Research Article

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Enhanced Entanglement in Hybrid Cavity Mediated by a Two-way Coupled Quantum Dot

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Abstract: We investigate theoretically the entanglement in a hybrid Fabry-Perot cavity system. A membrane in the cavity acts as a mechanical resonator, and a two-level quantum dot is coupled to both the cavity mode and the mechanical resonator. The entanglements between the cavity field and the mechanical resonator, between the mechanical resonator and the quantum dot, as well as between the cavity field and the quantum dot are observed. The logarithmic negativities in the first two subsystems are much larger than those in the system without two-way coupled quantum dot, and the entanglements are robust against the thermal temperature (entanglements still exist in tens of Kelvin). We also find that without direct coupling between the cavity field and the mechanical resonator, one can still observe effective entanglement between them in our system. Our work is helpful and may have potential applications in the research of multipartite entanglement in physical system.

Keywords: quantum entanglement; quantum dot; coherent manipulation

1 Introduction

Entanglement [1], a fundamental phenomenon of quantum mechanics, has been considered to be a crucial resource for quantum communication [2] and information processing [3]. Up to now, quantum entanglement has been prepared and manipulated in many physical systems, such as photons [4, 5], individual atoms [6], ions [7], between optical photon and solid-state spin qubit [8], etc. Optomechanical systems offer new types of control over the quantum states of both light and matter. By creating

entanglement between mechanical modes, hybrid optical cavity systems have good potential applications in quantum information processing. Vitali *et al.* [9] first show the stationary entanglement between an optical cavity field mode and a macroscopic vibrating mirror in a Fabry-Perot cavity. Many other types of entanglement are, afterwards, also studied and experimentally demonstrated, such as entanglement between exciton and mechanical modes [10], between two vibrating mirrors [11], between two remote micromechanical oscillators [12], etc. Hybrid optical cavities provide more controllable degrees of freedom; help us investigate various properties of the compound system, for instances, investigating the nonlinear properties of cavity optomechanics by introducing into the setup a quantum two-level system [13, 14], studying the atom-cavity entanglement with low excitation limit approximation of the atom ensembles [15], analyzing the entanglement between the optical cavity and the nanomechanical resonator beam via a quantum dot [16]. By including an optical parametric amplifier in an optical cavity, Yang *et al.* [40], Ahmed *et al.* [41] present schemes to enhance the optomechanical entanglement and the entanglement of two optical modes.

The typical schematic to study the interaction between optical cavities and mechanical objects is a Fabry-Perot cavity in which one of the cavity's mirrors is free to move. In fact, in Fabry-Perot cavity and other analogous cavities, the movable mirror must provide both optical confinement and mechanical pliability. Membrane-in-Cavity designs remove the need to integrate good mirrors into good mechanical devices [17, 18]. Meanwhile, varieties of devices integrating quantum dots and membranes are investigated and experimentally demonstrated [19–23]. And many theoretical study and experiments of the coupling between two-level system and mechanical resonator have proposed and demonstrated, such as strong coupling between a single trapped atom and a mechanical oscillator [24], coupling of an atomic force microscope cantilever to a quantum dot [25], nitrogen-vacancy defect coupled to a nanomechanical oscillator [26], exciton-phonon coupling with carbon nanotubes [27], coupling Josephson junction quantum circuits to micromechanical resonator [28], quantum dots in graphene nanoribbon and graphene sheet [29, 30],

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exciton–phonon coupling with a quantum dot embedded within a nanowire [31–33], spin-mechanical coupling of a quantum dots integrated into a cantilever resonator [34].

Motivated by these developments, we consider a hybrid Membrane-in-Cavity with a quantum dot coupling both with the optical cavity and the resonator. We study the entanglement of the system in three subsystems: cavity field and mechanical resonator subsystem, mechanical resonator and quantum dot subsystem, cavity field and quantum dot subsystem. We find that there are effective entanglements in these subsystems, and the entanglements in the first two subsystems are much strong compared with those in the system without the two-way coupled quantum dot. What's more, the entanglements in the first two subsystems are robust to temperature.

