Research Article

Jörn Schliewe*

Electrodynamics in Euclidean Space Time Geometries

<https://doi.org/10.1515/phys-2019-0077>

Received May 07, 2019; accepted Oct 29, 2019

Abstract: In this article it is proven that Maxwell's field equations are invariant for a real orthogonal Cartesian space time coordinate transformation if polarization and magnetization are assumed to be possible in empty space. Furthermore, it is shown that this approach allows wave propagation with finite field energy transport. To consider the presence of polarization and magnetization an alternative Poynting vector has been defined for which the divergence gives the correct change in field energy density.

Keywords: EM fields, theory of relativity, relativistic field transformation, wave propagation, ray of light

PACS: 03.50.De, 03.30.+p, 03.65.Pm

1 Introduction

The foundation of the electromagnetic theory is the work of many people and culminated in the publications of J. C. Maxwell in 1865 [1] and 1873 [2]. The field equations in the modern form, known as Maxwell's equations, are validated in countless applications, like antennas, radar and electronic components. Around 1900 H. A. Lorentz [3] and others [4] tried to explain the behavior of electromagnetic fields in moving matter. Background was the idea of an ether which transports the field energy of light. In 1905 A. Einstein published his famous theory of special relativity [5]. This was the start of a new way of thinking. Electromagnetic energy moves without any carrier charges through empty space. Einstein's theory which is based on the geometrical properties of the Lorentz transformation is enforced by Maxwell's electromagnetic theory of empty space, e.g. [6, p. 660], [7, p. 645] and [8, p. 595]. The criticism on Einstein's initial postulate of constant velocity of light has been refuted by the fact that Maxwell's equations of empty space are invariant under Lorentz transfor-

mation. W. Pauli said: "It is clear that one would then have to abandon not only the idea of the existence of an aether but also Maxwell's equations for the vacuum, so that the whole of electrodynamics would have to be constructed anew" [4, p. 6]. Attempts to find new electromagnetic theories like the one of Ritz [9] failed to explain all experiments. Nevertheless, the theory of Ritz has been considered to be noteworthy as Einstein's initial postulate of constant velocity of light, even though supported by experiments (e.g. [10, 11]), is quite contrary to our experience with mechanical or acoustical waves [12, p. 75]. However, from a theoretical point of view, it would be preferable if this constant velocity would be a result and not an initial postulate of the theory.

Starting only from the principle of relativity and basic geometrical considerations [13, 14] or [15] showed that a set of transformation groups satisfies the principle of relativity without the initial assumption of a constant velocity of light. Different transformations can be distinguished by a universal space time constant. These transformations include the Galilean transformation as a trivial solution where the velocity of light tends towards infinity [16], the Lorentz transformation, but also a transformation based on a simple orthogonal Cartesian rotation in space time. As the Newtonian mechanics are invariant under a Galilean transformation rigorous discussions to validate the Lorentz transformation based theory of relativity against the Galilean transformation can be found in literature, e.g. [11, 16]. However, a rigorous comparison of the Lorentz transformation with a simple orthogonal Cartesian rotation in space time cannot be found.

Beginning around 1949 the emptiness of the space was somehow relativized by the possibility of vacuum fluctuations which allow the creation of particle-antiparticle pairs of virtual particles in empty space [17]. Quantum electrodynamics which successfully predicted the Landé factor of magnetic moment of electrons relies on the concept of virtual particles [18, 19].

The main aim of this article is to give a theoretical justification for the necessity of a comparison of the Lorentz transformation based theory of relativity with the simple orthogonal Cartesian rotation in space time. The two main theoretical statements presented in this article support

*Corresponding Author: Jörn Schliewe: Steinheim, Germany;
Email: joern.schliewe@ieee.org

this necessity. Firstly, it will be proven that Maxwell's field equations are invariant under a simple orthogonal Cartesian rotation in space time if electric and magnetic polarization are considered to happen in space. Secondly, it will be shown that this approach leads to bounded wave solutions which contains finite field energy transport in contrast to the plane wave solution.

2 Definitions

In this section the most important definitions are discussed on the basis of Maxwell's field equations [6, p. 34], [7, p. 31], [20, p. 425] or [8, p. 3]. Ampers law 1 and the law of induction 2 can be written in the following form:

$$\text{rot}(\vec{H}) = \vec{J} + \epsilon \frac{\partial (\vec{P} + \vec{E})}{\partial t} \quad (1)$$

$$\text{rot}(\vec{E}) = -\mu \frac{\partial (\vec{M} + \vec{H})}{\partial t} \quad (2)$$

$\text{rot}()$ is the curl, $\vec{H}$ is magnetic field strength, $\vec{J}$ is current density, $\vec{P}$ is polarization, $\vec{E}$ is electric field strength, $\vec{M}$ is magnetization, ϵ is the permittivity of empty space which gives the ability for polarization and finally μ is the permeability of empty space which gives the ability for magnetization.

Using these definitions the electric flux density results in

$$\vec{D} = \epsilon (\vec{P} + \vec{E}) \quad (3)$$

and the magnetic flux density results in

$$\vec{B} = \mu (\vec{M} + \vec{H}) \quad (4)$$

These equations imply the basic definitions of the polarization and magnetization which are used throughout this paper. The permittivity ϵ and permeability μ describe the basic properties of empty space. Using both properties the velocity of light c and the impedance of empty space Z_0 can be derived:

$$c = \frac{1}{\sqrt{\epsilon\mu}} \quad (5)$$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}. \quad (6)$$

Both quantities are properties of propagating electromagnetic field waves and show directly that the properties of empty space define the properties of wave propagation.

3 Orthogonal Cartesian Space Time Transformation

As a most basic approach this transformation can be derived from general group-theoretical considerations like in [13], [14] or [15]. Considering two systems moving relative to each other in z direction this leads to [4, p. 11]:

$$x' = x \quad (7)$$

$$y' = y \quad (8)$$

$$z' = \frac{1}{\sqrt{1 - \alpha \frac{v^2}{c^2}}} (z - v \cdot t) \quad (9)$$

$$t' = \frac{1}{\sqrt{1 - \alpha \frac{v^2}{c^2}}} \left(t - \alpha \frac{v}{c^2} z \right) \quad (10)$$

For $\alpha = 0$ this represents the Galilean transformation, for $\alpha = 1$ this gives the Lorentz transformation and finally with $\alpha = -1$ this results in an orthogonal Cartesian space time transformation:

$$x' = x \quad (11)$$

$$y' = y \quad (12)$$

$$z' = \frac{1}{\sqrt{1 + \frac{v^2}{c^2}}} (z - v \cdot t) \quad (13)$$

$$t' = \frac{1}{\sqrt{1 + \frac{v^2}{c^2}}} \left(t + \frac{v}{c^2} z \right) \quad (14)$$

In the following γ will be defined as $\gamma = \frac{1}{\sqrt{1 + \frac{v^2}{c^2}}}$.

Figure 1 shows the geometrical representation of an orthogonal Cartesian space time transformation relation between two systems moving relative to each other in z direction. It can be seen that the orthogonal relation between time and z is ensured which represents a simple rotation in an Euclidean space. In reflection of this simple rotation neither the Lorentz transformation nor the Galilean transformation is orthogonal Cartesian in four dimensional space.

