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From Numerical Considerations to Theoretical Solutions: Rational Design of a Debtor Creditor Agreement

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Abstract: This paper illustrates how the rational design of an international agreement can be investigated using software and numerical solutions as preliminary steps to a formal modelling of the agreement. The paper examines the repetition of a Called Bluff game interpreted as representing the stylized incentives of a debtor-creditor relationship.

Keywords: game theory, politics, international relations

1 Introduction

The process by which game theoretic models are born can exploit numerical solutions and the ease with which these can be manipulated using software. This does not guarantee quality or insight, but the pursuit of a rigorous creative process that begins with numerical solutions can enlighten the researcher on the potential of her ideas as well as their scope. We illustrate the numerical step in the creative process using the software GamePlan developed by Jean-Pierre Langlois.¹ This is not the only game theory software in town. We are aware of the existence of such tools as Gambit or Game Theory Explorer.² But GamePlan is unique in that it allows for the solving of discounted repeated games and stochastic games. Numerical experimentation with these types of games prior to

1 The GamePlan software is available for download free of charge at <http://userwww.sfsu.edu/langlois/>

2 A list of game theory software can be found at <http://www.gametheorysociety.org/resources.html>

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formal mathematical modeling provides valuable direction to the researcher. The development of a rational design for a debtor-creditor agreement illustrates the potential.

2 A debtor-creditor stage game

Aggarwal (1996) investigates sovereign debt games and the possible incentives at play between creditor and debtor. He identifies as Called Bluff games cases in which default actually forces the creditor to renegotiate and forgive part of the debt rather than let the debtor collapse with dire economic consequences and a failure to repay any portion of extended credit. As Aggarwal and Dupont (2013) state, Called Bluff political economy games illustrate the “tyranny of weakness.” The protracted negotiations between Greece and its creditors is a case in point. Repeated attempts by the Greek Government to soften the terms of debt repayment are met with ambiguous responses from creditors who fear a collapse of the Greek economy and a withdrawal of Greece from the European Union. How can such games be handled when successive payment deadlines determine a repetition of the Called Bluff incentives?

Consider a game between a sovereign Debtor and a Creditor. Following our example we can focus on the game between Greece and its creditors viewed as a unitary actor.³ Meeting scheduled payments ensures collaboration from the Creditor who will be willing to extend further credit as long as servicing of current debts complies with negotiated terms. However, if the Debtor defects, the Creditor is better off cooperating with the Debtor to alleviate economic pressures brought about by renegotiating the terms of the loan rather than recalling the funds forcing the Debtor’s sovereign default and the subsequent economic disruption that follows. Knowing this, can the Debtor have any incentive, in multiple rounds of the game, to cooperate rather than defect on its international financial obligations? The constituent game is represented numerically in Figure 1 below.

The Debtor has a dominant strategy-to defect, failing to comply with negotiated terms. The Creditor’s best response in that case is to cooperate and to offer renegotiated terms and possibly some debt forgiveness. How can a cooperative agreement be structured to move the Debtor to compliance and cooperation?

³ This is a simplifying assumption of course and the model could be extended to distinguish between main European creditors and the IMF for example.

Figure 1: The constituent game.

3 Identifying relevant states of the game

The identification of relevant states of the game is critical to the establishment of cooperation in a repeated Called Bluff game. Suppose we considered two states of the game—*compliance* defined as having both parties comply with the negotiated terms of the loan and *default* reached if one or both parties deviated from cooperation. The game would look like this:

In the *Compliance* state the hope is that both parties adopt cooperative strategies if they start the game doing so. In case of deviation the parties find themselves in a state of *Default* from which they can return to *Compliance* spontaneously by both adopting cooperative strategies. In any other case the parties are referred to a review panel that decides probabilistically to return the parties to *Compliance*. In the Greek debt crisis case, the panel could represent the Eurogroup of Finance Ministers or, alternatively the European Central Bank whose influence on the process as the largest creditor is high. Figure 2 is the simplest possible representation of the repeated game but it is likely to be insufficient because the incentives of the players are very different. As we pointed out earlier, if the Creditor deviates from cooperation it would be a simple matter for Debtor to simple revert to its dominant strategy. It would be strictly better off doing so. But if it is the Debtor that defects, punishment by the Creditor requires that it take a hit. So the merging of the two players’

Figure 2: Repeated game with two states.

non-compliant behaviors to define a single state of *default* is unlikely to produce the most interesting results. We should therefore be prepared to introduce three states of the game to identify the defector. It turns out that the game of Figure 2 has a unique equilibrium in which the players repeat the Nash Equilibrium of the constituent game in perpetuity. There is no scope for cooperation.

Consider a three state discounted repeated game that distinguishes between *Compliance*, the Debtor has defected and the Creditor has defected. The parties begin the game in *Compliance*. If the Debtor defects the game moves to *Debtor Default*. If Creditor defects the game moves to *Creditor Default*. The game is represented in Figure 3 below.

