Research Article

Chuanzhi Cui, Zhongwei Wu*, Zhen Wang, Jingwei Yang, and Yingfei Sui

New prediction method for transient productivity of fractured five-spot patterns in low permeability reservoirs at high water cut stages

<https://doi.org/10.1515/phys-2018-0066>

Received Dec 13, 2017; accepted Jun 14, 2018

Abstract: Predicting the productivity of fractured five-spot patterns in low permeability reservoirs at high water cut stages has an important significance for the development and optimization of reservoirs. Taking the reservoir heterogeneity and uneven distribution of the remaining oil into consideration, a novel method for predicting the transient productivity of fractured five-spot patterns in low permeability reservoirs at high water cut stages is proposed by using element analysis, the flow tube integration method, and the mass conservation principle. This new method is validated by comparing with actual production data from the field and the results of a numerical simulation. Also, the effects of related parameters on transient productivity are analyzed. The results show that increasing fracture length, pressure difference and reservoir permeability correspond to an increasing productivity. The research provides theoretical support for the development and optimization of fractured five-spot patterns at the high water cut stage.

Keywords: low-permeability reservoirs; element analysis method; flow tube integration method; reservoir heterogeneity; productivity

PACS: 47.56.+r

1 Introduction

Water flooding is an efficient approach to maintain reservoir pressure and has been widely used to enhance oil

recovery. Currently, most water flooding reservoirs have been in a high water cut period in China [1, 2]. Due to reservoir heterogeneity, various regions in a reservoir have a different scouring intensity, which results in the uneven distribution of the remaining oil at high water cut stages [3, 4]. Therefore, the reservoir heterogeneity, especially for the remaining oil uneven distribution, must be taken into consideration for predicting the productivity of fractured five-spot patterns at the high water cut stage.

There are many research papers on analytic/semi-analytic methods for predicting the productivity of fractured vertical wells. All the research can be classified into two categories: (a) methods for predicting productivity of single phase flows and (b) methods for predicting productivity of oil-water two phase flows. For the scenario of single phase oil flow, Gringarten *et al.* [5, 6] and Goode *et al.* [7] gave a prediction method for the transient pressure and productivity for fractured vertical wells under different reservoir boundary conditions and fracture flow models. Wang *et al.* analyzed the asymptotic characteristics of the bottom hole pressure and derived an accurate production formula for radial flow in the mid period as well as a production formula for pseudo-steady state flow in the late period. Also, a modified Dupuit-type productivity formula for a fractured vertical well was derived [8]. Jacques established a steady-state flow model of vertical well with an infinite conductivity horizontal fracture by using steady-state flow theory and the principle of pressure superposition [9]. Xiong *et al.*, Wang *et al.*, Chen *et al.* [10–12] established a steady-state productivity model for a fractured-vertical well by using flow theory and conformal transformation, and analyzed the effects of the main influencing factors. The research topics mentioned above are all about predicting methods for the productivity of single phase flow. These prediction methods aren't applicable to the case of oil-water two phase flow at the high water cut stage. Regarding this scenario, Ji, *et al.*, Pu, *et al.* and Li, *et al.* [13–16] introduced a method for calculating the steady state productivity of different patterns by using the flow theory and the flow tube integral method. However, the method

*Corresponding Author: Zhongwei Wu: College of Petroleum Engineering, China University of Petroleum (East China), No.66 Changjiang west road, Qingdao, P.R. China;
Email: 327803426@qq.com

Chuanzhi Cui, Zhen Wang, Jingwei Yang, Yingfei Sui: College of Petroleum Engineering, China University of Petroleum (East China), No.66 Changjiang west road, Qingdao, P.R. China

didn't take the reservoir heterogeneity into consideration and can't be used to predict the transient productivity. Numerical simulation methods can be used to predict the transient productivity for oil-water two phase flow considering the reservoir heterogeneity. However, these methods are time-consuming.