The rest of this paper is organized as follows. In Sec. 2, we introduce the model and describe the Hamiltonian of our system. In Sec. 3, the quantum Langevin equations are provided and the drift matrix, which is crucial in the Lyapunov equation to calculate the logarithmic negativity, is figured out. In Sec. 4, we calculated numerically the logarithmic negativities in the three subsystems and analyses in detail the dependence of the effective entanglement on thermal bath temperature, pump power, detuning, coupling strength, etc. Conclusions are given in Sec. 5.

2 Model

The model considered here is schematically depicted in Figure 1. The membrane is suspended within a high-Q Fabry-Perot cavity which is driven by an intense pump laser of frequency ω_l . Both mirrors of the cavity are fixed, and the membrane with a quantum dot serves as a membrane resonator in the cavity. We consider a single cavity mode of frequency ω_c . This cavity mode is coupled to the

Figure 1: Schematic diagram of the optomechanical system. The two mirrors (blue) of the cavity are fixed. The membrane (green) inside the cavity oscillates at. The quantum dot (red) is coupled with both the cavity and the membrane. The cavity is also driven by a laser of frequency.

membrane resonator mode (MR) via radiation pressure depending on both the photon number of the cavity and the displacement of the MR. The two-level quantum dot interacts with both the cavity and the MR.

The total Hamiltonian of the system is given by

$$H = \hbar\omega_c c^\dagger c + \frac{1}{2}\hbar\omega_m(p^2 + q^2) + \frac{1}{2}\hbar\omega_e s^z \quad (1)$$

$$- \hbar g_{om} c^\dagger c q + \frac{1}{2}\hbar g_{em} s^z q + \hbar g_{oe}(c^\dagger + c)s_x$$

$$+ i\hbar\epsilon_l(c^\dagger e^{-i\omega_l t} - c e^{i\omega_l t})$$

The first term represents the free energy of the cavity mode where $c^\dagger(c)$ is the creation (annihilation) operator satisfying the commutation relation $[c, c^\dagger] = 1$. The second term is the energy of MR, described by the dimensionless position and momentum operators and q which satisfy the commutation relation $[q, p] = i$. p is defined by $p = \sqrt{\frac{1}{m\hbar\omega_m}}p_o$ and $q = \sqrt{\frac{m\omega_m}{\hbar}}q_o$ where $p_o(q_o)$ is the original position (momentum) operator. The Hamiltonian of the localized quantum dot in MR is described by $\hbar\omega_e s^z$ with the transition frequency ω_e . s^z , s_x , s^+ and s are operators used to describe the two-level quantum dot satisfying relations $[s^z, s^+] = 2s^+$, $[s^z, s] = -2s$, $[s^+, s] = s^z$ and $s_x = s^+ + s$. The fourth term describes the coupling of the cavity mode with the MR. The interaction between the resonator and quantum dot is given by the fifth term in Eqs. (1), and g_{em} is the coupling strength [27]. The sixth term describes the coupling between the two-level system and the cavity with the coupling strength g_{oe} [27, 28]. The last term describes the interaction of the cavity mode with the pump laser. The amplitude of the pump laser is defined by $\epsilon_l = \sqrt{2\kappa P_l/(\hbar\omega_l)}$, where κ being the cavity decay rate and P_l the input power.

In the frame rotating at the driving laser frequency ω_l , applying a unitary transformation $U = \exp[-\frac{i}{\hbar}(\hbar\omega_l c^\dagger c + \frac{1}{2}\hbar\omega_l s^z)t] = \exp(-i\omega_l c^\dagger ct - i\frac{1}{2}\omega_l s^z t)$ and applying the rotating wave approximation, one can obtain the Hamiltonian of the system as follows,

$$H = \hbar\Delta_c c^\dagger c + \frac{1}{2}\hbar\omega_m(p^2 + q^2) + \frac{1}{2}\hbar\Delta_e s^z \quad (2)$$

$$- \hbar g_{om} c^\dagger c q + \frac{1}{2}\hbar g_{em} s^z q + \hbar g_{oe}(c^\dagger s + c s^\dagger) + i\hbar\epsilon_l(c^\dagger - c),$$

where $\Delta_c = \omega_c - \omega_l$ and $\Delta_e = \omega_e - \omega_l$ are the detuning of the cavity mode and quantum dot with respect to the pump field.

