Research Article

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Three phase heat and mass transfer model for unsaturated soil freezing process: Part 1 - model development

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Abstract: A three-phase model capable of predicting the heat transfer and moisture migration for soil freezing process was developed based on the Shen-Chen model and the mechanisms of heat and mass transfer in unsaturated soil freezing. The pre-melted film was taken into consideration, and the relationship between film thickness and soil temperature was used to calculate the liquid water fraction in both frozen zone and freezing fringe. The force that causes the moisture migration was calculated by the sum of several interactive forces and the suction in the pre-melted film was regarded as an interactive force between ice and water. Two kinds of resistance were regarded as a kind of body force related to the water films between the ice grains and soil grains, and a block force instead of gravity was introduced to keep balance with gravity before soil freezing. Lattice Boltzmann method was used in the simulation, and the input variables for the simulation included the size of computational domain, obstacle fraction, liquid water fraction, air fraction and soil porosity. The model is capable of predicting the water content distribution along soil depth and variations in water content and temperature during soil freezing process.

Keywords: Lattice Boltzmann; unsaturated soil freezing; three phase model; water content

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1 Introduction

During soil freezing, both heat transfer and water migration coexist in the freezing process which occurs in a coupled manner. When a temperature gradient forms in the soil, the temperature gradient drives the heat to flow from the higher temperature zone towards the lower one and the pore water to migrate from the unfrozen zone to the freezing fringe, and then to the frozen front. The water migration from warmer unfrozen zone can influence the heat conduction process due to the effect of convection and latent heat of phase change, while the heat conduction may induce phase change and in turn change the hydraulic conductivity of the soil.

Because of the importance of the soil freezing process, many classical models have been developed, such as finite difference method (FDM), finite element method (FEM) and finite volume method (FVM), to describe the coupled heat-fluid transport phenomenon of the soil freezing process when ice lensing does not occur [1–6]. However, the traditional numerical methods are based on the discretization of macroscopic continuum equations. This scheme makes traditional numerical methods face great challenges with solving flow with complex interface or flow with complex boundary. Besides, the traditional numerical methods have difficulties dealing with the microscale force such as interactive force in the pre-melted film. Recently, numerical models based on LBM (Lattice Boltzmann method) for simulating the heat and mass transfer phenomena with phase transformation in frozen soil during freezing process are presented [7, 8]. However, the freezing fringe, the balance between surface tension and gravity, and the pre-melted film in frozen front were

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not considered. Here, in this work, based on the Shen-Chen model which gives a Lattice Boltzmann approach for multiphase fluid flows, and allows to calculate the temporal and spatial evolution of the density distribution functions f_i for an arbitrary number of components [9–12], we further developed a new Lattice Boltzmann model for the simulation of soil freezing. The suction of the pre-melted film in frozen front and freezing fringe was regarded as a kind of suction force, two kinds of resistance in unfrozen zone and freezing fringe were regarded as a kind of body force, and the balance between the surface tension and gravity was regarded as a kind of block force.

The aim of this study was to develop a comprehensive three-phase Lattice Boltzmann model capable of describing the heat and mass transfer during unsaturated soil freezing. The model is capable of predicting the water content distribution along the depth of soil and the water content variations with temperature in unfrozen zone, freezing fringe and frozen zone.

2 Basic models

2.1 Physical model of soil freezing

The physical model is schematically illustrated in Figure 1. During soil freezing, as temperature gradient forms in soil, heat moves gradually from high temperature to the lower one, and moisture migrates from unfrozen zone to the frozen zone. The heat and mass transfer caused by the temperature gradient can be divided into three parts (frozen zone, freezing fringe and unfrozen zone). In frozen zone, due to the intermolecular interactions, a certain amount of unfrozen water exists in the pre-melted film between the surfaces of soil grains and ice lens [13, 14], and the pre-melted film forms the channel for the migration of liquid water towards the solidification front in soils to supply the growth of ice lenses [15]. In freezing fringe, the temperature is lower than freezing point T_m , and higher than the ice entering point T_{ie} . A certain amount of unfrozen water exists between the surfaces of soil grains and ice grains due to the Gibbs-Thomson effect [16, 17], and the formation of the ice grains rises the resistance for the migration of liquid water. In unfrozen zone, water exists among the soil pore space where the adhesion force and water-air interface tension keep balance with gravity.

