*)\, dF(\omega) \end{pmatrix} \\
\omega_\alpha^* - c &= \frac{\beta}{1+\alpha\beta-\beta}\left(E\omega - \omega_\alpha^* - \int_0^{\omega_\alpha^*} (\omega-\omega_\alpha^*)\, dF(\omega)\right) \\
(1+\alpha\beta)\,\omega_\alpha^* - (1+\alpha\beta-\beta)\, c &= \beta E\omega - \beta\int_0^{\omega_\alpha^*} (\omega-\omega_\alpha^*)\, dF(\omega) \\
(1+\alpha\beta)\,\omega_\alpha^* - (1+\alpha\beta)\, c &= \beta\,(E\omega - c) + \beta\int_0^{\omega_\alpha^*} F(\omega)\, d(\omega) \\
\omega_\alpha^* - c &= \frac{\beta}{(1+\alpha\beta)}\left((E\omega - c) + \int_0^{\omega_\alpha^*} F(\omega)\, d(\omega)\right)
\end{aligned}$$

Now, by definition of a mean-preserving spread, $\int_0^{\omega_\alpha^*} (F'(\omega) - F(\omega))\, d\omega \geq 0$. Thus,

$$\omega_\alpha^* - c \leq \frac{\beta}{(1+\alpha\beta)}\left((E\omega - c) + \int_0^{\omega_\alpha^*} F'(\omega)\, d(\omega)\right).$$

Both sides are increasing in ω_α^*, but the right hand at a smaller rate. Thus, increasing ω_α^* increases the left hand side more than it does increase the right hand side and equates them. Thus,

$$\omega_\alpha^{*\prime} - c = \frac{\beta}{(1+\alpha\beta)}\left((E\omega - c) + \int_0^{\omega_\alpha^{*\prime}} F'(\omega)\, d(\omega)\right)$$

implies $\omega_\alpha^{*\prime} \geq \omega_\alpha^*$, with strict inequality if $\int_0^{\omega_\alpha^*} (F'(\omega) - F(\omega))\, d\omega > 0$ (that is, if the mean-preserving spread is strict at ω^*).

The welfare effect of a mean-preserving spread follows from Equation 13:

$$Ev = \frac{1}{(1-\beta)(\alpha\beta+1)}\left(\int_0^{\omega_\alpha^*} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega) + \beta\alpha c\right).$$

By definition, a mean-preserving spread does not effect $\int_0^M \omega dF(\omega)$ and increases $\int_0^{\omega_\alpha^*} F(\omega)\, d(\omega)$.

Combining this with the fact that $\omega_{\alpha}^{*\prime} \geq \omega_{\alpha}^{*}$ we obtain

$$\begin{aligned} & \frac{1}{(1-\beta)(\alpha\beta+1)}\left(\int_0^{\omega_{\alpha}^{*}} F(\omega)\, d(\omega) + \int_0^M \omega dF(\omega) + \beta\alpha c\right) \\ < \; & \frac{1}{(1-\beta)(\alpha\beta+1)}\left(\int_0^{\omega_{\alpha}^{*\prime}} F'(\omega)\, d(\omega) + \int_0^M \omega dF'(\omega) + \beta\alpha c\right), \end{aligned}$$

that is, Ev increases from F to F'. ■