3 Calculation

There are three main components optical cavity, mechanical resonator and quantum dot in our model. In this sec-

tion, we calculate the entanglement between each two components in the system. Here, we consider the entanglements under the condition of low excitation limit approximation of the quantum dot which is broadly used in the analysis of the behavior of a hybrid cavity [13–15].

In the low quantum dot excitation limit, one has $s_z \simeq \langle s_z \rangle \simeq -1$. And operator s and s^z satisfy the usual bosonic commutation relation $[s, s^z] = 1$. In this case, the quantum Langevin equations describing the dynamic of the system then can be written as,

$$\dot{q} = \omega_m p, \quad (3)$$

$$\dot{p} = -\gamma_m p - \omega_m q + g_{om} c^\dagger c + \frac{1}{2} g_{em} + \xi, \quad (4)$$

$$\dot{c} = -[\kappa + i(\Delta_c - g_{om} q)]c - i g_{oe} s + \varepsilon_l + \sqrt{2\kappa} c_{in}, \quad (5)$$

$$\dot{s} = -[\gamma_a + i(\Delta_e + g_{em} q)]s - i g_{oe} c + \sqrt{2\gamma_a} s_{in}. \quad (6)$$

Here γ_a is the decay rate of quantum dot. $\xi(t)$ is the quantum Brownian noise acting on the MR. $c_{in}(t)$ and $s_{in}(t)$ are input vacuum noise operators for the cavity and the quantum dot. They all have zero mean values $\langle \xi(t) \rangle = \langle c_{in}(t) \rangle = \langle s_{in}(t) \rangle = 0$. The non-zero correlation functions in time domain can be written as [35, 36]

$$\begin{aligned} \langle \xi(t) \xi(t') \rangle &= \frac{1}{2\pi} \frac{\gamma_m}{\omega_m} \int \omega e^{-\omega(t-t')} \left[1 \right. \\ &\quad \left. + \coth\left(\frac{\hbar\omega}{2k_B T}\right) \right] d\omega, \end{aligned} \quad (7)$$

$$\langle c_{in}(t) c_{in}^\dagger(t') \rangle = \delta(t - t'), \quad (8)$$

$$\langle s_{in}(t) s_{in}^\dagger(t') \rangle = \delta(t - t'), \quad (9)$$

where k_B is the Boltzmann constant and T the environment temperature of MR. Under the condition of large mechanical quality factor limit of $Q = \omega_m/\gamma_m \gg 1$, $\xi(t)$ becomes delta correlated,

$$\langle \xi(t) \xi(t') + \xi(t') \xi(t) \rangle / 2 \simeq \gamma_m (2n_{th} + 1) \delta(t - t') \quad (10)$$

where $n_{th} = [\exp(\hbar\omega_m/k_B T) - 1]^{-1}$ is the mean thermal excitation number.

We focus on the steady-state entanglement between different parts of our system, which can be analyzed from the dynamics of the fluctuations of the operators in the system. We first get the steady-state solutions of the operators

by setting the time derivatives to zero in Eqs. (3)–(6) and calculate the mean values of the operators from the algebraic equations,

$$p_s = 0, \quad (11)$$

$$q_s = \frac{g_{om} I_c + \frac{1}{2} g_{em}}{\omega_m}, \quad (12)$$

$$s_s = \frac{-i g_{oe}}{\gamma_a + i(\Delta_e + g_{em} q_s)} c_s, \quad (13)$$

$$\begin{aligned} c_s &= \frac{\varepsilon_l}{k + \frac{g_{oe}^2 \gamma_a}{\gamma_a^2 + (\Delta_e + g_{em} q_s)^2} + i \left[\Delta_c - g_{om} q_s - \frac{g_{oe}^2 (\Delta_e + g_{em} q_s)}{\gamma_a^2 + (\Delta_e + g_{em} q_s)^2} \right]} \end{aligned} \quad (14)$$

where $I_c = |c_s|^2$ is the mean intracavity photon number in the steady state.