A clear transition from *Compliance* to one of the default states is determined by the identity of the defector. From *Compliance*, if the Debtor fails to comply while the Creditor cooperates the parties move to the state *Debtor Default*. Similarly the Creditor's unilateral defection leads to *Creditor Defaults*. But what should be decided if both parties defect simultaneously? In the above

Figure 3: The three state repeated debtor-creditor game.

representation joint defection leaves the parties in the state they happen to have reached. The above representation illustrates a particular treatment of guilt. If both parties defect, then both and neither are deemed guilty. Mutual punishment is inflicted in this case so there seems to be no need to supplement it by moving the parties to one non-compliance state or another. A sensitivity analysis section will investigate modifications to this assumption.

4 How do the parties sustain cooperation in the three state game?

Consider the equilibrium of the three state game pictured in Figure 4 below. In the *Compliance* state, both parties cooperate with probability 1. If they begin by cooperating they continue to as long as no deviation occurs. This is because deviation imposes enough cost on the Debtor if it were to occur. Indeed, look at what happens if the Debtor defects. The game then reverts to the state *Debtor Defaults* in which the Creditor plays defect harming the Debtor in so doing. One turn leads to a chance to return to cooperation of 70% in this example. In terms of our example, Greece's delaying of payments on schedule represents a move to

Figure 4: Equilibrium in the three state game.

the *Debtor Defaults*. And with 70% probability, no hardening of terms is imposed on Greece by any of the creditors and the game returns to *Compliance*.

Similarly, if the Creditor were to defect, since after all she also has an incentive to do so if the Debtor is cooperating, then the Debtor punishes by reverting to his favorite and dominant strategy-defect. This would cover the unlikely scenario in which Greece's creditors would, for example, accelerate the debt payment schedule leading Greece to delay scheduled payments. Again the parties return to cooperation from non-compliance, returning to the prior status quo, with probability 70%. This scheme supports cooperation as long as the probability that the parties return to cooperation from punishment mode is not too high. Given the numerical values in this case, it must not exceed 88.8%. Proof that the scheme we have uncovered numerically is an equilibrium in the general case where players face the symbolic payoffs of a Called Bluff is provided in the appendix. The mathematical appendix therefore provides a generalization of the numerical result examined here as it illustrates formal methodology.

The cooperative scheme pictured in Figure 4 is not the only equilibrium of the three state sovereign debt game. Of course perpetual repetition of the Nash equilibrium of the constituent game is one. As for other possible equilibria, one can support cooperation with the Debtor declining to punish the Creditor with small probability in state *Creditor Default*. This is not fundamentally different from the

equilibrium of Figure 4. It is just a statement about how much punishment is enough to deter the Creditor. All other equilibria of the game fail to support cooperation. Their structure is typical: in *Compliance* the parties choose a mixed strategy. Therefore even if they cooperate at first they will inevitably deviate and end up in one of the default states where they play the Nash equilibrium of the constituent game. With strictly positive probability the parties will find themselves in *Debtor Default* or *Creditor Default* where they will remain, forever playing Nash.

5 Sensitivity analysis

Sensitivity analysis on the assumptions made in the three state game illustrates the delicate balance that must be reached in the rules of a debtor-creditor agreement for cooperation to be supported. The representation of Figure 3 emphasizes the identity of the defector as a key descriptor of the *default* state, and it assumes that *mutual* defection keeps the parties in the state they happen to be in. Responsibility for mutual defection is not assigned. But it could be. We could decide that mutual defection is both parties' fault and they share responsibility. In case of mutual defection, the parties would then be routed to *Debtor Default* or *Creditor Default* each with probability 50%. Or we could decide that mutual defection is all the Debtor's fault and mutual defection would route the parties to *Debtor Defaults* with probability 1. Such responsibility would be implicit in the Debtor-Creditor agreement. What happens to the equilibria of the game?

Blaming the Debtor altogether in case of mutual defection does not ruin the chances of cooperation although cooperation is only maintained and re-established probabilistically. Varying the level of responsibility that the Debtor must take in case of mutual defection yields some interesting insights. To support cooperation the debtor cannot be blamed too much but should shoulder enough responsibility in case of mutual defection for cooperation to be sustained. More precisely, given our parameter values, if less than 75% of the blame for mutual defection is attributed to the Debtor, cooperation cannot be sustained at all. But, given parameter values, full cooperation can be sustained if the guilt attributed to the Debtor ranges from 76% to 82%. Above 82% assigned blame to the debtor, cooperation can only be sustained sporadically.

6 Conclusion

This article shows how software and numerical solutions examined using software inform on the structure of a model as well as the nature of its solutions. For

example numerical exploration reveals that cooperation can only be sustained in the repetition of the Called Bluff game if three states of the game are distinguished. This is in contrast to the repetition of the Prisoner's Dilemma for which cooperation can be sustained when only two states of the game are defined. Defining three states in the repetition of the Called Bluff game attributes noncompliant behavior to one or the other actor and underscores the importance of guilt in the failure to abide by the terms of an international agreement.