For the proof of invariance the transformation of the differentials is essential. As space and time depend on each other the differentials have to be derived by use of the chain rule:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x'} \quad (15)$$

Figure 1: Geometrical representation of an orthogonal space time transformation relation between two relatively in z direction moving systems

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y'} \quad (16)$$

$$\frac{\partial}{\partial z} = \frac{\partial z'}{\partial z} \cdot \frac{\partial}{\partial z'} + \frac{\partial t'}{\partial z} \cdot \frac{\partial}{\partial t'} = \gamma \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) \quad (17)$$

$$\frac{\partial}{\partial t} = \frac{\partial t'}{\partial t} \cdot \frac{\partial}{\partial t'} + \frac{\partial z'}{\partial t} \cdot \frac{\partial}{\partial z'} = \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) \quad (18)$$

4 Proof of Invariance

4.1 Current density, polarization and magnetization

To prove Maxwell's equations to be invariant under the Euclidean space time transformation the role of polarization and magnetization has to be changed from an amplification of the field quantities by ϵ_r and μ_r to quantities which influence even the empty space. Polarization is the separation of virtual particles and magnetization is the rotation or spinning of virtual particles. With this definition the polarization and magnetization can exhibit a different distribution in space compared to the field quantities.

As explained in [6, p. 29] or [7, p. 27] the introduction of the time derivative of the electric flux density in ampers law 1 is necessary to fulfill the equation of continuity. Figure 2 illustrates the relation with a simple example. A wire connects two electrodes. If a current reduces the charges on the electrodes, the corresponding electric field should be reduced in the same way.

Figure 2: Example for the continuity equation

The divergence of the right hand side of Ampers law 1 should be equal to zero. This gives:

$$0 = \text{div} \left(\vec{J} + \epsilon \frac{\partial \vec{P}}{\partial t} + \epsilon \frac{\partial \vec{E}}{\partial t} \right) \quad (19)$$

As discussed in [6, p. 88] or [7, p. 87] the time derivative of the polarisation can be understood as densities of polarisation currents and consequently the so called bound charge densities can be introduced in contrast to the free charge densities ρ . As "bound" sounds too restricting they will be described as virtual charge densities ρ_v throughout this article.

In the following the electric field will be defined by $\text{div}(\epsilon \vec{E}) = \rho + \rho_v$ and the electric flux density by $\text{div}(\vec{D}) = \rho$. With the latter the equation of continuity is defined.

$$0 = \text{div}(\vec{J}) + \frac{\partial \rho}{\partial t} \quad (20)$$

Proving the equation of continuity for invariance gives:

$$\begin{aligned} 0 &= \text{div}(\vec{J}) + \frac{\partial \rho}{\partial t} \\ 0 &= \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} + \frac{\partial \rho}{\partial t} \\ 0 &= \frac{\partial J_x}{\partial x'} + \frac{\partial J_y}{\partial y'} + \gamma \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) J_z \\ &\quad + \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) \rho \\ 0 &= \frac{\partial J_x}{\partial x'} + \frac{\partial J_y}{\partial y'} + \frac{\partial}{\partial z'} \gamma (J_z - v\rho) + \frac{\partial}{\partial t'} \gamma \left(\rho + \frac{v}{c^2} J_z \right) \end{aligned}$$

According to this derivation the following transformation rules should apply:

$$J'_x = J_x \quad (21)$$

$$J'_y = J_y \quad (22)$$

$$J'_z = \frac{1}{\sqrt{1 + \frac{v^2}{c^2}}} (J_z - v\rho) \quad (23)$$

$$\rho' = \frac{1}{\sqrt{1 + \frac{v^2}{c^2}}} \left(\rho + \frac{v}{c^2} J_z \right) \quad (24)$$

A similar discussion is necessary for the magnetic flux density. The divergence of the right hand side of the law of induction 2 should be equal to zero. This gives:

$$0 = \operatorname{div}(\vec{B}) = \mu \cdot \operatorname{div}(\vec{M}) + \mu \cdot \operatorname{div}(\vec{H}) \quad (25)$$

In contrast to the usual approach $\operatorname{div}(\vec{M})$ and $\operatorname{div}(\vec{H})$ is not necessarily equal to zero. In fact this has been proven to exist in spin ice materials [21–23]. The physical manifestation of these divergences are not magnetic monopoles but the spin of virtual particles. That the magnetic field and the magnetization are not source free, is nothing unexpected for calculations of magnetostatic fields on boundaries of magnetizable materials [6, p. 303] or [7, p. 297].

4.2 Invariance of Maxwell's equations

The same method used for the equation of continuity is now used to prove the invariance of Amperes law.

$$\operatorname{rot}(\vec{H}) = \vec{J} + \epsilon \frac{\partial \vec{P}}{\partial t} + \epsilon \frac{\partial \vec{E}}{\partial t} \quad (26)$$

$$0 = \operatorname{rot}(\vec{H}) - \vec{J} - \epsilon \frac{\partial \vec{P}}{\partial t} - \epsilon \frac{\partial \vec{E}}{\partial t} \quad (27)$$

This can be separated to three equations:

$$0 = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - J_x - \epsilon \frac{\partial P_x}{\partial t} - \epsilon \frac{\partial E_x}{\partial t} \quad (28)$$

$$0 = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - J_y - \epsilon \frac{\partial P_y}{\partial t} - \epsilon \frac{\partial E_y}{\partial t} \quad (29)$$

$$0 = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_z - \epsilon \frac{\partial P_z}{\partial t} - \epsilon \frac{\partial E_z}{\partial t} \quad (30)$$

Firstly, transforming equation 28:

$$\begin{aligned} 0 &= \frac{\partial H_z}{\partial y'} - \gamma \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) \cdot H_y \\ &\quad - J_x - \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) \epsilon P_x \\ &\quad - \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) \epsilon E_x \\ &= \frac{\partial H_z}{\partial y'} - \frac{\partial}{\partial z'} \gamma (H_y - v \epsilon P_x - v \epsilon E_x) \\ &\quad - J_x - \gamma \frac{\partial \epsilon P_x}{\partial t'} - \epsilon \frac{\partial}{\partial t'} \gamma (E_x + v \mu \cdot H_y) \end{aligned} \quad (31)$$

To get the same field transformation rule for the electric field like Einstein, the equation has to be expanded to:

$$\begin{aligned} 0 &= \frac{\partial H_z}{\partial y'} - \frac{\partial}{\partial z'} \gamma (H_y - v \epsilon P_x - \epsilon E_x) \\ &\quad - J_x - \gamma \frac{\partial \epsilon P_x}{\partial t'} - \gamma \frac{v}{c^2} \frac{\partial}{\partial t'} (M_y + 2H_y) \\ &\quad - \epsilon \frac{\partial}{\partial t'} \gamma (E_x - v \mu (M_y + H_y)) \\ &= \frac{\partial H_z}{\partial y'} - \frac{\partial}{\partial z'} \gamma (H_y - v \epsilon (P_x + E_x)) \\ &\quad - J_x - \epsilon \frac{\partial}{\partial t'} \gamma (P_x + v \mu (M_y + 2H_y)) \\ &\quad - \epsilon \frac{\partial}{\partial t'} \gamma (E_x - v \mu (M_y + H_y)) \end{aligned} \quad (32)$$

Then equation 29:

$$\begin{aligned} 0 &= \gamma \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) \cdot H_x - \frac{\partial H_z}{\partial x'} \\ &\quad - J_y - \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) \epsilon P_y \\ &\quad - \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) \epsilon E_y \\ &= \frac{\partial}{\partial z'} \gamma (H_x + v \epsilon P_y + v \epsilon E_y) - \frac{\partial H_z}{\partial x'} \\ &\quad - J_y - \gamma \frac{\partial \epsilon P_y}{\partial t'} - \epsilon \frac{\partial}{\partial t'} \gamma (E_y - v \mu H_x) \\ &= \frac{\partial}{\partial z'} \gamma (H_x + v \epsilon P_y + v \epsilon E_y) - \frac{\partial H_z}{\partial x'} \\ &\quad - J_y - \gamma \frac{\partial \epsilon P_y}{\partial t'} + \gamma v \epsilon \mu \frac{\partial}{\partial t'} (M_x + 2 \cdot H_x) \\ &\quad - \epsilon \frac{\partial}{\partial t'} \gamma (E_y + v \mu (M_x + H_x)) \\ &= \frac{\partial}{\partial z'} \gamma (H_x + v \epsilon (P_y + E_y)) - \frac{\partial H_z}{\partial x'} \\ &\quad - J_y - \epsilon \frac{\partial}{\partial t'} \gamma (P_y - v \mu (M_x + 2 \cdot H_x)) \\ &\quad - \epsilon \frac{\partial}{\partial t'} \gamma (E_y + v \mu (M_x + H_x)) \end{aligned} \quad (33)$$