Figure 1: Schematic of physical model for unsaturated soil freezing

2.2 Soil structure

In nature, soils are porous media with stochastic particle size distribution which is an important soil characteristic, and this characteristic has significant effects on the heat and mass transfer during soil freezing process [18–20]. In this paper, random generated volume fraction for each lattice was used to describe this characteristic.

In theoretical and experimental work on fluid flow in porous media, it is typically attempted to find functional correlations between the particle size distribution and some other macroscopic properties of the porous medium. Among the most important of such properties are the porosity ϕ and the specific surface area S , which give the ratios of the total void volume and the total interstitial surface area to the bulk volume, respectively [21].

The value of porosity is equal to the probability that a given point in volume V is not overlapped by any of the obstacles, this can be calculated by:

$$\phi = (1 - f_v)^K \quad (1)$$

where K is the number of obstacles, $f_v = V_0/V$ is the average volume fraction of obstacles, V_0 is the average volume of obstacles.

The specific surface area is $S = A/V$ (A is the value of the total surface area of all obstacles), which is given by the total number of obstacles times the surface area of a single obstacle times the probability that a given point on the surface of an obstacle is not overlapped by any other

obstacles, i.e.,

$$A = KA_0(1 - f_v)^{K-1} \quad (2)$$

where A_0 is the average surface area of obstacles.

It is observed that both these two macroscopic properties are related to the average volume fraction of obstacles f_v . Similarly, we get the macroscopic properties (porosity and specific surface area) from the average volume fraction of obstacles f_{vl} and the average number of obstacles in a single lattice N , where $f_{vl} = Nf_v$. Consequently, it is reasonable to use the randomly generated volume fraction to indicate the stochastic particle size distribution.

2.3 Lattice model

The D2Q9 lattice model was used in this simulation, as shown in Figure 2. This model is very common, especially for solving fluid flow problems. It has high velocity vectors, with the central particle speed being zero, and the streaming velocity for the D2Q9 model takes the value:

$$e_i = \begin{cases} (0, 0) & i = 1 \\ (\pm 1, 0), (0, \pm 1), & i = 2 - 5 \\ (\pm 1, \pm 1), & i = 6 - 9 \end{cases} \quad (3)$$

The associated lattice weights w_i are:

$$w_i = \begin{cases} \frac{4}{9} & i = 1 \\ \frac{1}{9} & i = 2 - 5 \\ \frac{1}{36} & i = 6 - 9 \end{cases} \quad (4)$$

The sound speed (c_s) is equal to $1/\sqrt{3}$.

Figure 2: Lattice model

2.4 Enthalpy conservation

The conservation of enthalpy for the three phases is:

$$\frac{\partial(\rho_s C_s T_s)}{\partial t} = -k_s \nabla T_s \quad (5a)$$

$$\frac{\partial(\rho_l C_l T_l)}{\partial t} = \rho_l C_l T_l - k_l \nabla T_l \quad (5b)$$

$$\frac{\partial(\rho_g C_g T_g)}{\partial t} = \rho_g C_g T_g - k_g \nabla T_g \quad (5c)$$

where k_i and T_i respectively refer to the thermal conductivity and temperature of phase i , C_i is the specific heat, ρ_s , ρ_l and ρ_g are the local density of solid, liquid and gas, respectively.

2.4.1 Initial conditions

The initial conditions are as follows:

$$T = T_{cold} \quad (6)$$

2.4.2 Boundary conditions

The boundary conditions are as follows:

$$T_{up} = T_{cold} \quad \text{at } t = 0 \quad (7)$$

$$T_{down} = T_{hot} \quad \text{at } t = 0 \quad (8)$$

Periodic temperature boundary conditions are applied on the side boundaries, as shown in Figure 3.

2.4.3 Heat conduction with phase change in porous media

During soil freezing process, the thermal diffusion satisfies the usual Lattice Boltzmann equation:

$$g_i(\mathbf{X} + e_i \Delta t, t + \Delta t) = g_i(\mathbf{X}, t) \quad (9)$$

Figure 3: Temperature boundary conditions

$$-\frac{\Delta t}{\tau_h} [g_i(\mathbf{X}, t) - g_i^{eq}(\mathbf{X}, t)]$$

where g_i is the temperature distribution function in the i -th velocity direction, and τ_h is the relaxation time related to the thermal diffusion as $\alpha = c_s^2(\tau_h - 0.5)$. The equilibrium distribution function for g_i^{eq} can be calculated as usual as: $g_i^{eq}(\mathbf{X}, t) = w_i g$.

