Research Article Open Access

Yuri I. Gorobets and Volodymyr V. Kulish*

Spin waves in a ferromagnetic nanotube of an elliptic cross-section in the presence of a spin-polarized current

DOI 10.1515/phys-2015-0033
Received December 26, 2014; accepted July 13, 2015

Abstract: In the paper, spin waves in a ferromagnetic nanotube of an elliptic cross-section in the presence of a spin-polarized electric current are investigated. The linearized Landau-Lifshitz equation in the magnetostatic approximation is used, with the exchange interaction, the dipole-dipole magnetic interaction, the anisotropy effects and the dissipation effects taken into account; the influence of the spin-polarized current is considered by the Slonczewski-Berger term. After elimination of the magnetization density perturbation, an equation for the magnetic potential for the above-described spin excitations is obtained. From this equation, a dispersion relation for spin waves in the nanosystem described previously is obtained. Analysis of the dispersion relation shows that the presence of the spin-polarized current can strengthen or weaken the dissipation, creating an "effective dissipation"; the effect is analogous to the "effective dissipation" in a two-layer ferromagnetic film in the presence of a spin-polarized current. Depending on the direction and the density of the current the spin wave can decay faster or slower than in the absence of the current, transform into a self-sustained wave or grow in amplitude, thus leading to a spin wave generation.

Keywords: spin wave; non-circular ferromagnetic nanotube; dipole-exchange theory; spin-polarized current

PACS: 75.30.Ds, 75.75.+a, 75.90.+w

Yuri I. Gorobets: Institute of Magnetism, National Academy of Sciences of Ukraine, 36-b Vernadskogo st., 03142, Kyiv, Ukraine
***Corresponding Author: Volodymyr V. Kulish:** National Technical University of Ukraine "Kyiv Polytechnic Institute" 37 Peremogy prosp., 03056, Kyiv, Ukraine, E-mail: kulish_volv@ukr.net

1 Introduction

Spin waves in magnetically ordered materials have been a popular topic of research during the last few decades. Different aspects of spin waves in different types of media have been studied intensively; spin excitations in nanosystems are particularly promising, in particular, in the terms of technical applications. Among them, magnonics - a sub-field of modern solid state physics that studies spin waves in nanosystems [1] - and spintronics (spin electronics) - a sub-field of modern solid state physics that studies the properties of the electron spin and ways of its manipulation in solid-state devices [2] - are promising for creating new information storage, transmission, and processing devices [3–5].

It is known that the magnetic properties of nanostructures depend essentially on their size and shape. Therefore, spin waves have been studied in different types of nanosystems individually. While spin waves in thin magnetic films have been studied for several decades (see, e.g., [6]), spin waves in nanosystems of more complex geometries, in particular nanosystems with a one-axis translational symmetry, represent a relatively novel field of research. Spin waves in magnetic nanowires [7–9] and macroscopic magnetic wires with an elliptic cross-section [10] were studied in the past few years. Spin waves in magnetic nanotubes were also studied (see, e.g., [11, 12]), but currently they attract little attention. Known theoretical papers on the subject investigate mostly spin solitons [13] and waves on magnetic domains interfaces [14, 15].

Magnetic nanotubes has been synthesized only recently [16], however, they have already found a wide range of technical applications (especially in magnetobiology, see, e.g., [17, 18]). In the past few years, nanotubes of non-circular cross section were also synthesized (see, e.g., [19]); their properties differ from the properties of circular nanotubes (see, e.g., [20]). In particular, nanotubes of elliptic cross-section [21–23] represent a special area of interest for synthesis and research. (Note also that typical nanotubes synthesized currently often have an essentially non-circular cross-section, see, e.g., [24]. For most cases, the "elliptical cross-section" approximation is more precise for such nanotubes than the "circular cross-section" approximation.) However, spin waves in elliptic magnetic nanotubes remain practically unresearched, and known papers on the subject consider purely dipole-dipole spin waves (so the results cannot be used for nanoscale systems when the exchange interaction becomes essential) without account for the magnetic anisotropy, magnetic dumping, and spin-polarized current (see, e.g., [25]). Therefore, spin waves in elliptic magnetic nanotubes accounting for the above-mentioned effects present a topical field of research.

Dissipation effects can either essentially influence a spin wave pattern in a nanosystem or be negligible depending on the spin wave frequency as well as the nanosystem's size, shape, and material (see, e.g., [26]). Therefore, in general case, one must consider dissipation effects when investigating spin waves in nanosystems.

Magnetic nanostructures, in particular magnetic nanotubes, can be used as waveguides for spin waves. Therefore, a task of generating spin waves in magnetic nanostructures becomes essential. One of the ways of generating spin waves (usually in the microwave range) in magnetic nanostructures is using so-called spin-torque effect: change of the magnetization direction (switching or precession) in a thin layer of a ferromagnet as a spin-polarized current passes through it [27–29]. As is known (see, e.g., [27]), the influence of the spin-torque effect on the spin wave pattern of a nanosystem can be either negligible or essential depending on the current density and, therefore, must be considered in a general case. Therefore, investigation of spin waves in magnetic nanotubes (in particular, nanotubes of elliptic cross-section) in the presence of spin-polarized current and, in particular, investigation of the spin waves generation in such systems is a topical field of research.

This paper investigates dipole-exchange longitudinal spin waves and orthogonal spin excitations in a ferromagnetic nanotube with an elliptic cross-section in the presence of a spin-polarized current. The dipole-dipole magnetic interaction, anisotropy effects, dissipation effects ,and the influence of the spin-polarized current are taken into account. For a two-layer nanotube, an equation for the magnetic potential and a relation between the wave frequency and two wavenumber components for such spin wave are obtained. For a thin nanotube, a dispersion relation for such spin wave is also obtained. Effective dissipation - which can be either positive or negative depending on the direction and the density of the current - is observed; the spin wave generation condition is obtained. For a one-layer ferromagnetic nanotube with an elliptic cross-section in the absence of a spin-polarized current, an orthogonal wavenumber spectrum of spin excitations is also obtained.