To ensure invariance according to the real orthogonal spacetime for both equations, the following field transformation equations have to be used:

$$\begin{aligned} E'_x &= \gamma (E_x - v \mu (M_y + H_y)) \\ E'_y &= \gamma (E_y + v \mu (M_x + H_x)) \\ H'_x &= \gamma (H_x + v \epsilon (P_y + E_y)) \\ H'_y &= \gamma (H_y - v \epsilon (P_x + E_x)) \\ H'_z &= H_z \\ P'_x &= \gamma (P_x + v \mu (M_y + 2 \cdot H_y)) \\ P'_y &= \gamma (P_y - v \mu (M_x + 2 \cdot H_x)) \end{aligned} \quad (34)$$

Last but not least the relations in 34 can be applied on equation 30 and it can be seen that Amperes law is indeed

invariant to the real orthogonal space time transformation.

$$\begin{aligned}
 0 &= \frac{\partial}{\partial x'} \left(\frac{1}{\gamma} H'_y + v \epsilon P_x + v \epsilon E_x \right) \\
 &\quad - \frac{\partial}{\partial y'} \left(\frac{1}{\gamma} H'_x - v \epsilon P_y - v \epsilon E_y \right) \\
 &\quad - \left(\frac{1}{\gamma} J'_z + v \cdot \rho \right) - \epsilon \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) P_z \\
 &\quad - \epsilon \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) E_z \\
 &= \frac{1}{\gamma} \frac{\partial H'_y}{\partial x'} + v \epsilon \frac{\partial P_x}{\partial x'} + v \epsilon \frac{\partial E_x}{\partial x'} \\
 &\quad - \frac{1}{\gamma} \frac{\partial H'_x}{\partial y'} + v \epsilon \frac{\partial P_y}{\partial y'} + v \epsilon \frac{\partial E_y}{\partial y'} \\
 &\quad - \frac{1}{\gamma} J'_z - v \cdot \rho - \epsilon \gamma \frac{\partial P_z}{\partial t'} + \epsilon \gamma \cdot v \frac{\partial P_z}{\partial z'} \\
 &\quad - \epsilon \gamma \frac{\partial E_z}{\partial t'} + v \epsilon \gamma \frac{\partial E_z}{\partial z'} \\
 &= \frac{\partial H'_y}{\partial x'} - \frac{\partial H'_x}{\partial y'} - J'_z - \gamma \cdot v \cdot \rho \\
 &\quad + v \epsilon \gamma \frac{\partial P_x}{\partial x'} + v \epsilon \gamma \frac{\partial P_y}{\partial y'} + v \epsilon \gamma \frac{\partial E_x}{\partial x'} + v \epsilon \gamma \frac{\partial E_y}{\partial y'} \\
 &\quad - \epsilon \gamma^2 \frac{\partial P_z}{\partial t'} + \epsilon \gamma^2 \cdot v \frac{\partial P_z}{\partial z'} - \epsilon \gamma^2 \frac{\partial E_z}{\partial t'} + v \epsilon \gamma^2 \frac{\partial E_z}{\partial z'} \\
 &= \frac{\partial H'_y}{\partial x'} - \frac{\partial H'_x}{\partial y'} - J'_z - \gamma \cdot v \cdot \rho \\
 &\quad + v \epsilon \gamma \frac{\partial (P_x + E_x)}{\partial x} + v \epsilon \gamma \frac{\partial (P_y + E_y)}{\partial y} \\
 &\quad - \epsilon \gamma^2 \frac{\partial P_z}{\partial t'} - \epsilon \gamma^2 \frac{\partial E_z}{\partial t'} + \mu \epsilon \gamma^2 \cdot v \frac{\partial}{\partial z'} (P_z + E_z)
 \end{aligned} \quad (35)$$

Using the definition for the electric flux density $\vec{D} = \epsilon (\vec{P} + \vec{E})$ and introducing the $\text{div} (\vec{D})$ we get:

$$\begin{aligned}
 0 &= \frac{\partial H'_y}{\partial x'} - \frac{\partial H'_x}{\partial y'} - J'_z - \gamma \cdot v \cdot \rho \\
 &\quad + v \gamma \cdot \text{div} (\vec{D}) - v \epsilon \gamma \frac{\partial (P_z + E_z)}{\partial z} \\
 &\quad - \epsilon \gamma^2 \frac{\partial P_z}{\partial t'} - \epsilon \gamma^2 \frac{\partial E_z}{\partial t'} + \epsilon \gamma^2 \cdot v \frac{\partial}{\partial z'} (P_z + E_z) \\
 &= \frac{\partial H'_y}{\partial x'} - \frac{\partial H'_x}{\partial y'} - J'_z + \gamma \cdot v \cdot (\text{div} (\vec{D}) - \rho) \\
 &\quad - v \epsilon \gamma^2 \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) (P_z + E_z) \\
 &\quad - \epsilon \gamma^2 \frac{\partial P_z}{\partial t'} - \epsilon \gamma^2 \frac{\partial E_z}{\partial t'} + \epsilon \gamma^2 \cdot v \frac{\partial}{\partial z'} (P_z + E_z)
 \end{aligned} \quad (36)$$

Now realizing that $\text{div} (\vec{D}) = \rho$ and canceling the $\frac{\partial}{\partial z'}$ terms leads to:

$$\begin{aligned}
 0 &= \frac{\partial H'_y}{\partial x'} - \frac{\partial H'_x}{\partial y'} - J'_z \\
 &\quad - \epsilon \gamma^2 \frac{v^2}{c^2} \frac{\partial}{\partial t'} (P_z + E_z) - \epsilon \gamma^2 \frac{\partial}{\partial t'} (P_z + E_z)
 \end{aligned} \quad (37)$$

$$\begin{aligned}
 &= \frac{\partial H'_y}{\partial x'} - \frac{\partial H'_x}{\partial y'} - J'_z \\
 &\quad - \epsilon \gamma^2 \left(1 + \frac{v^2}{c^2} \right) \frac{\partial}{\partial t'} (P_z + E_z)
 \end{aligned}$$

And finally using $\gamma^2 = \frac{1}{1 + \frac{v^2}{c^2}}$ results in:

$$\begin{aligned}
 0 &= \frac{\partial H'_y}{\partial x'} - \frac{\partial H'_x}{\partial y'} - J'_z - \epsilon \frac{\partial}{\partial t'} (P_z + E_z) \\
 &= \frac{\partial H'_y}{\partial x'} - \frac{\partial H'_x}{\partial y'} - J'_z - \epsilon \frac{\partial P_z}{\partial t'} - \epsilon \frac{\partial E_z}{\partial t'}
 \end{aligned} \quad (38)$$

From this derivation the next field transformation rules are derived:

$$E'_z = E_z \quad (39)$$

$$P'_z = P_z \quad (40)$$

Now the same method is applied on the law of induction.