2.5 Mass and momentum conservation

2.5.1 Mass conservation

In a lattice with a random obstacle volume fraction f_v , the mass conservation for each of the two fluids and the solid fraction can be written in the following equations:

$$\frac{\partial \rho_g}{\partial t} + \nabla \cdot (\rho_g \mathbf{u}_g) = 0 \quad (10a)$$

$$\frac{\partial \rho_s}{\partial t} = -\Gamma_m \quad (10b)$$

$$\frac{\partial \rho_l}{\partial t} + \nabla \cdot (\rho_l \mathbf{u}_l) = \Gamma_m \quad (10c)$$

where Γ_m is the melting rate, $\mathbf{u}_g$ and $\mathbf{u}_l$ are the local fluid velocities of liquid and gas, respectively.

2.5.2 Momentum conservation

Momentum equations for the fluid mixture are seen as a single fluid:

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p + \nabla \cdot [\rho \nu (\nabla \mathbf{u} + \nabla \mathbf{u}')] + \rho \mathbf{g} \quad (11)$$

where $\rho = \sum_{\sigma} \rho_{\sigma}$ is the total density of the mixture. The total momentum of the fluid mixture is:

$$\rho \mathbf{u} = \sum_{\sigma} \sum_i f_i^{\sigma} v_i + 1/2 \sum_{\sigma} F_{\sigma} \quad (12)$$

2.5.3 Initial conditions

The initial conditions are as follows:

$$\begin{cases} \mathbf{u}_g = \mathbf{u}_l = 0 \\ \rho_s = 0 \\ \rho_g = \rho_{g0} \\ \rho_l = \rho_{l0} \end{cases} \quad (13)$$

2.5.4 Boundary conditions

Bounce back boundary conditions are applied on both top (down) boundaries and ice-fluid interfaces in order to maintain mass conservation. The periodic boundary conditions are applied on the side boundaries in order to maintain the continuity of flow, as shown in Figure 4.

Figure 4: Fluid flow boundary conditions

2.6 Multiphase fluid model

To our best of knowledge, the Shen-Chen model is the most widely used multiphase LB model for the simulation of multiphase fluid [9-12]. In Shen-Chen model, distribution function of all components in soil freezing satisfies the usual Lattice Boltzmann equation:

$$f_i^{\sigma}(\mathbf{X} + \mathbf{e}_i \Delta t, t + \Delta t) = f_i^{\sigma}(\mathbf{X}, t) - \frac{\Delta t}{\tau_{\sigma}} [f_i^{\sigma}(\mathbf{X}, t) - f_i^{\sigma,eq}(\mathbf{X}, t)] \quad (14)$$

where f_i^{σ} is the density distribution function of the σ -th component in the i -th velocity direction, and τ_{σ} is the relaxation time related to the kinematic viscosity for the component σ as $\nu_{\sigma} = c_s^2(\tau_{\sigma} - 0.5)$. The equilibrium distribution function for $f_i^{\sigma,eq}(\mathbf{X}, t)$ can be calculated as usual as:

$$f_i^{\sigma,eq}(\mathbf{X}, t) = \omega_i \rho_{\sigma} \cdot \left[1 + \frac{\mathbf{e}_i \cdot \mathbf{u}_{\sigma}^{eq}}{c_s^2} + \frac{(\mathbf{e}_i \cdot \mathbf{u}_{\sigma}^{eq})^2}{2c_s^4} - \frac{\mathbf{u}_{\sigma}^{eq2}}{2c_s^2} \right] \quad (15)$$

where the density and momentum of the σ fluid component, can be calculated through:

$$\rho_{\sigma} = \sum_i f_i^{\sigma} \text{ and } \rho_{\sigma} \mathbf{u}_{\sigma} = \sum_i V_i f_i^{\sigma} \quad (16)$$