In this paper, a method for rapid predicting transient productivity of fractured five-spot patterns at high water cut stages is proposed, which takes the reservoir heterogeneity and uneven remaining oil distribution into consideration. In the new method, by using element analysis, an injection-producing unit of a fractured five-spot pattern is divided into four sub-units (SUs), each of which is divided into three calculation units (CUs) in accordance with streamline distribution characteristics. The transient productivity of every CU is derived by the flow tube integration method and the mass conservation principle. The transient productivity for an injection-producing unit of a fractured five-spot pattern equals to the sum of productivity of all CUs. The new method for predicting transient productivity is validated by comparing with the actual production data from the field and the results of a numerical simulation. The effects of fracture length, pressure difference between injection well and production well, and reservoir heterogeneity on transient productivity are analyzed. The research results provide theoretical guidance for the development and optimization of fractured five-spot patterns at the high water cut stage.

2 Physical model for fractured five-spot patterns at high water cut stages

The schematic illustration (Figure 1a) depicts an injection-producing unit of a fractured five-spot pattern. The fracture half-length of the injection well is L_{fw} . The fracture half-length of the production well is equal to L_{fo} . The well spacing and array spacing of the well are equal to L_2 and L_1 , respectively. By applying element analysis, an injection-producing unit of a fractured five-spot pattern is divided into four SUs, each of which is divided into three CUs in accordance with streamline distribution characteristics. Therefore, an injection-producing unit of a fractured five-spot pattern is divided into 12 CUs in total, as shown in Figure 1b

The saturation distribution in the injection-producing unit is not uniform at the high water cut stage. The saturation of each SU is different, but saturation in a SU is

Figure 1: Illustration of an injection-producing unit of a fractured five-spot pattern and its CUs

the same. Gravity and capillary force are neglected, and the conductivity of all fractures is infinite. The productivity of an injection-production unit for a fractured five-spot pattern is equal to the sum of the productivity of all CUs. The method for predicting productivity of every CU is introduced in the next part.

3 Productivity of every CU

The productivity of a CU equals the sum of the production for flow through all flow tubes in the CU. Therefore, the production for flow through a flow tube is introduced below.

3.1 Production of a flow tube

A. Prada and F. Civan [17] proposed a corrected Darcy's law, which has widely been used to describe the flow in low permeability reservoirs [13–16, 18]. Based on Prada and Civan's research, the production for flow through a cross-section (Figure 2) of a flow tube can be calculated by

$$\Delta q_o = \gamma \frac{k_o(s_w)}{\mu_o} A(\xi) \left(\frac{dp_\xi}{d\xi} - G \right) \frac{1}{B_o} \quad (1)$$

where Δq_o is the production of a flow tube, sm^3/d ; γ is a unit conversion factor, taken to be 0.0864; k_o is the effective permeability of the oil phase, $10^{-3} \mu\text{m}^2$; s_w is the water saturation; μ_o is the oil viscosity, $\text{mPa}\cdot\text{s}$; B_o is the oil volume factor; ξ is the distance from the injection well along the centerline of the flow tubes, m ; $A(\xi)$ is the cross-sectional area at ξ , m^2 ; dp_ξ is the pressure difference between the cross section's left and its right, MPa ; and G is the threshold pressure gradient, MPa/m .

Figure 2: Illustration of a flow tube

Eq. (1) can be rewritten as follows:

$$dp_\xi = \left(\frac{\mu_o B_o \Delta q_o}{\gamma k_o(s_w)} \frac{1}{A(\xi)} + G \right) d\xi \quad (2)$$

Integrating Eq. (2) over ξ , an expression for calculating the production of a flow tube that relates to the pressure difference between the injection well and the production well is derived as follows:

$$\Delta q_o = \gamma \frac{k_o(s_w)}{\mu_o} (P_{in} - P_{pro} - Gl) \frac{1}{B_o} \int_l \frac{d\xi}{A(\xi)} \quad (3)$$

Here P_{in} is the bottom hole pressure of the injection well, MPa ; P_{pro} is the bottom hole pressure of the production well, MPa ; and l is the length of the centerline for a flow tube, m .

3.2 Productivity of the triangular CU

The method for calculating productivity of a triangular CU is the same. Therefore, we take CU 1 (Figure 3) as an example to present the method for calculating productivity of a triangular CU.