The dynamics of the fluctuation of the operators in the system is obtained from Eqs. (3)–(6) by decomposing each operator as the sum of its steady-state value and a small fluctuation, that is, $q = q_s + \delta q$, $p = p_s + \delta p$, $c = c_s + \delta c$, $s = s_s + \delta s$ and $s^z = s_s^z + \delta s^z$. Neglecting the higher-order fluctuations, we get the linearized quantum Langevin equations,

$$\delta \dot{q} = \omega_m \delta p, \quad (15)$$

$$\delta \dot{p} = -\gamma_m \delta p - \omega_m \delta q + g_{om} c_s^* \delta c + g_{om} c_s \delta c^\dagger + \xi, \quad (16)$$

$$\begin{aligned} \delta \dot{c} &= -[\kappa + i(\Delta_c - g_{om} q_s)] \delta c + i g_{om} c_s \delta q - i g_{oe} \delta s \\ &\quad + \sqrt{2\kappa} c_{in} \end{aligned} \quad (17)$$

$$\begin{aligned} \delta \dot{s} &= -[\gamma_a + i(\Delta_e + g_{em} q_s)] \delta s - i g_{em} s_s \delta q - i g_{oe} \delta c \\ &\quad + \sqrt{2\gamma_a} s_{in} \end{aligned} \quad (18)$$

We are interested in the entanglements in terms of quadrature operators which are defined by $\delta X_c = (\delta c^\dagger + \delta c)/\sqrt{2}$, $\delta Y_c = i(\delta c^\dagger - \delta c)/\sqrt{2}$, $\delta X_a = (\delta s^\dagger + \delta s)/\sqrt{2}$ and $\delta Y_a = i(\delta s^\dagger - \delta s)/\sqrt{2}$. The noises are defined in the similar way $X_{cin} = (c_{in}^\dagger + c_{in})/\sqrt{2}$, $Y_{cin} = i(c_{in}^\dagger - c_{in})/\sqrt{2}$, $X_{ain} = (s_{in}^\dagger + s_{in})/\sqrt{2}$ and $Y_{ain} = i(s_{in}^\dagger - s_{in})/\sqrt{2}$. Eqs. (15)–(18) can be rewritten in terms of δq , δp , δX_c , δY_c , δX_a and δY_a as follows.

$$\delta \dot{q} = \omega_m \delta p. \quad (19)$$

$$\delta \dot{p} = -\omega_m \delta q - \gamma_m \delta p + \frac{g_{om}(c_s^* + c_s)}{\sqrt{2}} \delta X_c \quad (20)$$

$$+ i \frac{g_{om}(c_s^* - c_s)}{\sqrt{2}} \delta Y_c + \xi.$$

$$\delta \dot{X}_c = \frac{ig_{om}(c_s - c_s^*)}{\sqrt{2}} \delta q - \kappa \delta X_c + (\Delta_c - g_{om}q_s) \delta Y_c \quad (21)$$

$$+ g_{oe} \delta Y_a + \sqrt{2\kappa} X_{cin}.$$

$$\delta \dot{Y}_c = \frac{g_{om}(c_s^* + c_s)}{\sqrt{2}} \delta q - (\Delta_c - g_{om}q_s) \delta X_c - \kappa \delta Y_c \quad (22)$$

$$- g_{oe} \delta X_a + \sqrt{2\kappa} Y_{cin}.$$

$$\delta \dot{X}_a = \frac{ig_{em}(s_s^* - s_s)}{\sqrt{2}} \delta q + g_{oe} \delta Y_c - \gamma_a \delta X_a \quad (23)$$

$$+ (\Delta_e + g_{em}q_s) \delta Y_a + \sqrt{2\gamma_a} X_{ain}.$$

$$\delta \dot{Y}_a = -\frac{g_{em}(s_s^* + s_s)}{\sqrt{2}} \delta q - g_{oe} \delta X_c - (\Delta_e + g_{em}q_s) \delta X_a \quad (24)$$

$$- \gamma_a \delta X_a + \sqrt{2\gamma_a} Y_{ain}.$$

By using the vector of quadrature fluctuations $U^T = (\delta q, \delta p, \delta X_c, \delta Y_c, \delta X_a, \delta Y_a)$ and the noise vector $N^T = (0, \xi, \sqrt{2\kappa}X_{cin}, \sqrt{2\kappa}Y_{cin}, \sqrt{2\gamma_a}X_{ain}, \sqrt{2\gamma_a}Y_{ain})$, we can write Eqs. (19)-(24) as