$\mathbf{u}_{\sigma}^{eq}$ in equation (15) is determined by the relation:

$$\mathbf{u}_{\sigma}^{eq} = \mathbf{u}' + \frac{\tau_{\sigma} \mathbf{F}_{\sigma}}{\rho_{\sigma}} \quad (17)$$

where u' is the common velocity of the fluid mixture, and it is calculated as:

$$\mathbf{u}' = \frac{\sum_{\sigma} \frac{\rho_{\sigma} \mathbf{u}_{\sigma}}{\tau_{\sigma}}}{\sum_{\sigma}} \quad (18)$$

and $\mathbf{F}_{\sigma} = \mathbf{F}_{\sigma}^{coh} + \mathbf{F}_{\sigma}^{ads} + \mathbf{F}_{\sigma}^b$ is the sum of the interactive force between fluid particles, $\mathbf{F}_{\sigma}^{coh}$ (responsible of cohesion force), between solid boundary and fluid particles, $\mathbf{F}_{\sigma}^{ads}$ (responsible of adhesion forces) and external body forces, $\mathbf{F}_{\sigma}^b$, such as gravity and buoyancy forces.

The interactive force acting on the particles of species σ located in position x can be calculated as:

$$\mathbf{F}_{\sigma}^{coh}(\mathbf{X}, t) = -\rho_{\sigma}(\mathbf{X}, t) G^{coh} \sum_i \omega_i \rho_{\bar{\sigma}}(\mathbf{X} + \mathbf{e}_i \Delta t) \mathbf{e}_i \quad (19)$$

where σ and $\bar{\sigma}$ are respectively the first and second components of the fluid mixture and G^{coh} is the parameter that can be used to tune the surface tension between the two modeled fluids. Analogously, the cohesion force between particles of the fluid σ and the solid boundary, can be evaluated as:

$$\mathbf{F}_{\sigma}^{ads}(\mathbf{X}, t) = -\rho_{\sigma}(\mathbf{X}, t) G_{\sigma}^{ads} \sum_i \omega_i s_i(\mathbf{X} + \mathbf{e}_i \Delta t) \mathbf{e}_i \quad (20)$$

where s is a flag variable that is equal to 1 if the lattice node i belongs to the solid boundary and it is equal to 0 if s_i points a fluid lattice node.

Eventually, the action of a constant body force that mimics the effect of a buoyancy force can be added as:

$$\mathbf{F}_{\sigma}^b(\mathbf{X}, t) = -\Delta \rho \mathbf{g} \quad (21)$$

where $\mathbf{g}$ is the gravity and $\Delta \rho$ the difference in density between the two components.

3 Developed models

3.1 Melting problem in soil freezing

For bulk water freezing, the problem of half space conduction melting with thermal diffusivity which is called Neumann-Stefan problem, has been solved analytically in 1860 by Neumann. Here, the enthalpy-based method by Jiaung *et al.* [22, 23], which has been successfully used by Huo and Rao [24] for solid-liquid phase change phenomenon of phase change material under constant heat flux, was modified for the LB approaches of melt-solid moving boundary in porous media.

In enthalpy-based method, the melting term is introduced as a source (crystallization) or sink (melting) term

in the collision step. In summary, at the time-step n and iteration kn , the macroscopic temperature is calculated by:

$$T^{n,k} = \sum_i g_i^{n,k} \quad (22)$$

where $T_{n,kn} \equiv T_{kn}(t = n)$. The local enthalpy is obtained by:

$$En^{n,kn} = cT^{n,kn} + L_f f_l^{n,kn-1} \quad (23)$$

with the liquid fraction f_l of the previous iteration. Finally, the enthalpy is used to linearly interpolate the melt fraction

$$f_l^{n,kn} = \begin{cases} 0 & En^{n,kn} < En_s = cT_{m0}, \\ \frac{En^{n,kn} - En_s}{En_l - En_s} & En_s \leq En^{n,kn} \leq En_s + L_f, \\ 1 & En^{n,kn} > En_s + L_f; \end{cases} \quad (24)$$

where En_s and En_l are the enthalpies of the solid and liquid at the melting temperature T_m .