2 Setting of the problem

Let us consider a two-layer ferromagnetic nanotube with an elliptical cross-section (elliptical cylinder) and a spin-polarized current passing through it. We assume that one layer of the nanotube is "fixed" in the sense of the magnetization direction, the second - "free", so that a spin-polarized current can pass through the "free" layer. (Later in the paper we will also consider the case of a single-layer ferromagnetic nanotube placed between two nonmagnetic metal surfaces.) Let us denote the "free" layer semiaxes as a_2, b_2 (for the outer surface) and a_1, b_1 (for the inner surface), see Figure 1.

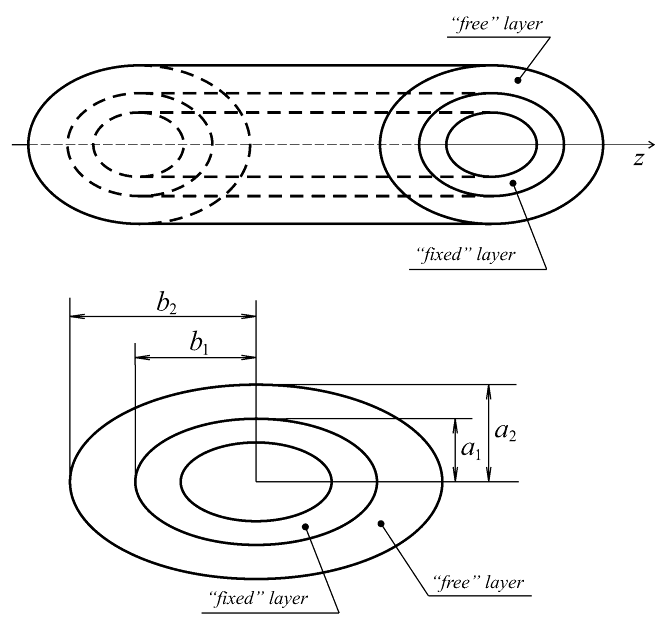

Figure 1: A nanotube modelled as per the article.

Let us assume that the free layer is composed of an "easy axis" uniaxial ferromagnet with the following parameters: the exchange constant α, the uniaxial anisotropy parameter β, and the gyromagnetic ratio γ. We also assume that the ferromagnet anisotropy axis is directed along the symmetry axis of the system.

Let us consider the case when the saturation magnetization of the "free" layer and the magnetization of the "fixed" layer are both directed along the translational symmetry axis of the system and let us choose this direction as the *Oz* axis of our coordinate system. Let us consider a spin wave propagating in the "free" layer of the above-described system along the axis *Oz*. Considering a typical nanotube length and corresponding wavenumber limitations, we have to take into account both the magnetic dipole-dipole interaction and the exchange interaction in the Landau-Lifshitz equation for a typical nanotube. We also have to keep the anisotropy addend in this equation as we consider a uniaxial ferromagnet.

We consider the magnetization $\vec{m}$ and the magnetic field $\vec{h}$ of the spin wave as a small perturbation of the overall magnetization density $\vec{M}$ and the overall internal magnetic field $\vec{H}^{(i)}$, correspondingly. Thus, we can write down $|\vec{m}| \ll \left|\vec{M}_0\right|$, $\left|\vec{h}\right| \ll \left|\vec{H}_0^{(i)}\right|$ (where $\vec{M}_0$ and $\vec{H}_0^{(i)}$ are the ground state magnetization and the internal magnetic field, correspondingly, so $\vec{M} = \vec{M}_0 + \vec{m}$, $\vec{H}^{(i)} = \vec{H}_0^{(i)} + \vec{h}$) and apply the linearized spin wave theory.

Our task is to obtain the dispersion relation for the above-described spin waves and determine the condition of spin wave generation in the "free" layer.

3 System of equations for a spin wave in the "free" nanotube layer

Let us write down a linearized Landau-Lifshitz equation for the "free" nanotube layer described in the previous section. In the absence of a spin-polarized current this equation can be written as follows [30]:

$$\frac{\partial \vec{m}}{\partial t} = \gamma \left(\vec{M}_0 \times \left(\vec{h} + \alpha \sum_i \frac{\partial^2 \vec{m}}{\partial x_i^2} + \beta \vec{n} \left(\vec{m}\vec{n} \right) - \frac{1}{M_0^2} \left(\vec{M}_0 \vec{H}_0^{(i)} + \beta \left(\vec{M}_0 \vec{n} \right)^2 \right) \vec{m} \right) \right), \quad (1)$$

here $\vec{n}$ is the unit vector along the anisotropy axis of the system (for our system, it coincides with the unit vector $\vec{e}_z$). To consider dissipation effects, let us use a damping term in the Gilbert form

$$\vec{T}_G = \frac{\alpha_G}{M} \left[\vec{M} \times \frac{\partial \vec{M}}{\partial t} \right], \quad (2)$$

where α_G is the Gilbert damping constant of a "free" layer ferromagnet. In the linearized form of the Landau-Lifshitz equation, this term can be rewritten as follows:

$$\vec{t}_G = \alpha_G \left[\vec{M}_0 \times \frac{\partial \vec{m}}{\partial t} \right]. \quad (3)$$