$$\dot{U}(t) = AU(t) + N(t), \quad (25)$$

where A is the drift matrix, and can be expressed as

$$\begin{pmatrix} 0 & \omega_m & 0 \\ -\omega_m & -\gamma_m & \frac{g_{om}(c_s^* + c_s)}{\sqrt{2}} \\ \frac{ig_{om}(c_s - c_s^*)}{\sqrt{2}} & 0 & -\kappa \\ \frac{g_{om}(c_s^* + c_s)}{\sqrt{2}} & 0 & -(\Delta_c - g_{om}q_s) \\ \frac{ig_{em}(s_s^* - s_s)}{\sqrt{2}} & 0 & 0 \\ -\frac{g_{em}(s_s^* + s_s)}{\sqrt{2}} & 0 & -g_{oe} \\ 0 & 0 & 0 \\ \frac{ig_{om}(c_s^* - c_s)}{\sqrt{2}} & 0 & 0 \\ \Delta_c - g_{om}q_s & 0 & g_{oe} \\ -\kappa & -g_{oe} & 0 \\ g_{oe} & -\gamma_a & \Delta_e + g_{em}q_s \\ 0 & -(\Delta_e + g_{em}q_s) & -\gamma_a \end{pmatrix} \quad (26)$$

The stability conditions of the system can be derived by applying the Routh-Hurwitz criteria [37]. However, it is too cumbersome to list out here. We instead guarantee the stability by numerically calculating the eigenvalues of the drift matrix A . When the real parts of all the eigenvalues of matrix A are negative, the system is stable. The quantum steady state for the fluctuations is fully characterized by the correlation matrix $V_{ij} =$

$(\langle U_i(\infty)U_j(\infty) + U_j(\infty)U_i(\infty) \rangle)/2$. When the system is stable, the correlation matrix satisfies Lyapunov equation $AV + VA^T = -D$. The elements of matrix D are obtained using the correlations of the noise operators [9]. Here, D . The Lyapunov equation is a linear equation for the covariance matrix V and can be solved straightforwardly. Here, we can write the covariance matrix V as

$$V = \begin{bmatrix} V_m & V_{cm} & V_{md} \\ V_{cm}^T & V_c & V_{cd} \\ V_{md}^T & V_{cd}^T & V_d \end{bmatrix} \quad (27)$$

Each element of matrix V in Eq. (27) is a matrix, and the elements V_m , and V_d on the main diagonal represent the variance of the three components in our system, i.e., mechanical resonator, cavity field and quantum dot. And the elements off diagonal indicate the covariance between each two components. To study the entanglement properties of the steady states of the cavity and the mechanical resonator, we extract a reduced correlation matrix V_r from V , i.e., $V_r = \begin{pmatrix} V_m & V_{cm} \\ V_{cm}^T & V_c \end{pmatrix}$. We adopt logarithmic negativity [38, 39] which is widely used in numerical modeling because it is easy to calculate, measure the entanglement for mixed states, and multipartite states. For continuous variables, logarithmic negativity is defined as

$$EN = \max[0, -\ln 2\zeta], \quad (28)$$

where $\zeta = 2^{-1/2}[\sigma - \sqrt{\sigma^2 - 4 \det V_r}]^{1/2}$, with $\sigma = \det V_c + \det V_m - 2 \det V_{cm}$. The optical cavity and the mechanical resonator modes are entangled when the logarithmic negativity EN is positive. Namely, a Gaussian state is entangled if and only if $\zeta < 1/2$, and that is consistent with Simon's theory [42]. The reduced correlation matrices for MR and quantum dot, cavity field and quantum dot are extracted, and their logarithmic negativities are calculated, in the same way.

4 Entanglement between different components of the system

We solve the Lyapunov equation and plot the logarithmic negativity of cavity field-MR subsystem EN_{cm} , MR-quantum dot subsystem EN_{md} and cavity field-quantum dot subsystem EN_{cd} as a function of temperature, of the input laser power, of the cavity field-pump laser detuning, of dot-pump laser detuning, of the cavity field-MR coupling strength, and of the coupling strength between MR and quantum dot, which are shown in Figure 2-7. The parameters are chosen with reference to the recent experiments.