However, in freezing fringe, according to Gibbs-Thomson equation, the melting temperature in porous media is related to the pore throat, that is to say, the particle size distribution plays an important role in the pore water melting temperature [25]. According to Saruya *et al.* [25], the difference between the warmest temperature T_f at which ice can first form and the normal bulk melting temperature $T_{m0} \approx 273.15$ K can be obtained by $\Delta T_f = T_{m0} - T_m = 2\gamma_{sl}T_m/(\rho L_f R_p)$, where, R_p is the characteristic radius of a pore throat, $\rho L_f \approx 3.1 \times 10^8$ J/m³ is the latent heat of fusion per unit volume and $\gamma_{sl} \approx 29 \times 10^{-3}$ J/m² is the ice-liquid surface energy, ΔT_f which describes the difference between the warmest temperature T_f at which ice can first form and the normal bulk melting temperature $T_{m0} \approx 273.15$ K. So, the actual water melting temperature in porous media is:

$$T_m = T_{m0} - \frac{2\gamma_{sl}T_{m0}}{\rho L_f R_p} \quad (25)$$

The characteristic radius of pore throat is $R_p = \alpha_p R$, where, R is the particle radius, and $\alpha_p = 0.7$ is a correlation coefficient. The average characteristic radius of a single lattice in the simulation can be calculated as:

$$R_p = \alpha_p \left(\frac{3f_v}{4\pi} \right)^{\frac{1}{3}} \quad (26)$$

In frozen zone, due to the intermolecular interactions between the particles, liquid, and ice (e.g., van der Waals forces), interfacial melting between ice surface and soil particle surface below bulk melting point T_m can be found, and the thickness of this pre-melted film upon approaching the melting point T_m follows in a logarithmic growth law [26]:

$$L(T) = a(0) \ln \frac{T_m - T_0}{T_m - T} \quad (27)$$

Figure 5: Schematic of pre-melted film

where $T_0 \approx T_m - 17$ K. The constant $a(0) \approx 0.84$ nm corresponds to the decay length of the non-ordering (average) density.

As shown in Figure 5, the average thickness of pre-melted film in a single lattice is also related to the average obstacle volume fraction f_v , and the relationship is:

$$L = \left[\frac{3}{4\pi} (1 - f_i + f_i f_v) \right]^{\frac{1}{3}} \quad (28)$$

Combine Eq. (27) with (28), we get the relationship between the average obstacle volume fraction f_v and melting temperature of water pre-melted film:

$$T_{mi} = T_m - \frac{T_m - T_0}{e \left(\left[\frac{3}{4\pi} (1 - f_i + f_i f_v) \right]^{\frac{1}{3}} \right)} \quad (29)$$

In the developed model, the bulk melting temperature T_m in equation (24) was replaced by the variable melting temperature (T_{mi}). Consequently, the formation of pre-melted is possible.

The collision is calculated by:

$$g_i^{n,k+1}(\mathbf{X} + \mathbf{e}_i) = g_i(\mathbf{X}) - \frac{1}{\tau_h} (g_i(\mathbf{X}) - g_i^{eq}(\mathbf{X})) - t_i \frac{L_f}{c} (f_l^{n,k}(\mathbf{X}) - f_l^{n-1}(\mathbf{X})) \quad (30)$$

where τ_h is the relaxation time related to the thermal diffusion for the component σ as $k_\sigma = c_s^2(\tau_h - 0.5)$.

Then, a sub-loop is introduced into every time step until the temperature and the melt fraction field converge to within a set tolerance. Finally, the macroscopic temperature can be calculated as:

$$T = \sum_i T_i \quad (31)$$

3.2 Shen-Chen model for multiphase fluid in soil freezing

During soil freezing, water in the freezing fringe and the unfrozen zone moves towards the frozen front by the suction in pre-melting film and freezing fringe. At the same time, the resistance along the path in the unfrozen zone and the freezing fringe blocks the movement, as shown in Figure 1. Similar to the adhesion force between fluid particles and solid boundaries in the Shen-Chen model, the suction in frozen front can be thought as a kind of interactive force between ice and water particles $\mathbf{F}_\sigma^{fre}$. According to Darcy law, we can infer that the resistance along the path in unfrozen region can be described by a body force $\mathbf{F}_\sigma^{blo}$, which is related to the distance from freezing fringe. Consequently, the sum of the interactive force for mixture fluid flow in soil freezing is:

$$\mathbf{F}_\sigma = \mathbf{F}_\sigma^{coh} + \mathbf{F}_\sigma^{ads} + \mathbf{F}_\sigma^{fre} + \mathbf{F}_\sigma^{blo} + \mathbf{F}_\sigma^b \quad (32)$$