Figure 3: Illustration of CU 1

From Eq. (3), the production of one flow tube in CU 1 can be obtained by

$$\Delta q_{o1} = \gamma \frac{k_o(s_w)}{\mu_o} (P_{in} - P_{pro} - Gl_1) \frac{1}{B_o} \int_{l_1} \frac{d\xi}{A(\xi)} \quad (4)$$

where l_1 is the length of centerline of one flow tube in CU 1, m ; and Δq_{o1} is the production of one flow tube for CU 1, sm^3/d .

From the geometric relationship in Figure 3, the following expressions can be obtained:

$$AC = AD \cos \alpha_{11} + DC \cos \beta_{11} \quad (5)$$

$$AC^2 = AB^2 + BC^2 \quad (6)$$

$$AD \sin \alpha_{11} = DC \sin \beta_{11} \quad (7)$$

$$l_1 = AD + DC - r_w - w_f \quad (8)$$

$$\frac{\alpha_1}{\beta_1} = \frac{\alpha_{11}}{\beta_{11}} = \frac{\Delta \alpha_{11}}{\Delta \beta_{11}} = \text{ratio1} \quad (9)$$

Here AC is the distance between A and C , m ; AD is the distance between A and D , m ; DC is the distance between D and C , m ; AB is the distance between A and B , and is equal to L_1 , m ; BC is the distance between B and C , and is equal to $\frac{L_2}{2} - L_{fw}$, m ; r_w is the well radius, m ; w_f is the fracture width, m ; α_1 and β_1 are the angles of the production well and the injection well in CU 1, respectively, $^\circ$; α_{11} and β_{11} are the angles of the flow tube for the production well and the injection well in CU 1, respectively, $^\circ$; $\Delta \alpha_1$ and $\Delta \beta_1$ are the angle increments of the flow tubes for the production well and the injection well in CU 1, respectively, $^\circ$; and ratio1 is a constant which depends on the shape of CU 1.

Substituting Eqs. (5) through (7) into Eq. (8), the expression for l_1 is as follows:

$$l_1 = \sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fw} \right)^2} \frac{(\sin \alpha_{11} + \sin \beta_{11})}{\sin(\alpha_{11} + \beta_{11})} \quad (10)$$

Figure 4: Illustration of CU 2 and its flow tube in the situation where $L_{wf} \neq L_{wo}$

$$-r_w - w_f$$

From the geometric relationship in Figure 3, the expression for $A(\xi)$ is as follows:

$$A(\xi) = \begin{cases} 2h\xi \tan(\frac{\Delta\alpha_1}{2})r_w \leq \xi \leq AD \\ 2h\left(\sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fw}\right)^2} \frac{(\sin\alpha_{11} + \sin\beta_{11})}{\sin(\alpha_{11} + \beta_{11})} - \xi\right) \cdot \tan(\frac{\Delta\beta_1}{2})AD \leq \xi \leq AD + DC - w_f \end{cases} \quad (11)$$

Substituting Eqs. (10) and (11) into Eq. (4), the production expression for one flow tube in CU 1 is as follows:

$$\Delta q_{o1} = \gamma \frac{\frac{k_o(s_w)}{\mu_o}(P_{in} - P_{pro} - G(\sqrt{L_1^2 + (\frac{L_2}{2} - L_{fw})^2} \frac{(\sin\alpha_{11} + \sin\beta_{11})}{\sin(\alpha_{11} + \beta_{11})} - r_w - w_f))}{\int_{r_w}^{AD} \frac{d\xi}{2h\xi \tan(\frac{\Delta\alpha_1}{2})} + \int_{AD}^{AD+DC-w_f} \frac{d\xi}{2h(\sqrt{L_1^2 + (\frac{L_2}{2} - L_{fw})^2} \frac{(\sin\alpha_{11} + \sin\beta_{11})}{\sin(\alpha_{11} + \beta_{11})} - \xi) \cdot \tan(\frac{\Delta\beta_1}{2})}} \cdot \frac{1}{B_o} \quad (12)$$