In order to consider the spin-polarized current, let us use the Slonczewski-Berger spin-transfer term. We assume that the "free" nanotube layer is thin enough to use the form of the term obtained for a flat film [27]:

$$\vec{T}_s = \frac{\varepsilon \gamma \hbar J}{2eM_0^2 d} [\vec{M} \times [\vec{M} \times \vec{e}_p]], \quad (4)$$

where ε is the dimensionless spin-polarization efficiency, J is the electric current density (is considered constant), μ_B is the Bohr magneton, e is the modulus of the electron charge, $d = ((b_2 - a_2) + (b_1 - a_1))/2$ is the mean "free" layer thickness (we assume that the relative change of the thickness is small enough), and $\vec{e}_p$ is the unit vector of the magnetization direction in the "fixed" layer (in our case, $\vec{e}_p = \vec{e}_z$). This Slonczewski–Berger term takes the following form in the linearized Landau-Lifshitz equation:

$$\vec{t}_s = \frac{\varepsilon \gamma \hbar J}{2eM_0^2 d} [\vec{M}_0 \times [\vec{m} \times \vec{e}_z]], \quad (5)$$

where we considered $\vec{M}_0 || \vec{e}_z$, $\vec{m} \perp \vec{e}_z$. Therefore, the linearized Landau-Lifshitz equation with the terms that consider the energy dissipation and the spin-polarized current influence can be rewritten as follows:

$$\frac{\partial \vec{m}}{\partial t} = \gamma \left(\vec{M}_0 \times \left(\vec{h} + \alpha \sum_i \frac{\partial^2 \vec{m}}{\partial x_i^2} + \beta \vec{n} \left(\vec{m}\vec{n} \right) - \frac{1}{M_0^2} \left(\vec{M}_0 \vec{H}_0^{(i)} + \beta \left(\vec{M}_0 \vec{n} \right)^2 \right) \vec{m} + \frac{\alpha_G}{\gamma M_0} \frac{\partial \vec{m}}{\partial t} + \frac{\varepsilon \hbar J}{2eM_0^2 d} \left[\vec{m} \times \vec{e}_z \right] \right) \right). \quad (6)$$

In particular, for perturbations in the form of oscillations periodic in time,

$$\vec{m}\left(\vec{r}, t\right) = \vec{m}_0\left(\vec{r}\right) \exp\left(i\omega t\right), \vec{h}\left(\vec{r}, t\right) = \vec{h}_0\left(\vec{r}\right) \exp\left(i\omega t\right), \quad (7)$$

the linearized Landau-Lifshitz equation (after using the system symmetry properties) can be rewritten as follows:

$$i\omega \vec{m}_0 = \gamma \left(M_0 \vec{e}_z \times \left(\vec{h}_0 + \alpha \Delta \vec{m}_0 - \left(\beta + \frac{H_0^{(e)}}{M_0} - i \frac{\alpha_G}{\gamma M_0} \omega \right) \vec{m}_0 + \frac{\varepsilon \hbar J}{2eM_0^2 d} \left[\vec{m}_0 \times \vec{e}_z \right] \right) \right), \quad (8)$$

where $\vec{H}_0^{(e)}$ is the external magnetic field.

In order to solve the Landau-Lifshitz equation, we need one more relation between the magnetization and the magnetic field. Let us use the magnetostatic approximation [30]. In this approximation, the magnetic field perturbation $\vec{h}$ is a potential field: $\vec{h} = -\nabla \Phi$, $\vec{h}_0 = -\nabla \Phi_0$, where Φ is a magnetic potential and $\Phi = \Phi_0\left(\vec{r}\right) \exp\left(i\omega t\right)$. In this way, we can obtain the sought relation from the Maxwell equation $div\vec{h} = -4\pi \cdot div\vec{m}$. After introducing the magnetic potential, it can be rewritten as $\Delta\Phi - 4\pi div\vec{m} = 0$, or

$$\Delta\Phi_0 - 4\pi div\vec{m}_0 = 0 \qquad (9)$$

for perturbations in the form of oscillations periodic in time (7).

The equations (8), (9) provide the necessary relationship between $\vec{m}$ and $\vec{h}$. Using this system of equations, we can find the equation for the magnetic potential Φ.

4 Equation for the magnetic potential

In order to obtain the equation for the magnetic potential of a spin wave in the system, let us eliminate the magnetization perturbation in the system of equations (8), (9).

For the system we consider, it is convenient to use the elliptic cylindrical coordinates (u,v,z) with the following relations describing the transition to Cartesian coordinates:

$$\begin{cases} x = \frac{d}{2} ch(u)\cos(v) \\ y = \frac{d}{2} sh(u)\sin(v) \\ z = z \end{cases} . \qquad (10)$$

The equation u=const describes an elliptic cylinder with semiaxes $\frac{d}{2}ch(u)$, $\frac{d}{2}sh(u)$. Therefore, the nanotube boundaries can be specified by the equations $u = u_1$, $u = u_2$ (here $ch(u_1) = 2b_1/d$, $sh(u_1) = 2a_1/d$, $ch(u_2) = 2b_2/d$, $sh(u_2) = 2a_2/d$).

After substituting the Maxwell equation (9) into the equation (8) and considering $m_{0z} = 0$ we obtain

$$-\frac{i}{\gamma M_0}(\omega \pm i\kappa)\, div\left[\vec{e}_z \times \vec{m}_0\right] = -\Delta\Phi_0 + \frac{\partial^2\Phi_0}{\partial z^2} + \frac{1}{4\pi}\left(\alpha\Delta - \tilde{\beta}\right)\Delta\Phi_0, \qquad (11)$$

where $\kappa = \frac{\gamma\varepsilon\hbar|J|}{2eM_0 d}$, $\tilde{\beta} = \beta + H_0^{(e)}/M_0 - i\alpha_G\omega/\gamma M_0$. The "+" sign in the left side of the equation corresponds to the current passing from the "fixed" nanotube layer to the "free" one: J>0, the "-" sign - vice versa. After applying the operator $\alpha\Delta - \tilde{\beta}$ to the equation (11) and making certain transformations we finally obtain the equation for the magnetic potential that does not contain $\vec{m}$:

$$\left(\frac{(\omega \pm i\kappa)^2}{\gamma^2 M_0^2} - \left(\tilde{\beta} - \alpha\Delta\right)\left(4\pi + \tilde{\beta} - \alpha\Delta\right)\right)\Delta\Phi_0 + 4\pi\left(\tilde{\beta} - \alpha\Delta\right)\frac{\partial^2\Phi_0}{\partial z^2} = 0. \qquad (12)$$

Note that if κ=0 (no spin-polarized current), the equation (12) can be used for a one-layer ferromagnetic nanotube or nanowire of an arbitrary cross-section. In particular, the equation (12) with κ=0 and α_G=0 (no dissipation) becomes similar to the known equation for the magnetic potential of spin waves in a ferromagnetic cylinder, see, e.g., [7].