$$\text{rot} (\vec{E}) = -\mu \frac{\partial \vec{M}}{\partial t} - \mu \frac{\partial \vec{H}}{\partial t} \quad (41)$$

$$0 = \text{rot} (\vec{E}) + \mu \frac{\partial \vec{M}}{\partial t} + \mu \frac{\partial \vec{H}}{\partial t} \quad (42)$$

The three separated equations are as follows:

$$0 = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \mu \frac{\partial M_x}{\partial t} + \mu \frac{\partial H_x}{\partial t} \quad (43)$$

$$0 = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} + \mu \frac{\partial M_y}{\partial t} + \mu \frac{\partial H_y}{\partial t} \quad (44)$$

$$0 = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \mu \frac{\partial M_z}{\partial t} + \mu \frac{\partial H_z}{\partial t} \quad (45)$$

The transformation of the first equation 43 leads to:

$$\begin{aligned}
 0 &= \frac{\partial E_z}{\partial y'} - \gamma \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) E_y \\
 &\quad + \mu \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) (M_x + H_x) \\
 &= \frac{\partial E_z}{\partial y'} - \frac{\partial}{\partial z'} \gamma (E_y + v \mu (M_x + H_x)) \\
 &\quad + \mu \gamma \frac{\partial H_x}{\partial t'} + \mu \frac{\partial}{\partial t'} \gamma (M_x - v \epsilon E_y)
 \end{aligned} \quad (46)$$

Transforming this with the corresponding rules of 34 results in:

$$\begin{aligned}
 0 &= \frac{\partial E'_z}{\partial y'} - \frac{\partial E'_y}{\partial z'} + \frac{\partial}{\partial t'} (H'_x - \gamma v \epsilon P_y - \gamma v \epsilon E_y) \\
 &\quad + \mu \frac{\partial}{\partial t'} \gamma (M_x - v \epsilon E_y)
 \end{aligned} \quad (47)$$

$$= \frac{\partial E'_z}{\partial y'} - \frac{\partial E'_y}{\partial z'} + \mu \frac{\partial}{\partial t'} \gamma (M_x - v\epsilon (P_y + E_y)) + \frac{\partial H'_x}{\partial t'}$$

And the second equation 44 gives:

$$\begin{aligned} 0 &= \gamma \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) E_x - \frac{\partial E_z}{\partial x'} \\ &\quad + \mu \gamma \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) (M_y + H_y) \\ &= \frac{\partial}{\partial z'} \gamma (E_x - v\mu (M_y + H_y)) - \frac{\partial E_z}{\partial x'} \\ &\quad + \mu \gamma \frac{\partial H_y}{\partial t'} + \mu \frac{\partial}{\partial t'} \gamma (M_y + v\epsilon E_x) \end{aligned} \quad (48)$$

Transforming this with the corresponding rules of 34 results in:

$$\begin{aligned} 0 &= \frac{\partial E'_x}{\partial z'} - \frac{\partial E'_z}{\partial x'} + \mu \frac{\partial}{\partial t'} (H'_y + \gamma v\epsilon P_x + \gamma v\epsilon E_x) \\ &\quad + \mu \frac{\partial}{\partial t'} \gamma (M_y + v\epsilon E_x) \\ &= \frac{\partial E'_x}{\partial z'} - \frac{\partial E'_z}{\partial x'} + \mu \frac{\partial}{\partial t'} \gamma (M_y + v\epsilon P_x + 2v\epsilon E_x) + \mu \frac{\partial H'_y}{\partial t'} \end{aligned} \quad (49)$$

From these results it can be seen that the field transformation equations, derived from Amperes law, transform the first two equations of the law of induction, if the following additional transformation rules are introduced:

$$M'_x = \gamma (M_x - v\epsilon (P_y + 2E_y)) \quad (50)$$

$$M'_y = \gamma (M_y + v\epsilon (P_x + 2E_x)) \quad (51)$$

Last but not least the transformation rules can be applied on equation 45.

$$\begin{aligned} 0 &= \frac{\partial}{\partial x'} \left(\frac{1}{\gamma} E'_y - v\mu (M_x + H_x) \right) \\ &\quad - \frac{\partial}{\partial y'} \left(\frac{1}{\gamma} E'_x + v\mu (M_y + H_y) \right) \\ &\quad + \gamma \left(\frac{\partial}{\partial t'} - v\mu \frac{\partial}{\partial z'} \right) (M_z + H_z) \\ &= \frac{\partial E'_y}{\partial x'} - \frac{\partial E'_x}{\partial y'} - \mu \gamma v \frac{\partial}{\partial x'} (M_x + H_x) \\ &\quad - \mu \gamma v \frac{\partial}{\partial y'} (M_y + H_y) \\ &\quad + \gamma^2 \left(\frac{\partial}{\partial t'} - \mu v \frac{\partial}{\partial z'} \right) (M_z + H_z) \end{aligned} \quad (52)$$

Using $\vec{B} = \mu (\vec{M} + \vec{H})$ and $\text{div}(\vec{B}) = 0$ leads to:

$$\begin{aligned} 0 &= \frac{\partial E'_y}{\partial x'} - \frac{\partial E'_x}{\partial y'} - \gamma v \cdot \text{div}(\vec{B}) + \mu \gamma v \frac{\partial}{\partial z} (M_z + H_z) \\ &\quad + \mu \gamma^2 \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) (M_z + H_z) \\ &= \frac{\partial E'_y}{\partial x'} - \frac{\partial E'_x}{\partial y'} + \mu \gamma^2 v \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) (M_z + H_z) \end{aligned} \quad (53)$$

$$+ \mu \gamma^2 \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) (M_z + H_z)$$

And finally using $\gamma^2 = \frac{1}{1+\frac{v^2}{c^2}}$ result in:

$$\begin{aligned} 0 &= \frac{\partial E'_y}{\partial x'} - \frac{\partial E'_x}{\partial y'} + \mu \gamma^2 \left(1 + \frac{v^2}{c^2} \right) \frac{\partial}{\partial t'} (M_z + H'_z) \\ &= \frac{\partial E'_y}{\partial x'} - \frac{\partial E'_x}{\partial y'} + \mu \frac{\partial M_z}{\partial t'} + \mu \frac{\partial H'_z}{\partial t'} \end{aligned} \quad (54)$$

This proves the invariance of the field equations to the real orthogonal spacetime. From this derivation the last field transformation rule has been found:

$$M'_z = M_z \quad (55)$$

In this proof it is shown that the following field transformation rules have to be applied:

$$E'_x = \gamma (E_x - v\mu (M_y + H_y)) \quad (56)$$

$$E'_y = \gamma (E_y + v\mu (M_x + H_x))$$

$$E'_z = E_z$$

$$H'_x = \gamma (H_x + v\epsilon (P_y + E_y)) \quad (57)$$

$$H'_y = \gamma (H_y - v\epsilon (P_x + E_x))$$

$$H'_z = H_z$$

Both rules are identical to the transformation rules derived by A. Einstein [5], except that γ has changed. This is a very important result, as it shows that our well established electromagnetic theory is in fact invariant under orthogonal Cartesian space time transformation. However, in addition to the field transformation rules 56 and 57 additional transformation rules for the polarization and magnetization have to be considered.

$$P'_x = \gamma (P_x + v\mu (M_y + 2 \cdot H_y)) \quad (58)$$

$$P'_y = \gamma (P_y - v\mu (M_x + 2 \cdot H_x))$$

$$P'_z = P_z$$

$$M'_x = \gamma (M_x - v\epsilon (P_y + 2 \cdot E_y)) \quad (59)$$

$$M'_y = \gamma (M_y + v\epsilon (P_x + 2 \cdot E_x))$$

$$M'_z = M_z$$

This is a new and very useful insight.