Simplifying Eq. (12), the expression of Δq_{o1} can be rewritten as follows:

$$\Delta q_{o1} = \gamma \frac{\frac{k_o(s_w)}{\mu_o}(P_{in} - P_{pro} - G(\sqrt{L_1^2 + (\frac{L_2}{2} - L_{fw})^2} \frac{(\sin\alpha_{11} + \sin\beta_{11})}{\sin(\alpha_{11} + \beta_{11})} - r_w - w_f))}{\frac{1}{2h \tan(\frac{\Delta\alpha_1}{2})} \ln\left(\frac{\sqrt{L_1^2 + (\frac{L_2}{2} - L_{fw})^2} \sin\beta_{11}}{\sin(\alpha_{11} + \beta_{11})r_w}\right) + \frac{1}{2h \tan(\frac{\Delta\beta_1}{2})} \ln\left(\frac{\sqrt{L_1^2 + (\frac{L_2}{2} - L_{fw})^2} \sin\alpha_{11}}{\sin(\alpha_{11} + \beta_{11})w_f}\right)} \cdot \frac{1}{B_o} \quad (13)$$

The productivity of CU 1 equals to the sum of the production for flow through all the flow tubes in CU 1, so the

productivity of CU 1 can be calculated by

$$Q_{o1} = \int_0^{\alpha_1} \left(\lim_{\Delta\alpha_1 \rightarrow 0} \frac{\Delta q_{o1}}{\Delta\alpha_1} \right) d\alpha_{11} \quad (14)$$

where Q_{o1} is the productivity of CU 1, sm^3/d .

Substituting Eq. (13) into Eq. (14), and simplifying by using Eq. (9), we can obtain the expression of the productivity of CU 1 as follows:

$$Q_{o1} = \int_0^{\alpha_1} \gamma \frac{\frac{k_o(s_w)}{\mu_o}h(P_{in} - P_{pro} - G(\sqrt{L_1^2 + (\frac{L_2}{2} - L_{fw})^2} \frac{(\sin\alpha_{11} + \sin\frac{\alpha_{11}}{\text{ratio1}}) - r_w - w_f))}{\ln\left(\frac{\sqrt{L_1^2 + (\frac{L_2}{2} - L_{fw})^2} \sin\frac{\alpha_{11}}{\text{ratio1}}}{\sin(\alpha_{11} + \frac{\alpha_{11}}{\text{ratio1}})r_w}\right) + \text{ratio1} \cdot \ln\left(\frac{\sqrt{L_1^2 + (\frac{L_2}{2} - L_{fw})^2} \sin\alpha_{11}}{\sin(\alpha_{11} + \frac{\alpha_{11}}{\text{ratio1}})w_f}\right)} \cdot \frac{1}{B_o} d\alpha_{11} \quad (15)$$

3.3 Production of the quadrilateral CU

The productivity calculating method of a quadrilateral CU is the same. Therefore, taking CU 2 (Figure 4) as an example to present the method for calculating productivity of the quadrilateral CU.

From Eq. (3), the production of one flow tube in CU 2 can be obtained by

$$\Delta q_{o2} = \gamma \frac{\frac{k_o(s_w)}{\mu_o}(P_{in} - P_{pro} - Gl_2)}{\int_{l_2} \frac{d\xi}{A(\xi)}} \cdot \frac{1}{B_o} \quad (16)$$

where Δq_{o2} is the production of one flow tube in CU 2, sm^3/d ; and l_2 is the centerline length of one flow tube, m.

Here are two situations in which to calculate $A(\xi)$. One situation is that the fracture length of the injection well does not equal the fracture length of the production well (seen in Figure 4). From the geometric relationship in Figure 4, the following expressions are obtained:

$$l_2 = \sqrt{0.5(L_2 - L_{fw} - L_{fo})} \quad (17)$$

$$A(\xi) = 2h \left(A'F' - \frac{(A'F' - C'E')}{l_2} \xi \right) \quad (18)$$

$$\frac{L_{fo}}{L_{fw}} = \frac{\Delta L_{fo}}{\Delta L_{fw}} = \text{ratio2} \quad (19)$$

Here $A'F'$ is the distance between A' and F' , and is equal to ΔL_{fo} , m; $C'E'$ is the distance between C' and E' , and is equal to ΔL_{fw} , m; and ratio2 is a constant which depends on the shape of CU 2.