5 Dispersion relation and condition of spin wave excitation

Now we can find the dispersion relation for spin waves in the "free" nanotube layer.

First, let us note that the equation (12) in the above-described elliptic cylindrical coordinates admits a solution in the following form:

$$\Phi = \left(C_1 Ce_m(u,\alpha)\, ce_m(v,\alpha) + C_2 Se_m(u,\alpha)\, se_m(v,\alpha)\right)\exp\left(i\left(\omega t - k_{||}z\right)\right) \qquad (13)$$

where C_1, C_2 are constants, Ce_m, ce_m, Se_m, and se_m are Mathieu functions of the order m (these functions are solutions of the two-dimensional Helmholtz equation in the plane xOy: $\Delta_\perp F - k_\perp^2 F = 0$, where $k_\perp$ is the orthogonal wavenumber), $k_{||}$ is the longitudinal wavenumber and $\alpha = k_\perp^2 d^2/16$. After substituting the solution (13) into the equation (12) we obtain a dispersion equation in the following form:

$$\left(\frac{(\omega \pm i\kappa)^2}{\gamma^2 M_0^2} - \left(\tilde{\beta} + \alpha k^2\right)\left(4\pi + \tilde{\beta} + \alpha k^2\right)\right)k^2 + 4\pi\left(\tilde{\beta} + \alpha k^2\right)k_{||}^2 = 0, \qquad (14)$$

where the general wavenumber $k^2 = k_\perp^2 + k_{||}^2$.

As we can see, the dispersion equation (14) contains two wavenumber components, orthogonal and longitudinal. Therefore, in general case, we have to solve the equation (12) with boundary conditions for the magnetization in order to obtain the dispersion relation. However, we can eliminate the orthogonal wavenumber component from (14) after noting that a typical nanotube thickness has the same order as the characteristic exchange interaction length l_{ex}. Therefore, we can consider the case when the "free" layer thickness is less than the exchange length and, therefore, neglect the radial dependence of the magnetic potential: $k_\perp = 0$. Thus, we can put $n = 0$ and transform the equation (14) for $k \neq 0$ in the following way:

$$\frac{(\omega \pm i\kappa)^2}{\gamma^2 M_0^2} - \left(\tilde{\beta} + \alpha k^2\right)\left(4\pi + \tilde{\beta} + \alpha k^2\right) + 4\pi\left(\tilde{\beta} + \alpha k^2\right) = 0. \qquad (15)$$

From the equation (15), we obtain the sought relations between k and ω:

$$k^2 = \frac{1}{\alpha}\left(\frac{\omega\left(1 - i\alpha_G\right) \pm i\kappa}{\gamma M_0} - \beta - \frac{H_0^{(e)}}{M_0}\right) \qquad (16)$$

and the dispersion relation

$$\omega = \frac{1}{1+\alpha_G^2}\left(\gamma M_0\left(\alpha k^2+\beta+\frac{H_0^{(e)}}{M_0}\right)\pm\kappa\right.$$
$$\left.+i\left(\alpha_G\gamma M_0\left(\alpha k^2+\beta+\frac{H_0^{(e)}}{M_0}\right)\mp\kappa\right)\right). \quad (17)$$

Note that if κ=0, the dispersion relation (17) can also be used for a nanotube of another cross-section (in the case when a coordinate system that correspond to the nanotube symmetry allows separation of the variables in the equation (12), and the resulting equation in the plane orthogonal to Oz has a form of a two-dimensional Helmholtz equation in these coordinates).

6 Orthogonal wavenumber spectrum for a one-layer nanotube in the absence of the spin-polarized current

If the “free” layer thickness cannot be considered small (so we cannot put $k_\perp = 0$), the dispersion relation (17), in a general case, should be complemented with the orthogonal wavenumber spectrum. (As the nanotube is considered long compared to the exchange length l_{ex}, the longitudinal wavenumber can be considered to change continuously; on the other hand, the nanotube thickness is of the same order of magnitude with l_{ex}, so the orthogonal wavenumber has an essentially discrete spectrum). This spectrum can be found, for instance, after applying boundary conditions for the magnetic potential on the “free” layer boundaries.

In a general case, we have to solve the equation (12) both inside and outside the “free” layer and match the obtained solutions using the above-mentioned boundary conditions. Because of the presence of the spin-polarized current and the corresponding additional magnetic field, the task of finding the orthogonal wavenumber spectrum is rather complicated. However, for a one-layer ferromagnetic nanotube in the absence of the current (κ=0) the task simplifies significantly. In particular, when the “free” ferromagnetic layer is bounded by nonmagnetic metal surfaces composed of a high-conductivity metal (so the conductivity can be considered infinitely high when we write down the boundary conditions), the boundary conditions reduce to the condition of a zero normal derivative of the magnetic potential on the surface of the ferromagnet:

$$\nabla\Phi\vec{n}_0 = 0, \quad (18)$$

here $\vec{n}_0$ is a unit vector of the interface normal. As the elliptic coordinates are normal coordinates, the condition (18) can be rewritten as

$$\left.\frac{\partial\Phi}{\partial u}\right|_{u=u_1,u_2} = 0, \quad (19)$$

or

$$C_1 Ce'_m(u_1,\alpha)\,ce_m(v,\alpha) + C_2 Se'_m(u_1,\alpha)\,se_m(v,\alpha)$$
$$= C_1 Ce'_m(u_2,\alpha)\,ce_m(v,\alpha) + C_2 Se'_m(u_2,\alpha)\,se_m(v,\alpha) = 0. \quad (20)$$