Figure 2: (a) logarithmic negativity EN_{cm} of the cavity field-mechanical resonator subsystem; (b) EN_{md} of the mechanical resonator-quantum dot subsystem; (c) EN_{cd} of the cavity field-quantum dot subsystem as a function of the temperature for input laser power $P_l = 20\mu W$, $P_l = 30\mu W$, and $P_l = 40\mu W$ respectively. Other parameters are $\omega_m/2\pi = 60\text{MHz}$, $\kappa = 0.1\omega_m$, $g_{om} = 3000\text{Hz}$, $\gamma_m = 227\text{Hz}$, $g_{em} = 0.01\omega_m$, $g_{oe} = 0.06\omega_m$, $\gamma_a = 10\text{MHz}$, $\Delta_e = 0.03\omega_m$ and $\Delta_c = -0.1\omega_m$.

Figure 3: (a) logarithmic negativity EN_{cm} of the cavity field-mechanical resonator subsystem; (b) EN_{md} of the mechanical resonator-quantum dot subsystem; (c) EN_{cd} of the cavity field-quantum dot subsystem as a function of the input laser power for thermal bath temperature $T = 0.1\text{K}$, $T = 0.5\text{K}$, and $T = 1\text{K}$ respectively. Other parameters are the same as in Figure 2.

The frequency of MR is $\omega_m/2\pi = 60\text{MHz}$, and the quality of MR $Q \approx 10^6$.

In Figure 2, the cavity is weakly driven by a pump laser of input power of $20\mu W$, $30\mu W$ and respectively, and the cavity field-MR coupling strength is $g_{om} = 3000$, quantum dot-MR coupling strength is $g_{em} = 0.01\omega_m$. We find that the entanglement of cavity field-MR subsystem and MR-quantum dot subsystem are much stronger than that of cavity field-quantum dot subsystem. This can be explained from the elements in the drift Matrix A . The effective interaction strength in the cavity field-MR subsystem is altered from g_{om} to $\frac{g_{om}(c_s^* + c_s)}{\sqrt{2}}$ and $\frac{ig_{om}(c_s^* - c_s)}{\sqrt{2}}$, and the effective coupling strength in the MR-quantum dot subsystem is also changed into and $-\frac{g_{em}(s_s^* + s_s)}{\sqrt{2}}$. Nevertheless, the effective coupling strength in cavity field-quantum dot subsystem remain the same as g_{oe} . Thus, the effective coupling strength is enhanced in the first two subsystems, while the

third one not. And an enhanced coupling strength is in favor of strong entanglement in most cases. One can also see from Figure 2 that the entanglement of MR-quantum dot subsystem is more robust against thermal bath temperature than those of the other two subsystems. These entanglements are, as expected, decrease with the increasing thermal bath temperature. The critical temperature, at which the entanglement disappears, increases as the increasing of input laser power as shown in Figure 2(a) and 2(b). However, it is not the case in Figure 2(c), which means there may be an optimal input laser power for the fixed detuning.

In Figure 3, we plot the logarithmic negativity of the three subsystems as a function of the input laser power for the fixed cavity field-pump laser detuning $\Delta_c = -0.1\omega_m$ and quantum dot-pump laser detuning $\Delta_e = 0.03\omega_m$. For the same input laser power, the entanglement decreases with the increasing of the temperature which is entirely

Figure 4: (a) Logarithmic negativity of the cavity field-mechanical resonator subsystem EN_{cm} ; (b) of the mechanical resonator-quantum dot subsystem EN_{md} ; (c) of the cavity field-quantum dot subsystem EN_{cd} as a function of the scaled cavity-pump laser detuning Δ_c/ω_m for fixed quantum dot-pump laser detuning Δ_e/ω_m , fixed pump laser power $P_l = 50\mu\text{w}$ and different temperatures $T = 0.1\text{K}$, $T = 2\text{K}$, and $T = 15\text{K}$. (d) Logarithmic negativity of three subsystems for input power $P_l = 30\mu\text{w}$ and $T = 1\text{K}$. Other parameters are the same as in Figure 2.