Substituting Eqs. (17) and (18) into Eq. (16), then simplifying, the production calculating expression of one flow tube is as follows:

$$\Delta q_{o2} = \gamma \quad (20)$$

$$\frac{k_o(s_w)}{\mu_o} \left(P_{in} - P_{pro} - G \left(\sqrt{0.5(L_2 - L_{fw} - L_{fo})} \right) \right) \frac{1}{\frac{\sqrt{0.5(L_2 - L_{fw} - L_{fo})}}{2h(\Delta L_{fo} - \Delta L_{fw})} \ln\left(\frac{\Delta L_{fo}}{\Delta L_{fw}}\right)} B_o$$

The productivity of CU 2 is equal to the sum of the production of all flow tubes in CU 2, so the productivity of CU 2 can be obtained by

$$Q_{o2} = \int_0^{L_{fw}} \left(\lim_{\Delta L_{fw} \rightarrow 0} \frac{\Delta q_{o1}}{\Delta L_{fw}} \right) dL \quad (21)$$

where Q_{o2} is the productivity for CU 2 in the situation where $L_{wf} \neq L_{wo}$, sm^3/d . L is the distance from injection well along the fracture of injection well, m.

Substituting Eq. (20) into (21), and then simplifying by Eq. (19), the expression of Q_{o2} can be rewritten as follows:

$$Q_{o2} = \gamma \frac{\frac{k_o(s_w)}{\mu_o} \left(P_{in} - P_{pro} - G \left(\sqrt{0.5(L_2 - L_{fw} - L_{fo})} \right) \right)}{\frac{\sqrt{0.5(L_2 - L_{fw} - L_{fo})}}{2h(\text{ratio2} - 1)} \ln(\text{ratio2})} \frac{1}{B_o} L_{fw} \quad (22)$$

Another situation is that the fracture length of the injection well is equal to the fracture length of the production well. Thus, the value of ratio2 is as follows:

$$\text{ratio2} = 1 \quad (23)$$

Substituting Eq. (23) into Eq. (22), we can find that the denominator ($\text{ratio2} - 1$) is zero, so the expression is not

Figure 5: Illustration of CU 2 in the situation where $L_{wf} = L_{wo}$

a legal equation. Therefore, under the situation where the fracture length of the injection well is equal to that of the production well (Figure 5), Eq. (22) can't be used and the whole of CU 2 can be considered as a flow tube.

The following expressions for describing the geometric relationship in Figure 5 are as follows:

$$A(\xi) = hL_{fo} \quad (24)$$

$$l_2 = \sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fo} \right)^2} \quad (25)$$

Substituting Eqs. (24) and (25) into Eq. (16), the productivity of CU 2 can be calculated by

$$Q'_{o2} = \Delta q_{o2} \quad (26)$$

$$= \gamma \frac{\frac{k_o(s_w)}{\mu_o} \left(P_{in} - P_{pro} - G \left(\sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fo} \right)^2} \right) \right)}{hL_{fo} \sqrt{L_1^2 + \left(\frac{L_2}{2} - L_{fo} \right)^2}} \frac{1}{B_o}$$

where Q'_{o2} is the productivity of CU 2 in the situation where $L_{wf} = L_{wo}$, sm^3/d .

The productivity for a fractured five-spot pattern is equal to the sum of the productivity of all CUs, so the productivity for a fractured five-spot pattern can be obtained by

$$Q_o = \sum_{i=1}^{12} Q_{oi} \quad (27)$$

where Q_o is the productivity for a fractured five-spot pattern, sm^3/d ; i is the number of CU; and Q_{oi} is the productivity of CU i , sm^3/d .

Equation (27) is used to calculate the steady state productivity. How to obtain the transient productivity by using the steady state productivity mentioned above will be introduced below.

4 Transient productivity prediction method for fractured five-spot patterns

It is well known that the process of oil-water two-phase flow is unsteady at the high water cut stage, and one transient flow process can be considered as the superposition of many steady state flow process [19, 20]. Therefore, the unsteady flow process can be divided into many steady state flow processes by separating the time. When the discrete time is very small, the flow process in every discrete time can be seen as steady state. By using a mass conservation principle, the expression of average water saturation between two adjacent processes is as follows:

$$\bar{s}_{wj}^{n+1} = \bar{s}_{wj}^n + \frac{Q_{osj}\Delta t B_o}{V_{\phi j}} \quad (28)$$