For arbitrary directions the transformations 56 to 59 have the form of:

$$\vec{E}' = \gamma \left(\vec{E} + \vec{v} \times \mu (\vec{M} + \vec{H}) \right) + (1 - \gamma) \frac{\vec{v} \circ \vec{E}}{|\vec{v}|^2} \vec{v} \quad (60)$$

$$\vec{H}' = \gamma \left(\vec{H} - \vec{v} \times \epsilon (\vec{P} + \vec{E}) \right) + (1 - \gamma) \frac{\vec{v} \circ \vec{H}}{\|\vec{v}\|^2} \vec{v} \quad (61)$$

$$\vec{P}' = \gamma \left(\vec{P} - \vec{v} \times \mu \left(\vec{M} + 2\vec{H} \right) \right) + (1 - \gamma) \frac{\vec{v} \circ \vec{P}}{|\vec{v}|^2} \vec{v} \quad (62)$$

$$\vec{M}' = \gamma \left(\vec{M} + \vec{v} \times \epsilon \left(\vec{P} + 2\vec{E} \right) \right) + (1 - \gamma) \frac{\vec{v} \circ \vec{M}}{\|\vec{v}\|^2} \vec{v} \quad (63)$$

To get the transformation rules for moved bulky bodies like Einstein defined in [24], we can add equations 61 and 63, or 62 and 60 to get the transformation rules for the flux densities.

$$\vec{E}' = \gamma \left(\vec{E} + \vec{v} \times \vec{B} \right) + (1 - \gamma) \frac{\vec{v} \circ \vec{E}}{|\vec{v}|^2} \vec{v} \quad (64)$$

$$\vec{H}' = \gamma \left(\vec{H} - \vec{v} \times \vec{D} \right) + (1 - \gamma) \frac{\vec{v} \circ \vec{H}}{\|\vec{v}\|^2} \vec{v} \quad (65)$$

$$\vec{D}' = \gamma \left(\vec{D} - \frac{\vec{v}}{c^2} \times \vec{H} \right) + (1 - \gamma) \frac{\vec{v} \circ \vec{D}}{|\vec{v}|^2} \vec{v} \quad (66)$$

$$\vec{B}' = \gamma \left(\vec{B} + \frac{\vec{v}}{c^2} \times \vec{E} \right) + (1 - \gamma) \frac{\vec{v} \circ \vec{B}}{\|\vec{v}\|^2} \vec{v} \quad (67)$$

These equations are the same as for the Lorentz transformation based field transformation [24], except the γ and the signs in front of the $\frac{\vec{v}}{c^2}$ terms in the flux density transformation rules.

For velocities much smaller than c this simplifies to:

$$\vec{E}' \approx \vec{E} + \vec{v} \times \vec{B} \quad (68)$$

$$\vec{H}' \approx \vec{H} - \vec{v} \times \vec{D} \quad (69)$$

$$\vec{D}' \approx \vec{D} \quad (70)$$

$$\vec{B}' \approx \vec{B} \quad (71)$$

which is in perfect agreement with our experience and valid for all technical applications.

4.3 The full set of wave equations

When polarization and magnetization can happen in empty space, wave equations for all quantities which are invariant under orthogonal Cartesian transformations are needed. This invariant wave operator is:

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \quad (72)$$

Applying the transformation rules for differentials 18 results in:

$$\begin{aligned} & \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} + \frac{1}{c^2} \frac{\partial^2}{\partial t'^2} \\ &= \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} \\ &+ \gamma^2 \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) \left(\frac{\partial}{\partial z'} + \frac{v}{c^2} \frac{\partial}{\partial t'} \right) \\ &+ \gamma^2 \frac{1}{c^2} \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) \left(\frac{\partial}{\partial t'} - v \frac{\partial}{\partial z'} \right) \\ &= \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} \\ &+ \gamma^2 \left(\frac{\partial^2}{\partial z'^2} + 2 \frac{v}{c^2} \frac{\partial^2}{\partial t' \partial z'} + \frac{v^2}{c^2} \frac{\partial^2}{\partial t'^2} \right) \\ &+ \gamma^2 \left(\frac{1}{c^2} \frac{\partial^2}{\partial t'^2} - 2 \frac{v}{c^2} \frac{\partial^2}{\partial z' \partial t'} + \frac{v^2}{c^2} \frac{\partial^2}{\partial z'^2} \right) \\ &= \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \gamma^2 \left(1 + \frac{v^2}{c^2} \right) \frac{\partial^2}{\partial z'^2} \\ &+ \gamma^2 \left(1 + \frac{v^2}{c^2} \right) \frac{1}{c^2} \frac{\partial^2}{\partial t'^2} \\ &= \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} + \frac{1}{c^2} \frac{\partial^2}{\partial t'^2} \end{aligned} \quad (73)$$

This proves the invariance of the new wave operator 72.

Applying the standard derivation for the wave equation like in [6, S. 414], with an additional curl on the field equations, leads to:

$$\vec{\Delta} \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \quad (74)$$

$$\mu \frac{\partial \vec{J}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2} + \mu \frac{\partial \text{rot}(\vec{M})}{\partial t} + \text{grad}(\text{div}(\vec{E}))$$

$$\vec{\Delta} \vec{H} - \frac{1}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} =$$

$$\frac{1}{c^2} \frac{\partial^2 \vec{M}}{\partial t^2} - \text{rot}(\vec{J}) - \epsilon \frac{\partial \text{rot}(\vec{P})}{\partial t} + \text{grad}(\text{div}(\vec{H}))$$

Introducing wave equations which are invariant to the orthogonal Cartesian space time transformation 74 gives:

$$\vec{\Delta} \vec{E} + \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad (75)$$

$$\vec{\Delta} \vec{H} + \frac{1}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad (76)$$

To get the identities of 75 and 76 the following equations should be satisfied:

$$0 = 2 \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \mu \frac{\partial \vec{J}}{\partial t} + \frac{1}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2} \quad (77)$$

$$\begin{aligned}
& + \mu \frac{\partial \operatorname{rot}(\vec{M})}{\partial t} + \operatorname{grad}(\operatorname{div}(\vec{E})) \\
0 = & 2 \frac{1}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} + \frac{1}{c^2} \frac{\partial^2 \vec{M}}{\partial t^2} - \operatorname{rot}(\vec{J}) \\
& - \epsilon \frac{\partial \operatorname{rot}(\vec{P})}{\partial t} + \operatorname{grad}(\operatorname{div}(\vec{H}))
\end{aligned}$$

which is

$$\begin{aligned}
\frac{\partial \operatorname{rot}(\vec{M})}{\partial t} = & -\frac{\partial \vec{J}}{\partial t} - \epsilon \frac{\partial^2 \vec{P}}{\partial t^2} \\
& - \frac{1}{\mu} \operatorname{grad}(\operatorname{div}(\vec{E})) - 2\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}
\end{aligned} \quad (78)$$

$$\begin{aligned}
\frac{\partial \operatorname{rot}(\vec{P})}{\partial t} = & -\frac{1}{\epsilon} \operatorname{rot}(\vec{J}) + \mu \frac{\partial^2 \vec{M}}{\partial t^2} \\
& + \frac{1}{\epsilon} \operatorname{grad}(\operatorname{div}(\vec{H})) + 2\mu \frac{\partial^2 \vec{H}}{\partial t^2}
\end{aligned} \quad (79)$$

These equations give very interesting relations. Please note that they are only valid for oscillating fields where the field quantities fulfill the new wave equations. Using them for static cases will possibly lead to wrong results. Applying the curl on the first equation and replacing $\operatorname{rot}(\vec{P})$ and $\operatorname{rot}(\vec{J})$ by the second equation gives, with an integration with respect to time, the following non invariant wave equation for the magnetic moments:

$$\begin{aligned}
\Delta \vec{M} - \frac{1}{c^2} \frac{\partial^2 \vec{M}}{\partial t^2} = & \operatorname{grad}(\operatorname{div}(\vec{M})) \\
& + \operatorname{grad}(\operatorname{div}(\vec{H})) \\
& + 2 \frac{1}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} + \int \operatorname{rot} \left(\frac{1}{\mu} \operatorname{grad}(\operatorname{div}(\vec{E})) + 2\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \right) \partial t
\end{aligned} \quad (80)$$