In order for the conditions (20) to be satisfied for an arbitrary v we have to put $C_1 = 0$ or $C_2 = 0$. Thus, we obtain two classes of solutions:

$$\left\{\begin{array}{l} F(u,v,k_\perp) = C_1 Ce_m\left(u,\frac{k_\perp^2 d^2}{16}\right) ce_m\left(v,\frac{k_\perp^2 d^2}{16}\right), \\ Ce'_m\left(u_1,\frac{k_\perp^2 d^2}{16}\right) = Ce'_m\left(u_2,\frac{k_\perp^2 d^2}{16}\right) = 0 \\ F(u,v,k_\perp) = C_2 Se_m\left(u,\frac{k_\perp^2 d^2}{16}\right) se_m\left(v,\frac{k_\perp^2 d^2}{16}\right), \\ Se'_m\left(u_1,\frac{k_\perp^2 d^2}{16}\right) = Se'_m\left(u_2,\frac{k_\perp^2 d^2}{16}\right) = 0. \end{array}\right. \quad (21)$$

The system (21) defines the sought wavenumber spectrum.

7 Discussion

Thus, we applied a theory of linear spin waves in the presence of a spin-polarized current to a previously uninvestigated case - dipole-exchange spin waves in a ferromagnetic nanotube with an elliptic cross-section. In the resulting dispersion relation, we obtained addends that are both mathematically analogous to those known for other nanosystems, as well as different ones.

Let us compare the obtained dispersion relation (17) with the dispersion relation obtained in the earlier paper of the authors [31] for a cylindric nanotube without dissipation and spin-polarized current. As we can see, the expression (17) for κ=0 and α_G=0 is mathematically similar to the dispersion relation presented in [31] despite different geometry of the system. However, the expressions for the orthogonal wavenumber spectrum for an elliptic nanotube (21) and cylindric nanotube [31] differ; therefore the wavenumber spectrum depends on the nanotube geometry. Consideration of the dissipation leads to the appearance the damping (non-zero imaginary part of the frequency) and also changes the real part of the frequency by the factor $1/(1+\alpha_G^2)$. Consideration of the spin-polarized current changes the real part of the frequency by the addend $\pm\kappa/(1+\alpha_G^2)$ and also changes the imaginary part of the frequency, leading to the appearance of the "effective dissipation".

Let us analyze the influence of the spin-polarized current on imaginary part of the frequency given by the dispersion relation (17).

As we can see from (17), the imaginary part of the spin wave frequency contains a positive (damping) addend that describes the spin wave dissipation and an addend that describes the effects of the spin-polarized current; the latter can be positive or negative depending on the direction of the current. The resulting imaginary part of the frequency can be positive or negative ("effective dissipation") - and, therefore, the excitation or damping process can dominate so that the spin wave can grow or attenuate in amplitude with time, correspondingly. Let us analyze the sign of the imaginary part of the frequency depending on the value of the spin-polarized current.

In order for the spin wave to grow with time, the following conditions must fulfill: the current should be is positive (J>0), and the value $\kappa > \alpha_G \gamma M_0 \left(\alpha k^2 + \beta + H_0^{(e)}/M_0\right)$. These two conditions together are equivalent to the following condition on the value of the current:

$$J > J_{cr} = \frac{2e\alpha_G M_0^2}{\varepsilon\hbar} d \left(\alpha k^2 + \beta + \frac{H_0^{(e)}}{M_0}\right). \qquad (22)$$

In this case, a spin wave generation takes place. If the current is positive, but its absolute value is less than the value in the right side of (22), so that the condition $0<J< J_{cr}$ is fulfilled, the process of damping dominates over the process of generation, but the spin-polarized current weakens the effective dissipation. Finally, for the case of a negative current (J<0) the presence of the spin-polarized current increases the damping. If the wave attenuates, the characteristic relaxation time (for the case of a real wavenumber) can be written in the following form:

$$\tau = \frac{2\pi}{Im\omega} = \frac{2\pi\left(1+\alpha_G^2\right)}{\alpha_G\left|\gamma\right| M_0 \left(\alpha k^2 + \beta + \frac{H_0^{(e)}}{M_0}\right) \mp \kappa}. \qquad (23)$$

We can also use this expression for a growing wave. However, in this case the relaxation time obtained from (23) has a negative value.

Note that if the current J is exactly equal to the expression in the right hand part of the equation (22), the damping and generation processes become balanced and, therefore, the spin wave does not grow or attenuate with time. Therefore, a self-sustained magnetization precession occurs.

Analogous effects have been obtained by Slavin and Tiberkevich [32] for a simpler system - a ferromagnetic nanodisc with a spin-polarized current passing through it. The disc is thin enough for the magnetization precession to be uniform on all of the nanodisc thickness, so its amplitude does not depend on the longitudinal coordinate: only uniform magnetization precession is observed and no wave is generated. This uniformity changes the entire scheme of the investigation; however, in such a thin nanodisc analogous "effective damping" – which can be either positive or negative depending on the value and direction of the current – is also present, and generation of spin oscillations takes place when the value of the current exceeds certain critical value. Analogous effects has been also described in the earlier paper by Slavin and Tiberkevich [27] for a thin film. This verifies the above-obtained results.