Here, j is the number of SU, $\bar{s}_{wj}^n$ and $\bar{s}_{wj}^{n+1}$ are the average water saturation of SU j at the time t_n and t_{n+1} , respectively, Δt is the time difference between t_n and t_{n+1} , d. $V_{\phi j}$ is the pore volume for SU j , and Q_{osj} is the steady productivity for SU j at t_n , sm^3/d . Expressions for Q_{osj} ($j=1, 2, 3$ or 4) are as follows:

$$Q_{os1} = \sum_{j=1}^3 Q_{oj} \quad (29)$$

$$Q_{os2} = \sum_{j=4}^6 Q_{oj} \quad (30)$$

$$Q_{os3} = \sum_{j=7}^9 Q_{oj} \quad (31)$$

$$Q_{os4} = \sum_{j=10}^{12} Q_{oj} \quad (32)$$

The relationship for the expression between the average water saturation of the reservoir and the water saturation in the production well is as follows [21, 22]:

$$s_w = \frac{3\bar{s}_w}{2} - \frac{1 - s_{or}}{2} \quad (33)$$

The procedure for the transient productivity prediction method is as follows: ① with the initial basic parameters, the initial time steady state productivity of fractured five-spot patterns can be calculated by Eq. (27). ② In the next moment water saturation of each SU in production well is obtained by using Eqs. (28)-(34). ③ Substituting the next moment water saturation into Eq. (27), the next moment state productivity can be obtained. Repeating ② and ③, transient productivity can finally be obtained.

5 Productivity prediction method validation

5.1 Reservoir basic parameters

The input values of relevant basic parameters for a fractured five-spot patterns in a water-flooding reservoir of Daqing oilfield are shown in Table 1

Table 1: The input value of relevant basic parameters for a fractured five-spot pattern

Parameters	value
Oil viscosity under reservoir's condition, μ_o (mPa·s)	6.7
Bottom hole flow pressure of injection well, P_{in} (MPa)	18.08
Bottom hole flow pressure of production well, P_{pro} (MPa)	8
Absolute permeability of SU 1, k_1 ($10^{-3} \mu\text{m}^2$)	32.7
Absolute permeability of SU 2, k_2 ($10^{-3} \mu\text{m}^2$)	30.1
Absolute permeability of SU 3, k_3 ($10^{-3} \mu\text{m}^2$)	25.5
Absolute permeability of SU 4, k_4 ($10^{-3} \mu\text{m}^2$)	29.4
Average water saturation of SU 1, s_{w1} (fraction)	0.65
Average water saturation of SU 2, s_{w2} (fraction)	0.60
Average water saturation of SU 3, s_{w3} (fraction)	0.55
Average water saturation of SU 4, s_{w4} (fraction)	0.57
Reservoir thickness of fractured five-spot patterns, h (m)	14
Oil volume factor, B_o (fraction)	1.15
The fracture half-length of injection well, L_{fw} (m)	120
The fracture half-length of production well, L_{fo} (m)	100
Well spacing, L_2 (m)	400
Array spacing of the well, L_1 (m)	200
Fracture width, w_f (m)	0.02
Wellbore radius, r_w (m)	0.2
Porosity, ϕ (fraction)	0.17

The relative permeability curve of the reservoir is shown in Figure 6.

The threshold pressure gradient [23] is given by

$$G = \varepsilon \left(\frac{k}{\mu_w^{S_{wn}} \mu_o^{1-S_{wn}}} \right)^{-n} \quad (34)$$

The parameters ε and n are regression coefficients. In this paper, the value of ε and n are 1.2427 and 0.9753, respectively. S_{wn} is the normalized water saturation, which can

Figure 6: Relative permeability curves

be obtained by using the following expression:

$$S_{wn} = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}$$

where S_{wc} and S_{or} are the irreducible water saturation and the residual oil saturation of the reservoir, respectively; and S_{wn} is the normalized water saturation.

5.2 Productivity prediction method validation

With the method and data mentioned above, the transient productivity has been calculated. The comparison of the results of the novel method with the actual production data from the field is shown in Figure 7.

Figure 7: Comparison of results from the new model with actual production data

As seen from Figure 7, we can get that the result of the new method is consistent with the actual production data from the field. Therefore, we can conclude that the new

method can accurately predict the transient productivity of fractured five-spot patterns in low permeability reservoirs at high water cut stages.