Applying the curl on the second equation and replacing $\frac{\partial \operatorname{rot}(\vec{M})}{\partial t}$ by the first gives wave equations for the polarization and current densities:

$$\begin{aligned}
\frac{\partial}{\partial t} \left(\Delta \vec{P} - \frac{1}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2} \right) + \frac{1}{\epsilon} \left(\Delta \vec{J} - \frac{1}{c^2} \frac{\partial^2 \vec{J}}{\partial t^2} \right) = & \\
\operatorname{grad} \left(\operatorname{div} \left(\frac{1}{\epsilon} \vec{J} + \frac{\partial \vec{P}}{\partial t} \right) \right) + \frac{\partial \operatorname{grad}(\operatorname{div}(\vec{E}))}{\partial t} & \\
+ \frac{2}{c^2} \frac{\partial^3 \vec{E}}{\partial t^3} - \operatorname{rot} \left(\frac{1}{\epsilon} \operatorname{grad}(\operatorname{div}(\vec{H})) + 2\mu \frac{\partial^2 \vec{H}}{\partial t^2} \right) &
\end{aligned} \quad (81)$$

To get the corresponding invariant wave equations

$$\Delta \vec{M} + \frac{1}{c^2} \frac{\partial^2 \vec{M}}{\partial t^2} = 0 \quad (82)$$

$$\Delta \vec{P} + \frac{1}{c^2} \frac{\partial^2 \vec{P}}{\partial t^2} = 0 \quad (83)$$

$$\Delta \vec{J} + \frac{1}{c^2} \frac{\partial^2 \vec{J}}{\partial t^2} = 0 \quad (84)$$

the following equations should be satisfied:

$$\begin{aligned}
0 = & 2 \frac{1}{c^2} \frac{\partial^2 \vec{M}}{\partial t^2} + \operatorname{grad}(\operatorname{div}(\vec{M})) + \operatorname{grad}(\operatorname{div}(\vec{H})) \quad (85) \\
& + 2 \frac{1}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} + \int \operatorname{rot} \left(\frac{1}{\mu} \operatorname{grad}(\operatorname{div}(\vec{E})) + 2\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \right) \partial t \\
0 = & 2 \frac{1}{\epsilon c^2} \frac{\partial^2 \vec{J}}{\partial t^2} + 2 \frac{1}{c^2} \frac{\partial^3 \vec{P}}{\partial t^3} \\
& + \operatorname{grad} \left(\operatorname{div} \left(\frac{1}{\epsilon} \vec{J} + \frac{\partial \vec{P}}{\partial t} \right) \right) + \frac{\partial \operatorname{grad}(\operatorname{div}(\vec{E}))}{\partial t} \\
& + \frac{2}{c^2} \frac{\partial^3 \vec{E}}{\partial t^3} - \operatorname{rot} \left(\frac{1}{\epsilon} \operatorname{grad}(\operatorname{div}(\vec{H})) + 2\mu \frac{\partial^2 \vec{H}}{\partial t^2} \right)
\end{aligned}$$

From $\operatorname{div}(\vec{B}) = 0$ follows $\operatorname{div}(\vec{M}) + \operatorname{div}(\vec{H}) = 0$. Furthermore:

$$\begin{aligned}
\operatorname{div}(\vec{J}) = & -\frac{\partial \rho}{\partial t} = -\frac{\partial \operatorname{div}(\vec{D})}{\partial t} \\
= & -\epsilon \left(\frac{\partial \operatorname{div}(\vec{P})}{\partial t} + \frac{\partial \operatorname{div}(\vec{E})}{\partial t} \right)
\end{aligned} \quad (86)$$

Applying $\operatorname{rot}(\operatorname{div}(\dots)) = 0$ in addition, the remaining parts give:

$$\begin{aligned}
0 = & 2 \frac{1}{c^2} \frac{\partial^2 \vec{M}}{\partial t^2} + 2 \frac{1}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} + \int \operatorname{rot} \left(2\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \right) \partial t \quad (87) \\
0 = & 2 \frac{1}{\epsilon c^2} \frac{\partial^2 \vec{J}}{\partial t^2} + 2 \frac{1}{c^2} \frac{\partial^3 \vec{P}}{\partial t^3} + \frac{2}{c^2} \frac{\partial^3 \vec{E}}{\partial t^3} - \operatorname{rot} \left(2\mu \frac{\partial^2 \vec{H}}{\partial t^2} \right)
\end{aligned}$$

or

$$\begin{aligned}
0 = & \frac{2}{c^2} \frac{\partial}{\partial t} \left(\operatorname{rot}(\vec{E}) + \mu \frac{\partial \vec{M}}{\partial t} + \mu \frac{\partial \vec{H}}{\partial t} \right) \quad (88) \\
0 = & \frac{-2}{c^2} \frac{\partial^2}{\partial t^2} \left(\operatorname{rot}(\vec{H}) - \vec{J} - \epsilon \frac{\partial \vec{P}}{\partial t} - \epsilon \frac{\partial \vec{E}}{\partial t} \right)
\end{aligned}$$

Both identities are satisfied if Maxwell's field equations 1 and 2 are satisfied. Now it has been proven that a set of solutions for the field equations satisfies wave equations for $\vec{E}$, $\vec{B}$, $\vec{M}$, $\vec{P}$ and $\vec{J}$. This does not mean that every solution of a wave equation fulfills the field equations. However, the wave equations have to be fulfilled to find a solution for the field equations.

For these wave equations the simple plane wave solution is not possible. Two solutions for these equations have been found which are worth to mention. Assuming propagation in z direction, the first solution shows what happens if a plane of electromagnetic fields is applied:

$$f(x, y, z, t) = \sin(\omega t) \cdot e^{-kz} \quad (89)$$

This solution explains the effect of tunneling. An oscillating field ($\omega = k \cdot c$) leads to a penetration of energy into empty space.

The second solution uses the perpendicular dimensions to compensate for the propagation direction and the time dimensions:

$$f(x, y, z, t) = \sin(k(z - ct)) \cdot e^{-k(|x|+|y|)} \quad (90)$$

This solution is a traveling wave with finite field energy transport due to the exponential decay in the perpendicular directions.

4.4 Propagation of Rays of Light

Assume the following vector potential $\vec{A}_v$ of a linearly polarized ray of light propagating in z direction:

$$\vec{A}_v = \begin{pmatrix} \frac{E_0}{c \cdot k} \cdot \sin(k(z - ct)) \cdot e^{-k(|x|+|y|)} \\ 0 \\ 0 \end{pmatrix} \quad (91)$$

This vector potential is not divergence free, resulting in the following non zero virtual charge density distribution:

$$\rho_v = -2 \frac{E_0 \cdot k}{Z_0 \cdot c} \cdot \frac{x}{|x|} \cdot \cos(k(z - ct)) \cdot e^{-k(|x|+|y|)} \quad (92)$$

Figure 3 shows the virtual charge density distribution necessary to fulfill the conservation of charge rule. The virtual charges are divided on both sides of the polarization axes x . The movement of the charges is therefore perpendicular to the propagation direction and the wave is of transversal nature accordingly.