Let us compare the expression for the critical current obtained in the current paper with the one obtained by Slavin and Tiberkevich [32] for a ferromagnetic nanodisc. An expression for the critical current in the abovementioned paper has the following form:

$$J_{cr}^d = \frac{2e\alpha_G M_0 \left(H_0^{(e)} - 4\pi M_0\right) d}{\varepsilon\hbar}. \qquad (24)$$

The critical current for the nanosystem investigated in the current paper (22) after omitting the exchange and anisotropy addends neglected in [32] can be written in the following form:

$$J'_{cr} = \frac{2e\alpha_G M_0 H_0^{(e)} d}{\varepsilon\hbar}. \qquad (25)$$

These expressions are similar (internal field $H_0^{(e)}$ for a nanotube corresponds to the internal field ($H_0^{(e)} - 4\pi M_0$) for a nanodisc), thus verifying the obtained results.

Let us make numerical evaluations of the spin wave frequency given by (17) in the absence of the external magnetic field, assuming that the wavenumber k is restricted, on the one hand, by the nanotube length (which makes unities or tens of micrometers for typical nanotubes) and, on the other hand, by the exchange interaction length (which has the order of several nanometers for typical ferromagnets). Thus, the wavenumber for a typical nanotube change from 10^2 cm^{-1} to 10^6 cm^{-1} by the order of magnitude.

Let us choose the following values for the "free" layer ferromagnet: β=1, α=10^{-12} cm^{-2}, γ=10^7 Hz/Gs, $M_0 = 10^3$ Gs (typical values for ferromagnets used in synthesized recently nanosystems, see, e.g., [33–35]). The Gilbert damping constant α_G for a typical ferromagnetic nanosystem used in experiments with a spin-polarized current can be choosen in the range of approximately 0.02-0.2, see, e.g., [36, 37]. Dependence of the real part of the frequency on the wavenumber for such nanotube parameters with α_G

=0.1 and in the absence of the spin-polarized current is given on the Figure 2; as we can see, the spin wave frequency has the order of magnitude of 10^{10} Hz throughout all the wavenumbers range. The characteristic time given by (23), therefore, has the order of magnitude 10^{-8}-10^{-9} s.

Now, let us consider the spin-torque term. As we can see from (17), the spin-torque component affects the effective dissipation (imaginary part of the frequency) essentially on much smaller values of the current than it starts to affect the real part of the frequency: the relation of corresponding critical values of the current density has the order of $\alpha_G \ll 1$. The critical value $\kappa_{cr} = \alpha_G \gamma M_0 \left(\alpha k^2 + \beta + H_0^{(e)}/M_0\right)$ – the value on which the sign of the dissipation changes – has the order of magnitude 10^8-10^9 Hz depending on the value of α_G, so the critical value of the current density J_{cr} has the order of 10^{16} Fr/(s $\cdot$ cm^2) (3$\cdot 10^6$ A/cm^2). These are typical values of the current density used in corresponding experiments (see, e.g., [28]), so the spin wave generation condition obtained above can be used for technical applications. Moreover, the limitations of the above-used linear theory on the value of J (the value $\frac{\varepsilon \hbar J}{2eM_0^2 d}$ must have the order of 1 or less) lead to the appearance of the upper limit of the current density of about $J_{max}=10^{17}$ Fr/(s $\cdot$ cm^2) (approximately 3$\cdot 10^7$ A/cm^2) which is much more than the critical value. Therefore, the above-mentioned theory is applicable to the spin wave generation on the current density interval from J_{cr} to J_{max}.

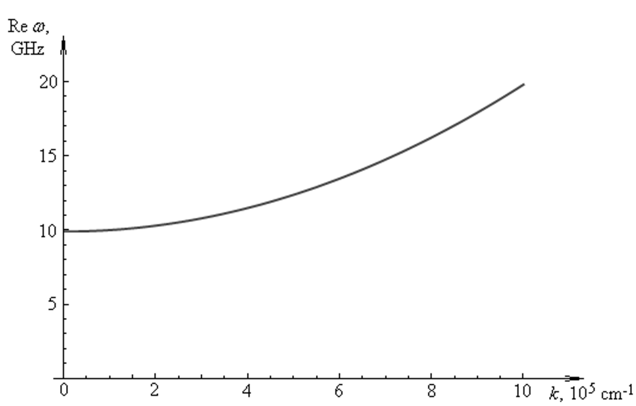

Figure 2: Dependence of Re ω on k for typical values of the nanotube parameters.

The graphical representation of the dependence of the relation of the imaginary (damping) part of the spin wave frequency to the real part on the current density for the obtained above limiting values of the wavenumber interval – k = 10^2 cm^{-1} and k = 10^6 cm^{-1} – is given on the Figure 3.

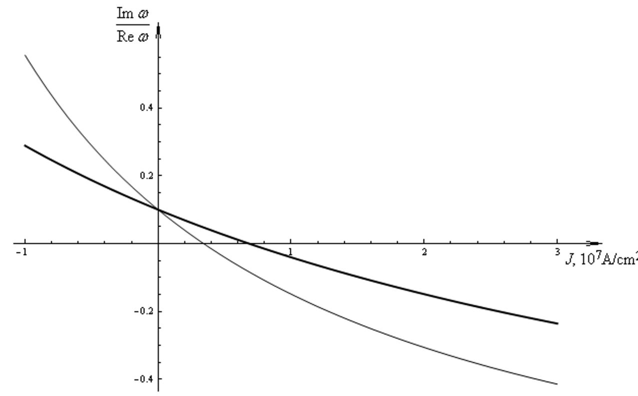

Figure 3: Dependence of Im ω/Re ω on J for typical values of the nanotube parameters. Thinner line represents the dependence for k = 10^2 cm^{-1}, thick line – for k = 10^6 cm^{-1}.