Figure 5: (a) logarithmic negativity of the cavity field-mechanical resonator subsystem EN_{cm} ; (b) of the mechanical resonator-quantum dot subsystem EN_{md} ; (c) of the cavity field-quantum dot subsystem EN_{cd} as a function of the scaled quantum dot-pump laser detuning Δ_e/ω_m for fixed cavity-pump laser detuning $\Delta_c = -0.2\omega_m$ and fixed input power $P_l = 50\mu\text{w}$. Other parameters are the same as in Figure 2.

Figure 6: (a) Logarithmic negativity of the cavity field-mechanical resonator subsystem EN_{cm} ; (b) of the mechanical resonator-quantum dot subsystem EN_{md} ; (c) of the cavity field-quantum dot subsystem EN_{cd} as a function of the cavity mode-MR coupling strength g_{om} for different temperatures $T = 0.1K$, $T = 2K$, and $T = 15K$. (d) Logarithmic negativity of third subsystem for temperatures $T = 1mK$, $T = 5mK$ and $T = 7mK$. The detuning are fixed at $\Delta_e = 0.1\omega_m$, $\Delta_c = -0.2\omega_m$ and the pump laser power is fixed at $P_l = 50\mu W$. Other parameters are the same as in Figure 2.

Figure 7: (a) logarithmic negativity of the cavity field-mechanical resonator subsystem EN_{cm} ; (b) of the mechanical resonator-quantum dot subsystem EN_{md} ; (c) of the cavity field-quantum dot subsystem EN_{cd} as a function of the scaled quantum dot-mechanical resonator coupling strength g_{em}/ω_m for fixed detuning $\Delta_c = -0.2\omega_m$, $\Delta_e = 0.1\omega_m$ and temperature $T = 0.5K$. Other parameters are the same as in Figure 2.

reasonable. Nevertheless, the entanglement does not increase linearly with the input power. There are two peaks in each curve in Figure 3(a) and 3(b). The first one locates at about $P_l \approx 10\mu w$ and the second one $P_l \approx 90\mu w$. And there is only one peak in each curve in Figure 3(c) located at $P_l \approx 25\mu w$. That is, on the basis of the parameters in Figure 3, the optimal pump powers for cavity field-MR entanglement, MR-quantum dot entanglement and cavity field-quantum dot entanglement are $90\mu w$, $90\mu w$ and $25\mu w$ respectively. When the pump power exceeds $145\mu w$, the system becomes unstable. What's more, the logarithmic negativity in cavity field-MR, MR-quantum dot subsystem can reach 1 and 2 respectively for the input power $P_l = 1\mu w$ at temperature $T = 0.1K$, while the entanglement in cavity field-quantum dot subsystem does not appear until the input power increases to about $10\mu w$ and sharply decreased to zero $P_l \approx 60\mu w$.

Figure 4 shows the entanglement of the three subsystems over the cavity field-pump laser detuning. We find that the main regime for effective entanglement in these three subsystems is within $[-1.3\omega_m, 5\omega_m]$, and the unstable regime lies mainly on the left side of the resonant point. The detuning regime for effective cavity field-quantum dot entanglement is limited to a small regime of negative detuning near resonant. The detuning regimes for cavity field-MR and MR-quantum dot subsystem are much larger than that of cavity field-quantum dot subsystem. The peaks on the curves of these two subsystem are related to three particular detuning points $\Delta_c = -\omega_m$, $\Delta_c = 0$, $\Delta_c = +\omega_m$. That is, when the pump laser is detuned to either the red sideband $\omega_l = \omega_c - \omega_m$ or the blue sideband $\omega_l = \omega_c + \omega_m$, one can get a local optimal entanglement of these subsystems. If the pump laser is resonant with the cavity, the entanglement increases gradually with the input power. However, the peaks at $\Delta_c = 0$ will split symmetrically when the input power exceeds $45\mu w$. And the split is much sharper in cavity field-MR subsystem than that in MR-quantum dot subsystem as depicted in Figure 4(a) and 4(b). If the input power increase to $70\mu w$, there will be a small unstable regime near the resonant point. This unstable regime expands slightly with the increasing of power, for instances the unstable regime for $P_l = 70\mu w$ is $[-0.0294\omega_m, 0.0475\omega_m]$, and $[-0.0586\omega_m, 0.0793\omega_m]$ for $P_l = 90\mu w$.