The corresponding fields still have some degrees of freedom which can be found by using inverse vector operators [25]. Field configurations have been chosen which lead to a sensible energy transport.

$$\vec{E} = E_0 \begin{pmatrix} 2 \cdot \cos(k(z - ct)) \\ \frac{x \cdot y}{|x| \cdot |y|} \cos(k(z - ct)) \\ -\frac{x}{|x|} \sin(k(z - ct)) \end{pmatrix} e^{-k(|x|+|y|)} \quad (93)$$

$$\vec{H} = \frac{E_0}{Z_0} \begin{pmatrix} \frac{x}{|x|} \frac{y}{|y|} \cos(k(z - ct)) \\ 2 \cos(k(z - ct)) \\ -\frac{y}{|y|} \sin(k(z - ct)) \end{pmatrix} e^{-k(|x|+|y|)} \quad (94)$$

Figure 4 shows the norm of the field distribution in the plane perpendicular to direction of propagation, which in this case is z . It can be seen that there are two orthogonal preferred axes which are in alignment with the two field

Figure 3: Illustration of the virtual charge density distribution of a linear polarized ray of light in the plane of propagation z and the polarisation axes x . Black illustrates maximal negative, gray zero and white maximal positive virtual charge density.

Figure 4: Illustration of the field distribution in the plane perpendicular to direction of propagation z (norm of $\vec{E}$ or $\vec{H}$). White is maximal intensity and black is zero intensity.

directions $\vec{E}$ and $\vec{B}$ in the linearly polarized example. It can be seen that this field configuration has finite energy in the x - y -plane.

From the equation of continuity, which ensures the conservation of charge, the density of polarization currents can be derived with the assumption that the free charge density is zero. This gives the solution corresponding to Amperes law if the field quantities 93 and 94 are

used.

$$\vec{J}_p = \frac{E_0 \cdot k}{Z_0} \begin{pmatrix} 1 \\ \frac{x \cdot y}{|x||y|} \\ 0 \end{pmatrix} \sin(k(z - ct)) \cdot e^{-k(|x|+|y|)} \quad (95)$$

The density of polarization currents shows 90° phase shift corresponding to the charge density of equation 92. This seems correct as the maximum velocity of the virtual charges corresponds to the minimum displacement and vice versa. Figure 5 shows the densities of polarization currents in the yx -plane at $t = T/4$ and $z = 0$. Keeping in mind that positive and negative virtual charges are involved and therefore negative virtual charges change the direction of the density of polarization current vectors with respect to the velocity of the moving virtual charge, it can be seen that the positive charges in the lower half space will move upwards (+x) and the negative charges in the upper half space will move downwards (-x) with 90° phase shift. At the same time the virtual charges will concentrate in the center of the y axes. The whole movement will be a continuous alternation of spreading the virtual charges in space and concentrate them back into the center.

Figure 5: Illustration of the density of polarization currents in y - x plane at $t = T/4$ and $z = 0$.

The solution corresponding to 93 and 94 for $\vec{M}$ is:

$$\vec{M} = -\frac{E_0}{c} \cdot \begin{pmatrix} \frac{x}{|x|} \frac{y}{|y|} \cos(k(z - ct)) \\ \cos(k(z - ct)) \\ 0 \end{pmatrix} \cdot e^{-k(|x|+|y|)} \quad (96)$$

Checking this set of solutions for conservation laws it has been found that $\text{div}(\vec{P}) + \text{div}(\vec{E}) = 0$ and $\text{div}(\vec{M}) + \text{div}(\vec{H}) = 0$ holds for every point in space.

Figure 6 shows the magnetic momenta caused by overlapping spin momenta of the involved virtual charge densities. Please note that alignment within the fields is possible, more or less independent for currents and spins.

Figure 6: Illustration of the magnetic momenta necessary to satisfy the ray equations

This derivation just shows what the ray of light solution could look like. This is no rigorously derived proof that it will indeed happen in this way. This proof would need a clear concept about virtual charges and how they behave. This would definitely go beyond the aim of this paper.

In contrast to the plane wave solution the proposed ray of light solution needs field components in direction of propagation to realize a finite but divergence free configuration. It is clear that finite fields can only be pure vortex fields if two space dimensions are involved. Figure 7 shows the vertices of the flux densities. It is easy to prove that these fields are divergence free and fulfill the new ray equations.

The flux densities are:

$$\vec{D} = \epsilon \cdot E_0 \begin{pmatrix} \cos(k(z - ct)) \\ 0 \\ -\frac{x}{|x|} \sin(k(z - ct)) \end{pmatrix} e^{-k(|x|+|y|)} \quad (97)$$

$$\vec{B} = \frac{E_0}{c} \begin{pmatrix} 0 \\ \cos(k(z - ct)) \\ -\frac{y}{|y|} \sin(k(z - ct)) \end{pmatrix} e^{-k(|x|+|y|)} \quad (98)$$

It can be seen that the flux densities are in phase but on orthogonal planes. Therefore a ray of light is a TEM wave. Finally, the ray of light looks like expected for electromagnetic waves, e.g. [6, p. 35, fig. 1.26] or [7, p. 33, fig. 1.26].

Figure 7: Illustration of the flux density vortices in direction of propagation z . The $\vec{D}$ and $\vec{B}$ fields look the same, however the shown planes are x - z for $\vec{D}$ and y - z for $\vec{B}$

4.5 Poynting vector and energy transport

Defining the Poynting vector as $\vec{S} = \vec{E} \times \vec{H}$ and applying the divergence $\text{div}()$ and Maxwell's equations leads to [6, p. 109], [7, p. 108]:

$$\begin{aligned} \text{div}(\vec{S}) &= \text{div}(\vec{E} \times \vec{H}) = \vec{H} \cdot \text{rot}(\vec{E}) - \vec{E} \cdot \text{rot}(\vec{H}) \quad (99) \\ &= -\vec{H} \cdot \frac{\partial \vec{B}}{\partial t} - \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} - \vec{E} \cdot \vec{J} \end{aligned}$$

This Poynting vector is usually interpreted as the energy flow of the electromagnetic field as equation 99 shows that the source of the energy flowing out of a volume is a change of energy within the volume or an ohmic source (or loss) [6, p. 110]. However, in the presence of virtual charges 99 is not equal to a change of field energy. To come up with a better solution a new definition of the Poynting vector is needed.

Lets start with $\text{div}(\vec{E} \times \vec{M})$. Replacing $\text{rot}(\vec{E})$ by 2 and $\text{rot}(\vec{M})$ by 79 simplifying by setting $\vec{J} = 0$ and $\text{grad}(\text{div}(E)) = 0$ gives:

$$\begin{aligned} \text{div}(\vec{E} \times \vec{M}) &\quad (100) \\ &= \vec{M} \cdot \text{rot}(\vec{E}) - \vec{E} \cdot \text{rot}(\vec{M}) \\ &= -\vec{M} \cdot \frac{\partial (\mu \vec{M} + 2\mu \vec{H})}{\partial t} + \vec{E} \cdot \frac{\partial (\epsilon \vec{P} + 2\epsilon \vec{E})}{\partial t} \end{aligned}$$

Now $\text{div}(\vec{P} \times \vec{H})$ is calculated. Replacing $\text{rot}(\vec{H})$ by 1 and $\text{rot}(\vec{P})$ by 80 simplifying by setting $\vec{J} = 0$ and $\text{grad}(\text{div}(H)) = 0$ gives:

$$\text{div}(\vec{P} \times \vec{H}) \quad (101)$$

$$\begin{aligned} &= \vec{H} \cdot \text{rot}(\vec{P}) - \vec{P} \cdot \text{rot}(\vec{H}) \\ &= \vec{H} \cdot \frac{\partial (\mu \vec{M} + 2\mu \vec{H})}{\partial t} - \vec{P} \cdot \frac{\partial (\epsilon \vec{P} + \epsilon \vec{E})}{\partial t} \end{aligned}$$