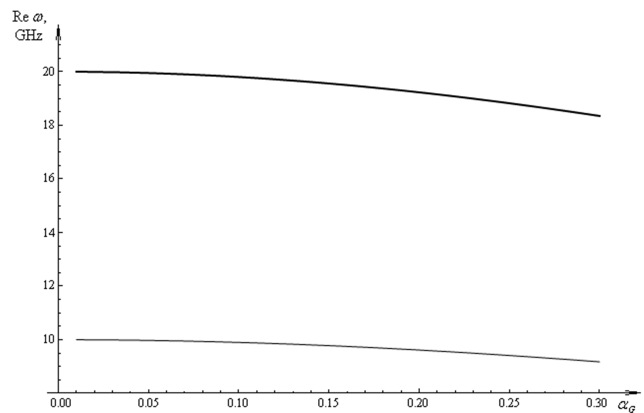

Figure 4: Dependence of Re ω on α_G for typical values of the nanotube parameters. The thinner line represents the dependence for k = 10^2 cm^{-1}, thick line – for k = 10^6 cm^{-1}.

The graphical representation of the dependence of the real part of the spin wave frequency given by (17) on the damping constant α_G for the obtained above limiting values of the wavenumber interval is given in Figure 4. As we can see, for typical values of the damping constants of ferromagnets used in experiments with spin-polarized current real part of the frequency only depends weakly on the damping constant throughout all of the admissible wavenumber interval.

The graphical representation of the dependence of the imaginary part of the spin wave frequency given by (17) on the damping constant α_G for the obtained above limiting values of the wavenumber interval is given in Figure 5. As we can see, for the imaginary part of the frequency we obtain a predictable result: it depends on the damping constant essentially. The latter dependence is close to linear on the same interval of damping constant values.

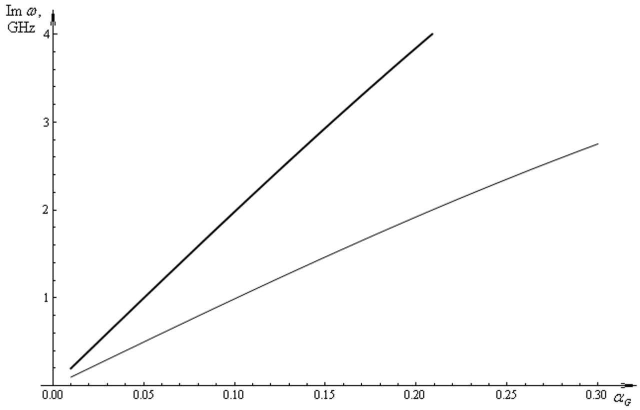

Figure 5: Dependence of Im ω on α_G for typical values of the nanotube parameters. The thinner line represents the dependence for $k = 10^2$ cm^{-1}, thick line – for $k = 10^6$ cm^{-1}.

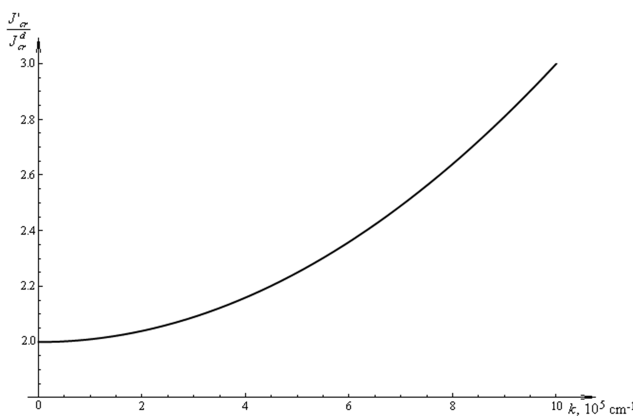

Figure 6: Relation of critical current for a nanotube to a critical current for a nanodisc for typical nanosystem parameters.

Finally, let us analyze the difference between the critical current value for spin waves in the nanosystem investigated in the current paper (22) and for spin oscillations in a ferromagnetic nanodisc (24) investigated in [32]. The relation between these critical currents for similar corresponding parameters of the nanosystems can be written (after considering difference in expressions for internal magnetic field) as follows:

$$\frac{J_{cr}}{J_{cr}^{d}} = \frac{\alpha k^2 + \beta + H_0^{(i)}/M_0}{H_0^{(i)}/M_0} \tag{26}$$

Let us make a graphical representation of that relation for a typical (given above) values of the parameters of the ferromagnetic nanosystems. In this part of the analysis we cannot neglect the internal magnetic field as for a disc the critical current corresponding to zero internal magnetic field is also equal to zero. Let us choose a typical value $H_0^{(i)} = 10^3$ Gs (see, e.g., [38]) for both cases (so we can consider $H_0^{(i)}/M_0 = 1$). The resulting dependence of the relation J_{cr}/J_{cr}^{d} on the wavenumber is given on the Fig. 6.

As we can see, the critical current for spin waves exceeds the critical current for spin oscillations. The cause for that is absence of space dependence in the magnetization density (and, therefore, the possibility to neglect the exchange and anisotropy effects) for uniform oscillations [32]. Even for the long waves ($k \sim 10^2$ cm^{-1}), when the exchange effects can be neglected, the critical current for spin waves exceeds the critical current for spin oscillations by approximately 2 times (for the given nanosystems parameters) because of the anisotropy effects.

8 Conclusions

In the paper, spin waves in a two-layer ferromagnetic nanotube with an elliptic cross-section in the presence of a spin-polarized current are investigated. One of the nanotube layers is considered "free" in the sense of the magnetization orientation (and is composed of an "easy axis" uniaxial ferromagnet), the other - "fixed", and the spin-polarized current passes through the "free" layer in the direction orthogonal to its surface. For such nanotube, a differential equation for the magnetic potential of the spin wave in the magnetostatic approximation – with account for the dipole-dipole magnetic interaction, anisotropy effects, dissipation effects, and the influence of the spin-polarized current – is obtained. The equation is solved and a relation between the wave frequency and two wavenumber components is obtained. For a thin shell, a dispersion relation for such spin waves is obtained.