For further verifying the correctness of the proposed method, a numerical simulation model is proposed. The reservoir size, fracture length and fluid properties used in the numerical simulation model are seen in Table 1. The reservoir is homogeneous and the permeability is equal to $30 \times 10^{-3} \mu\text{m}^2$. The fracture permeability is $10000 \times 10^{-3} \mu\text{m}^2$. The grid of the numerical simulation model is 40×41 , and the schematic of the grid can be seen in Figure 8.

With the data used in the numerical stimulation model, as well as the new method, the transient productivity after 90% water cut is obtained. The comparison of the

Figure 8: Schematic of the grid in the numerical simulation model

Figure 9: Comparison of the results from the new model with those from the numerical simulation

Figure 10: Effects of the fracture length of the injection well on productivity

Figure 11: Effects of the fracture length of the production well on productivity

results of the new method with those of numerical simulation is shown in Figure 9. From Figure 9, we obtain that the results of new method agree with that of numerical simulation. The maximum relative error is below 5%. So, the new method can accurately predict transient productivity and the productivity prediction result can meet the demand of field applications.

Using a computing program compiled by the 2013a version of MATLAB on a computer whose processor model is Intel(R) Core(TM) i7-6700 CPU @3.4GHz, it takes only 21.282 seconds to run the program for predicting transient productivity of a fractured five-spot pattern once. It is faster to predict the transient productivity by using the novel method than numerical simulation method.

6 Analysis of influencing factors

In this section, the effects of some relevant parameters for transient productivity are analyzed. The relevant basic parameters are listed in Table 1.

Figs. 10 and 11 illustrate the impacts of fracture length on productivity and cumulative production for a fractured five-spot pattern. From Figs. 10 and 11, the fracture length has a significant effect on productivity and cumulative production of fracture five-spot patterns. The longer the fracture length is, the higher the production will be. This is the case because that the longer the fracture length is, the smaller the flow resistance for flowing in the fractured five-spot pattern will be.

Figure 12 shows the impact of pressure difference between injection well and production well on productivity.

(a) Productivity

(b) Cumulative production

Figure 12: Effects of the pressure difference on productivity

(a) Productivity

(b) Cumulative production

Figure 13: Effects of reservoir heterogeneity on productivity

From Figure 12, the greater the is pressure difference, the higher the productivity will be.

In order to research the effect of reservoir heterogeneity on productivity, we chose three sets of permeability, which are (30.7, 28.1, 23.5 and 27.4), (32.7, 30.1, 25.5 and 29.4) and (34.7, 32.1, 27.5 and 31.4), the i -th value in the bracket is the respective permeability of the i -th SU. The other parameters are listed in Table 1. The effect of reservoir heterogeneity on productivity is analyzed. From Figure 13, we find that an increasing permeability of each SU corresponds to an increasing productivity and cumulative production. The reason for this case is that the flow resistance will decrease when the permeability of each SU increases.

7 Conclusions

With the element analysis method and flow tube integration method, a new method for rapid predicting transient productivity is established which considers the reservoir heterogeneity and remaining oil uneven distribution. The new method is verified by comparing the result of the new method with that of a numerical simulation and actual production data from field. The new method can accurately predict the productivity and the calculation speed of the novel method is faster than that of the numerical simulation method.

The effect of related parameters on transient productivity is analyzed. The results show that the fracture length, pressure difference, and reservoir heterogeneity have significant effects on productivity for fractured five-spot patterns. An increasing fracture length, pressure dif-

ference, and reservoir permeability correspond to an increasing productivity.

Acknowledgement: The paper is supported by the fund of National Massive Oil & Gas Field and Coal-bed Methane Development Program (2016ZX05010-002-007). The authors would like to thank the reviewers and editors whose critical comments were very helpful in preparing this article.