Finally $\text{div}(\vec{P} \times \vec{M})$ is calculated. Replacing $\text{rot}(\vec{P})$ by 80 and $\text{rot}(\vec{M})$ by 79 with same simplifications like above gives:

$$\begin{aligned} \text{div}(\vec{P} \times \vec{M}) &\quad (102) \\ &= \vec{M} \cdot \text{rot}(\vec{P}) - \vec{P} \cdot \text{rot}(\vec{M}) \\ &= \vec{M} \cdot \frac{\partial (\mu \vec{M} + 2\mu \vec{H})}{\partial t} + \vec{P} \cdot \frac{\partial (\epsilon \vec{P} + 2\epsilon \vec{E})}{\partial t} \end{aligned}$$

Redefining the pointing vector in the following form

$$\begin{aligned} \vec{S} &= -\frac{1}{2} \vec{P} \times \vec{H} - \frac{1}{2} \vec{E} \times \vec{M} - \frac{1}{2} \vec{P} \times \vec{M} \quad (103) \\ &= \frac{1}{2} \vec{E} \times \vec{H} - \frac{1}{2} c^2 \vec{D} \times \vec{B} \end{aligned}$$

the divergence of the vector relates directly to the change of field energy:

$$\begin{aligned} \text{div} \left(\frac{1}{2} \vec{E} \times \vec{H} - \frac{1}{2} c^2 \vec{D} \times \vec{B} \right) &\quad (104) \\ &= -\frac{1}{2} \mu (\vec{M} + \vec{H}) \cdot \frac{\partial \vec{H}}{\partial t} - \frac{1}{2} \mu \frac{\partial (\vec{M} + \vec{H})}{\partial t} \cdot \vec{H} \\ &\quad - \frac{1}{2} \epsilon (\vec{P} + \vec{E}) \cdot \frac{\partial \vec{E}}{\partial t} - \frac{1}{2} \epsilon \frac{\partial (\vec{P} + \vec{E})}{\partial t} \cdot \vec{E} \\ &= -\frac{1}{2} \frac{\partial \vec{H} \mu (\vec{M} + \vec{H})}{\partial t} - \frac{1}{2} \frac{\partial \vec{E} \epsilon (\vec{P} + \vec{E})}{\partial t} \end{aligned}$$

The sign and the 1/2 are chosen to ensure the same inherent energy definition like in 99, but 104 enables arbitrary orientations of the field and polarization vectors.

The newly defined Poynting vector for the ray of light results in:

$$\vec{S} = \frac{E_0^2}{Z_0} \begin{pmatrix} 0 \\ 0 \\ \cos(k(z-ct))^2 \end{pmatrix} e^{-2k(|x|+|y|)} \quad (105)$$

Please note that the time average of the divergence of this vector is zero because on every point the energy flows in and out.

This new Poynting vector can be used as an energy flow of electromagnetic field waves as it makes sense for many applications. However, the Poynting vector is not the only possible way to transfer energy. This is possible for magnetic or electric coupling without any Poynting vector in space.

5 Conclusion

In this article the invariance of Maxwell's field equations to an orthogonal Cartesian space time transformation has been proven if polarization and magnetization are allowed to happen in space. The ray of light solution acts as a prove of usefulness of this new approach. This solution gives a detailed description in terms of electromagnetic fields of what a ray of light could look like. This is in fact not possible with the plane wave solution which shows infinite field energy transport due to the unbounded fields perpendicular to the propagation direction. It is theoretically unsatisfactory that the theory of relativity gives local field solutions a prominent meaning as carrier of energy density and momentum but the plane wave solution carries infinite total field energy.

Possible applications of the bounded solution are diffraction theories and ray tracing. Even the effect of tunneling appears to be a field solution in the Euclidean space. The new role of the magnetization and polarization, used in this derivation, can help to find new description models of material properties on a microscopic scale.

Future work should be dedicated to compare the orthogonal Cartesian relativity with the Lorentz transformation based relativity as the Lorentz transformation acts as a standard reference to prove new theories. In this context it was important to show that both geometrical spaces give valid solutions to Maxwell's field equations and the principle of relativity. Finally, deducing the constance of velocity of light within the Euclidean space would be of great theoretical value.

References

- [1] Maxwell J. C., A dynamical theory of the electromagnetic field, *Philos. Trans. Royal Soc.*, 1865, 155, 459–512.
- [2] Maxwell J. C., A Treatise on Electricity and Magnetism, Macmillan, New York, 1873
- [3] Lorentz H. A., Versuch einer Theorie der elektrischen und magnetischen Erscheinungen in bewegten Körpern., E. J. Brill, Leiden, 1895
- [4] Pauli W., Theory of Relativity, Pergamon Press, Oxford England, 1958
- [5] Einstein A., Zur Elektrodynamik bewegter Körper, *Annalen der Physik*, 1905, 17, 891–921.
- [6] Lehner G., Elektromagnetische Feldtheorie, 6th ed., Springer-Verlag, Berlin Heidelberg, 2008
- [7] Lehner G., Electromagnetic field theory for engineers and physicists, Springer-Verlag, Berlin Heidelberg, 2010
- [8] Jackson J. D., Klassische Elektrodynamik, 4th ed., Walter de Gruyter, Berlin, 2006
- [9] W. Ritz, Recherches critiques sur l'electrodynamique generale, *Ann. Chim. Phys.*, 1908, 13, 145–275.
- [10] Lämmerzahl C., Special relativity and Lorentz invariance, *Annalen der Physik*, 2005, 14, 71–102.
- [11] Heintze J., Bock P., Lehrbuch zur Experimentalphysik, Springer Spektrum, Berlin Heidelberg, 2014
- [12] Stratton J. A., Electromagnetic Theory, reissue ed., John Wiley and Sons Inc., Hoboken New Jersey, 2007
- [13] Ignatowsky W., Das Relativitätsprinzip, *Archiv der Mathematik und Physik*, 1910, 17, 1–24.
- [14] Ignatowsky W., Das Relativitätsprinzip, *Archiv der Mathematik und Physik*, 1911, 18, 17–40.
- [15] Frank W., Rothe H., Über die Transformation der Raumzeitkoordinaten von ruhenden auf bewegte Systeme, *Annalen der Physik*, 1911, 34, 825–855.
- [16] Bartelmann M., Feuerbacher B., Krüger T., Lüst D., Rebhan A., Wipf A., Theoretische Physik, Springer, Berlin Heidelberg, 2015
- [17] Dyson F. J., The radiation theories of Tomonaga, Schwinger and Feynman, *Phys. Rev.*, 1949, 75, 486–502.
- [18] Feynman R. P., QED Die seltsame Theorie des Lichts und der Materie, Piper, München Zürich, 2006
- [19] Berestetzki W. B., Lifschitz E. M., Pitajewski L. P., Quantenelektrodynamik, 7th ed., Verlag Harri Deutsch, Frankfurt am Main, 2009
- [20] Griffiths D. J., Elektrodynamik, 3rd ed., Pearson, München, 2011
- [21] Tchernyshov O., Magnetism: Freedom for the poles, *Nature*, 2008, 451, 22–23.
- [22] Castelnovo C., Moessner R., Sondhi S. L., Magnetic monopoles in spin ice, *Nature*, 2008, 451, 42–45.
- [23] Gingras M. J. P., Observing monopoles in magnetic analog of ice, *Science*, 2009, 326, 375–376.
- [24] Einstein A., Laub J., Über die elektromagnetischen Grundgleichungen für bewegte Körper, *Annalen der Physik*, 1908, 26, 532–540.
- [25] Sahoo S., Inverse Vector Operations, arXiv:0804.2239 math-ph, 2010