For a one-layer ferromagnetic nanotube with an elliptic cross-section (in the absence of the spin-polarized current) the orthogonal wavenumber spectrum of such spin waves is also obtained (in a non-explicit form).

The presence of the spin-polarized current can strengthen or weaken the spin wave damping, creating the "effective dissipation" (analogous to the effective dissipation for a two-layer ferromagnetic film with a spin-polarized current). The effective dissipation can be positive or negative (depending on the direction and the density of the current); for the negative dissipation, the spin wave grows in amplitude with time, thus leading to a spin wave generation. The condition for this generation is found.

References

[1] V. V. Kruglyak, S. O. Demokritov, D. Grundler, J. Phys. D: Appl. Phys. 43, 264001 (2010)
[2] S. D. Bader, S. S. P. Parkin, Annual Review of Condensed Matter Physics 1, 71 (2010)
[3] T. Schneider, A. A. Serga, B. Leven, B. Hillebrands, R. L. Stamps, M. P. Kostylev, Appl. Phys. Lett. 92, 022505 (2008)
[4] M. R. Freeman, B. C. Choi, Science 294, 1484 (2001)
[5] S. Neusser, D. Grundler, Adv. Mat. 21, 2927 (2009)
[6] R. E. De Wames, T. Wolfram, Appl. Phys. Lett. 16, 305 (1970)
[7] R. Arias, D. L. Mills, Phys. Rev. B 63, 134439 (2001)
[8] R. Arias, D. L. Mills, Phys. Rev. B 67, 094423 (2003)
[9] R. Arias, D. L. Mills, Phys. Rev. B 72, 104418 (2005)
[10] I. V. Zavislyak, G. P. Golovach, M. A. Popov, V. F. Romanyuk, Journal of Communications Technology and Electronics 51, 203 (2006)
[11] T. K. Das, M. G. Cottam, J. Appl. Phys. 109, 07D323 (2011)
[12] A. Kozhanov, M. Popov, I. Zavislyak, D. Ouellette, D. W. Lee, S. X. Wang, M. Rodwell, S. J. Allen, J. Appl. Phys. 111, 013905 (2012)
[13] H. Leblond, V. Veerakumar, Phys. Rev. B 70, 134413 (2004)
[14] A. L. González, P. Landeros, Á. S. Núñez, J. Magn. Magn. Mater 322, 530 (2010)
[15] J. A. Otálora, J. A. López-López, A. S. Núñez, P. Landeros, J. Phys.: Condens. Matter 24, 436007 (2012)
[16] Y. Ye, B. Geng, Critical Reviews in Solid State and Materials Sciences 37, 75 (2012)
[17] A. K. Salem, P. C. Searson, K. W. Leong, Nat. Mater. 2, 668 (2003)
[18] C. C. Berry, A. S. G. Curtis, J. Phys. D: Appl. Phys. 36, R198 (2003)
[19] G. Xiao-Kun, C. Bing-Yang, Chinese Phys. 16, 3777 (2007)
[20] D. M. N. Hasan, M. N. Hossain, K. M. Mohsin, M. S. Alam, In: TENCON-2010 Computer, Circuit and Systems, Antennas and Propagation - 2010 IEEE Region 10 Conference, Nov. 21-24, 2010, Fukuoka, Japan (IEEE, Japan, 2010) 1915
[21] S. Dag, S. Ciraci, Phys. Rev. B. 71, 165414 (2005)
[22] R. L. D. Whitby, W. K. Hsu, Y. Q. Zhu, H. W. Kroto, D. R. M. Walton, Phil. Trans. R. Soc. Lond. A. 362, 2127 (2004)
[23] V. Holovatsky, V. Gutsul, Cond. Matt. Phys. 10, 61 (2007)
[24] Y. Ye, B. Geng, Crit. Rev. Solid State Mater. Sci. 37, 75 (2012)
[25] M. A. Popov, I. V. Zavislyak, Ukr. J. Phys. 53, 702 (2008)
[26] C. Wu, Ph.D. thesis, City University of New York (New York, USA, 2008)
[27] A. Slavin, V. Tiberkevich, IEEE Transactions on Magnetics 44, 1916 (2008)
[28] J. C. Slonczewski, J. Magn. Magn. Mater. 159, L1 (1996)
[29] L. Berger, Phys. Rev. B 54, 9353 (1996)
[30] A. I. Akhiezer, V. G. Barýakhtar, S. V. Peletminskiy, Spin waves, 1st edition (North-Holland, Amsterdam, 1968)
[31] Yu. I. Gorobets, V. V. Kulish, Ukr. Journ. Phys. 59, 541 (2014)
[32] A. Slavin, V. Tiberkevich, IEEE Transactions on Magnetics 45, 1875 (2009)
[33] H. Y. Liu, Z. K. Wang, H. S. Lim, S. C. Ng, M. H. Kuok, D. J. Lockwood, M. G. Cottam, K. Nielsch, U. Gösele, J. Appl. Phys. 98, 046103 (2005)
[34] C. Mathieu, J. Jorzick, A. Frank, S. O. Demokritov, A. N. Slavin, B. Hillebrands, Phys. Rev. Lett. 81, 3968 (1998)
[35] N. A. Usov, S. E. Peschany, Magn. Magn. Mater 118, L290 (1993)
[36] Y. Tserkovnyak, A. Brataas, G. E. W. Bauer, Phys. Rev. Lett. 88, 117601 (2002)
[37] S. J. Gamble, M. H. Burkhardt, A. Kashuba, R. Allenspach, S. S. P. Parkin, H. C. Siegmann, J. Stöhr, Phys. Rev. Lett. 102, 217201 (2009)
[38] W. H. Rippard, M. R. Pufall, S. Kaka, S. E. Russek, T. J. Silva, Phys. Rev. Lett. 92, 027201 (2004)