Mathematical Appendix

We will illustrate the modeling of the structure pictured in Figure 4. Players are denoted C for Creditor and D for Debtor. Payoffs of the constituent game are given in Table 1 below:

Table 1: The stage game.

C\D	Cooperate	Defect
Cooperate	$(0, 0)$	(f_c, t_d)
Defect	(t_c, f_d)	(s_c, s_d)

We assume that:

$$t_c > 0 > f_c > s_c \text{ for C and } t_d > 0 > s_d > f_d \text{ for D}$$

The unique Nash Equilibrium of the one shot game is therefore (Cooperate, Defect). States of the repeated game are denoted C for bilateral compliance, DN for the Debtor's non-compliance (or default) and CN for the Creditor's non-compliance. Review boards are denoted RD for the Debtor's review and RC for the Creditor's review. Probabilities of return to state C are denoted $(1-p)$ at state RC and $(1-q)$ at RD. The common discount factor is denoted d .

Expected utilities on the equilibrium path of the repeated game are denoted U_J^S for player $J \in \{C, D\}$ at state or node S. Expected equilibrium play is (Cooperate, Cooperate) at state C, (Cooperate, Defect) at state CN and (Defect, Cooperate) at state DN. To verify whether this forms a MPE we calculate expected payoffs for all combinations of moves. We have:

$$U_J^C = 0 + d * U_J^C = 0 \text{ for both } J \in \{C, D\}$$

$$U_c^{CN} = f_c + d * U_c^{RC} = f_c + d * (p * U_c^{CN} + (1-p) * U_c^C) = \frac{f_c}{1-dp}$$

$$U_D^{\text{CN}} = t_D + d * U_D^{\text{RC}} = t_D + d * (p * U_D^{\text{CN}} + (1-p) * U_D^{\text{C}}) = \frac{t_D}{1-dp}$$

$$U_C^{\text{DN}} = t_C + d * U_C^{\text{RD}} = t_C + d * (q * U_C^{\text{DN}} + (1-q) * U_D^{\text{C}}) = \frac{t_C}{1-dq}$$

$$U_D^{\text{DN}} = f_D + d * U_D^{\text{RD}} = f_D + d * (q * U_D^{\text{RD}} + (1-q) * U_D^{\text{C}}) = \frac{f_D}{1-dq}$$

We denote by W_J^S the expected payoff for defection by J from expected play at state S . For example W_C^{CN} results from C choosing *Defect* instead of *Cooperate* at state CN (while D plays the expected *Defect* at that state). We have:

$$W_C^{\text{CN}} = s_C + d * U_C^{\text{CN}} = s_C + \frac{df_C}{1-dp}$$

$$W_D^{\text{CN}} = 0 + dp * U_D^{\text{CN}} = \frac{dt_D}{1-dp}$$

and $W_D^{\text{CN}} = dp * U_D^{\text{CN}}$ if the transition were to review RC instead of directly to CN after play of (*Cooperate*, *Cooperate*). In either case $W_D^{\text{CN}} \leq U_D^{\text{CN}}$ holds since $0 \leq \frac{t_D}{1-dp}$ so that it is best for D to play *Defect* in state CN . Finally it is best for C to play *Cooperate* at CN if $W_C^{\text{CN}} \leq U_C^{\text{CN}}$ or if:

$$s_C \leq \frac{1-d}{1-dp} f_C$$

which always holds given payoff assumptions.

Similarly have at state DN where (*Defect*, *Cooperate*) is expected we have:

$$W_D^{\text{DN}} = s_D + d * U_D^{\text{DN}} = s_D + \frac{df_D}{1-dq}$$

$$W_C^{\text{DN}} = 0 + d * U_C^{\text{DN}} = \frac{dt_C}{1-dq}$$

or $W_C^{\text{DN}} = dq * U_C^{\text{DN}}$ if transition were to review RD instead of directly to DN after (*Cooperate*, *Cooperate*). In either case $W_C^{\text{DN}} \leq U_C^{\text{DN}}$ holds since $0 \leq \frac{t_C}{1-dq}$ so that it

is best for C to play *Defect* in state DN. Finally it is best for D to play *Cooperate* at DN if $W_D^{DN} \leq U_D^{DN}$ or if:

$$s_D \leq \frac{1-d}{1-dq} f_D$$

which does not always hold given payoff assumptions. We therefore need q to be chosen small enough so that:

$$(1-dq)s_D \leq (1-d)f_D \quad (1)$$

At state C we have:

$$W_C^C = t_C + d * U_C^{CN} = t_C + \frac{df_C}{1-dp} \leq U_C^C \text{ which holds if}$$

$$(1-dp)t_C + df_C \leq 0 \quad (2)$$

$$W_D^C = t_D + d * U_D^{DN} = t_D + \frac{df_D}{1-dq} \leq U_D^C \text{ which holds if}$$

$$(1-dq)t_D + df_D \leq 0 \quad (3)$$

Inequalities (1) (2) and (3) are the only requirements for this scheme to form an MPE.

References

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