3.2.1 Adhesion force

The interactive force (adhesion force $\mathbf{F}_\sigma^{ads}$) in soil is related to the soil particle distribution, so the adhesion force $\mathbf{F}_\sigma^{ads}$ can be calculated as:

$$\mathbf{F}_\sigma^{ads}(\mathbf{X}, t) = -\rho_\sigma(\mathbf{X}, t) G_\sigma^{ads} \sum_i \omega_i f_i(\mathbf{X} + \mathbf{e}_i \Delta t) \mathbf{e}_i \quad (33)$$

where f is a flag variable that is equal to the soil particle volume fraction of the lattice node i .

3.2.2 Suction force

According to the kinetics of ice growth in porous media [27], The suction force $\mathbf{F}_\sigma^{fre}$ is caused by the pore water pressure difference between frozen zone, freezing fringe and unfrozen zone. First, due to the pore water pressure decrease between ice grain surfaces and soil particle surfaces, water moves from unfrozen zone towards the freezing fringe. Then, because of the deep decrease of water pressure in pre-melted film in frozen front, water moves

again from the freezing fringe towards the frozen front. We can found that the suction only forms in the freezing fringe and pre-melted film near the ice particles. So, it is reasonable to consider the suction as an adhesion force between the ice particles and fluid particles, which can be calculated by:

$$\mathbf{F}_\sigma^{fre}(\mathbf{X}, t) = -\rho_\sigma(\mathbf{X}, t) G^{fre} \sum_i \omega_i l_i (\mathbf{X} + \mathbf{e}_i \Delta t) \mathbf{e}_i \quad (34)$$

where l is a flag variable that equals to f_l for the particles of fluid σ with phase change to happen near the phase change boundary, and it is equal to 0 for the particles of fluid σ without phase change to happen. G^{fre} is the parameter that describes the magnitude of suction force which is related to the pore water pressure drop in freezing fringe and pre-melted film.

According to the generalized Clapeyron equation:

$$P_{cl} = P - \rho L_f \frac{T_m - T}{T_m} \quad (35)$$

where P_{cl} is the pore water pressure in the freezing fringe and pre-melted film at temperature T , P is the soil particle pressure in warm region, T_m is the bulk water freezing point, L_f is the latent of ice melting, ρ is the ice density. If there is a supply of ground water at a pressure P_R , the pore water pressure difference between the ground water and pore water near ice particles is:

$$\Delta P = P_R - P + \rho L_f \frac{T_m - T}{T_m} \quad (36)$$

As we know, the fluid should move from the higher pressure to the lower one. So, in this study the parameter G^{fre} is related to the maximum pressure difference that locates in the frozen front where the temperature is equal to the ice entering temperature T_{ie} :

$$T_{ie} = T_m - \frac{T_m \gamma_{sl}}{\rho L_f R_p} \quad (37)$$

Consequently, the suction force can be described as:

$$\mathbf{F}_\sigma^{fre} = \frac{P_R - P + \frac{\gamma_{sl}}{R_p}}{A_c} \quad (38)$$

where A_c is the cross-sectional area of flow.

Therefore, G^{fre} is a parameter that relates to the surface tension at ice-water interface and the pore structure of soil particles.

3.2.3 Resistance along the path

In soil freezing, due to the suction, water moves from unfrozen zone to the freezing fringe, then to the frozen

front. According to the kinetics of ice growth in porous media [27], there are two kinds of resistance along the path way, one is the hydraulic resistance due to the flow in the porous medium, and the other is the resistance due to the flow in the thin films around the particles.

The hydraulic resistance is:

$$\mathbf{F}_\sigma^h = \frac{\mu h}{k_p} \quad (39)$$

where k_p is the permeability from Carman-Kozeny equation:

$$k_p = \frac{R^2(1-\phi)^3}{100\phi^2} \quad (40)$$

The resistance in the thin films is:

$$\mathbf{F}_\sigma^f = \frac{3k\phi\mu_p R^2}{2L^3} \quad (41)$$

where μ_p is the viscosity of water in thin films.