References

- [1] Feng Q., Wang S., Gao G., Li C., A new approach to thief zone identification based on interference test, *J. of Petrol. Sci. & Eng.*, 2010, 75, 13-18.
- [2] Cui C., Li K., Yang Y., Huang Y., Cao Q., Identification and quantitative description of large pore path in unconsolidated sandstone reservoir during the ultra-high water-cut stage, *J. of Petrol. Sci. & Eng.*, 2014, 122, 10-17.
- [3] Nie C., Ma S., Chen H., The research on improving the efficiency of injection-production of the lamadian oilfield in ultra-high water cut stage, *Adv. Mater. Res.*, 2013, 734-737, 1484-1487.
- [4] Pi Y., Guo X., Wang J., Jia Y., Experimental study on saturation distribution of heterogeneous Reservoir, *Proceedings of the 2015 Asia-Pacific Energy Equipment Engineering Research Conference*, 2015, 408-412
- [5] Gringarten A., Ramey H., Raghavan R., Unsteady-state pressure distributions created by a well with a single infinite-conductivity vertical fracture, *SPE J.*, 1974, 347-360
- [6] Gringarten A., Reservoir limit testing for fractured wells, the 53rd SPE Annual Fall Technical Conference and Exhibition, 1978, 1-14
- [7] Goode P., Kuchuk F., Inflow performance of horizontal wells, *SPE Res. Eng.*, 1991, 319-323
- [8] Wang X., Zhang Y., Liu C., Productivity evaluation and conductivity optimization for vertically fractured wells, *Petrol. Explor. and Dev.*, 2004, 31, 78-81
- [9] Hagoort J., The productivity of a well completed with a vertical penny-shaped fracture. *SPE J.*, 2011, 401-410
- [10] Xiong J., Wang X., Lu L., Productivity model for asymmetrical vertical fracture well in low-permeability oil reservoir, *Sci. & Tech. Rev.*, 2013, 31, 40-43
- [11] Wang F., Liu H., Lv G., Steady-state productivity prediction model for long-length fractured vertical well in low permeability oil reservoirs, *Petrol. Geol. and Recov. Effic.*, 2014, 21, 84-86
- [12] Chen Z., Liao X., Zhao X., Wang H., Ye H., Zhu X., *et al.*, Productivity model of oil /gas productivity of vertical wells in simulated reservoir volume, *Petrol. Geol. and Recov. Effic.*, 2015, 22, 121-126
- [13] Ji B., Li L., Wang C., Oil production calculation for areal well pattern of low-permeability reservoir with non-Darcy seepage flow, *Acta Petrol. Sin.*, 2008, 29, 258-261
- [14] Ji B., Wang C., Li L., Calculation method for production rate of rectangular well pattern and fracturing integration production mode in low-permeability reservoir, *Acta Petrol. Sin.*, 2009, 30, 578-582
- [15] Pu J., Liu C., Shang G., Non-steady water-flooding production model of areal well pattern based on flow line integral method, *J. of Southwest Petrol. Uni. (Sci. & Tech. Edition)*, 2016, 38, 97-106
- [16] Li R., Lyu A., Wang J., Li Yang, Zhan S., Wang Y., Productivity of the imitation horizontal well pattern in low permeability reservoirs, *Petrol. Geol. and Recov. Effic.*, 2016, 37, 439-443
- [17] Prada A., Civan F., Modification of darcy's law for the threshold pressure gradient, *J. of Petrol. Sci. & Eng.*, 1999, 22, 237-240
- [18] Cai J., A fractal approach to low velocity non-darcy flow in a low permeability porous medium, *Chinese Phys. B*, 2014, 23, 385-389
- [19] Zhang J., Du D., Hou J., Lei G., Lyv A., *Oil and gas flow mechanics*, Dong Ying: China University of Petroleum Press, 2010
- [20] Zhang J., Lei G., Zhang Y., *Oil and gas flow mechanics*, Dong Ying: China University of Petroleum Press, 1998
- [21] Gao W., Xu J., Theoretical study on common waterdrive characteristic curves, *Acta Petrol. Sin.*, 2007, 28, 89-92
- [22] Cui C., Xu J., Wang D., Yang Y., A new water flooding characteristic curve at ultra-high water cut stage, *Acta Petrol. Sin.*, 2015, 36, 1267-1271
- [23] Deng Y., Liu h., Experiment on starting pressure gradient of oil and water two phases in low permeability core, *Oil Drill. & Prod. Tech.*, 2006, 28, 37-40