Finally, the whole resistance along the path is:

$$\mathbf{F}_\sigma^{blo} = \mathbf{F}_\sigma^h + \mathbf{F}_\sigma^f \quad (42)$$

3.2.4 Block force

Before soil freezing, as shown in Figure 6 (a), there is no pressure gradient between pore water and ground water, and to keep the pore water stand still in the space among soil grains, the adhesion force and surface tension at the water-air interface need to keep balance with gravity ($F_t = \rho g h A$). In soil freezing, as shown in Figure 6 (b), pressure gradient starts to form in the pore water, the tension force is gradually replaced by the pressure gradient to keep the balance with gravity, and the water flow begins after the pressure gradient overcomes the gravity. So, this varying force can be thought as a block force instead of the body force $\mathbf{F}_\sigma^b$, and it can be calculated as:

$$\mathbf{F}_\sigma^b = \begin{cases} 0 & \rho_\sigma - \rho_{\sigma,initisl} < 3 * \rho_\sigma g h \\ \rho_\sigma g h & \rho_\sigma - \rho_{\sigma,iniysl} > 3 * \rho_\sigma g h \end{cases} \quad (43)$$

Figure 6: The stress of fluid particles in unfrozen zone

4 Conclusions

Based on the Shen-Chen model, this paper presented a new Lattice Boltzmann model for the simulation of soil freezing. In this model, the suction of the pre-melted film in freezing fringe was regarded as a kind of suction force, the adjustment coefficient of the suction force was a parameter that related to the particle size, water-air surface tension, and ice entering temperature. Two kinds of resistance were regarded as a kind of body force related to the water films between the ice grains and soil grains, and a block force instead of gravity was introduced to keep balance with gravity before soil freezing.

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Nomenclature

ϕ porosity of porous medium (dimensionless)
 S specific surface area (m^2/m^3)
 T_m bulk water melting temperature (K)
 T_{ie} ice entering temperature in porous medium (K)
 K number of obstacles
 f_v average volume fraction of a single obstacles (dimensionless)
 V_0 average volume of obstacles (m^3)
 A_0 average surface area of a single obstacle (m^2)
 f_{vl} average volume fraction of obstacles in a single lattice (dimensionless)
 N average number of obstacles in a single lattice
 c_s sound speed (1/s)
 e discrete velocity (dimensionless)
 w lattice weight (dimensionless)
 k thermal diffusion ($\text{W}/\text{m K}$)
 T temperature (K)
 C specific heat ($\text{J}/\text{kg K}$)
 ρ density (kg/m^3)
 T_{cold} fixed temperature on up boundary (K)
 T_{hot} fixed temperature on down boundary (K)
 τ relaxation time (s)
 α thermal diffusion ($\text{W}/\text{m K}$)
 g temperature distribution function (dimensionless)
 f_l liquid fraction (dimensionless)
 En_s enthalpy of the solid (J/kg)
 En_l enthalpy of the liquid (J/kg)

R_p characteristic radius of a pore throat (m)
 R particle radius (m)
 α_{pa} correlation coefficient (dimensionless)
 γ_{sl} ice-liquid surface tension (N/m)
 L_f latent heat of ice ($\text{J}/\text{kg K}$)
 L film thickness (m)
 u velocity of fluid (m/s)
 F^{coh} cohesion force (N)
 F^{adh} adhesion force (N)
 F^{fre} suction force (N)
 F^{blo} resistance along path way (N)
 F^b body force (N)
 ν kinematic viscosity ($\text{kg}/\text{m s}$)
 f density distribution function (dimensionless)
 P pressure (Pa)
 P_R ground water pressure (Pa)
 P_{cl} pore water pressure by Clapeyron equation (Pa)
 G^{coh} parameter control the surface tension between the two modeled fluids (dimensionless)
 G^{ads} parameter control the adhesion force between fluid and soil particles (dimensionless)
 G^{fre} parameter control the suction force between ice and fluids (dimensionless)
 μ bulk water viscosity ($\text{kg}/\text{m s}$)
 μ_p viscosity of water in thin films ($\text{kg}/\text{m s}$)
 k_p permeability (dimensionless)

Subscripts

σ fluid component
 i index of speed direction of lattice

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