The influence of quantum dot-pump laser detuning Δ_e is demonstrated in Figure 5. Similar to that of Δ_c , the stable detuning regimes of Δ_e for effective entanglement in cavity field-MR subsystem and MR-quantum dot subsystem are much wider than that in cavity field-quantum dot subsystem. And the optimal detuning region of the third subsystem is somewhat opposite to those of the former two sub-

systems. That is, the entanglement in the third subsystem ascends gradually from $\Delta_e \approx 0.1\omega_m$, and reach the maximum, and then declines to zero at $\Delta_e \approx 0.4\omega_m$, while the direction of the curves in the first two subsystems is completely opposite in that detuning region.

In Figure 6, we plot the logarithmic negativities of the three subsystems as a function of the cavity field-MR coupling strength g_{om} . We find that the entanglement in MR-quantum dot subsystem is the most robust against g_{om} which is mainly due to the direct interaction between the mechanical resonator and quantum dot. There are two peaks on each curve of the first two subsystems at low temperatures in the stable regime for effective entanglement as shown in Figure 6(a) and 6(b). Nevertheless, the entanglement in cavity field-MR subsystem is much more sensitive to the change in temperature. When the thermal bath temperature increases to above $2K$, there is only one peak remained on each curve in Figure 6 (a) with the location at $g_{om} \approx 12.5kHz$. Interestingly, we also find that the entanglement in cavity field-MR subsystem still exists when there is no direct coupling between the cavity field and MR, i.e., $g_{om} = 0$ under the condition of low temperature $T < 1K$. The logarithmic negativity EN_{cm} is about 0.06 for $g_{om} = 0$ and $T = 0.1K$. In Figure 6(d), we can see that the logarithmic negativity in cavity field-MR subsystem is over 0.1 if the temperature is under 7 milli-Kelvin with $g_{om} = 0$. And it increases to 0.35 when $T = 1mK$. That means under the condition of low temperature, without direct coupling between the cavity field and mechanical resonator, the entanglements in cavity field-MR and MR-quantum dot subsystem are still robust. Thus, if we just consider the entanglement in the first two subsystems, the mechanical resonator in our system can have varieties of alternatives including those not so suitable for coupling by optical radiation pressure, such as graphene membrane, cantilever, carbon nano tube, etc.

The dependence of the entanglement of the three subsystems on the coupling strength between MR and quantum dot is depicted in Figure 7. In the stable regime, the optimal coupling strength for effective entanglement EN_{cm} , EN_{md} and EN_{cd} are $g_{em} \approx 0.04\omega_m$, $g_{em} \approx 0.05\omega_m$ and $g_{em} \approx 0.01\omega_m$ with the input power $P_l = 50\mu w$. The optimal coupling strength g_{em} decreases with the increase of the input power as shown in Figure 7, and the stable coupling regime also shrinks. Actually, each of the parameters in our system makes contribution to the effective entanglement in these three subsystems. All the parameters should be set a value in a stable regime. To get the desired entanglement in these subsystems, one should make a trade off among these parameters.

5 Conclusions

In conclusion, we have studied in detail the stationary continuous-variable entanglement between the cavity field and the nanomembrane resonator, entanglement between the mechanical resonator and the two-level quantum dot, as well as entanglement between the cavity field and the quantum dot. The cavity with a membrane in it is weakly driven (with pump power less than $100\mu W$) by a coherent light and mediated by a quantum dot. The quantum dot interacts both with the optical cavity and the membrane resonator.

We find that there are effective entanglements in all three subsystems. The entanglements in the first two subsystems are much stronger than that in the third subsystem mainly due to the enhanced interaction between the cavity field and the mechanical resonator, and the enhanced interaction between the mechanical resonator and the quantum dot. The logarithmic negativities in the first two subsystems can be up to about 7 which is much stronger than most of those in previous studies (often less than 1).

And the entanglements in the first two subsystems are robust to thermal bath temperature with effective entanglements exists at tens of Kelvin. We also find that the effective entanglement still exists when there is no direct optical-mechanical coupling at all in our system, which means one can obtain optical-mechanical entanglement through the help of coupling two-level system with mechanical resonator. What's more, the key to our system is a two-level system coupled to both the cavity field and the mechanical resonator, thus our model can be expanded to other physical systems which can accomplish this property. Our work is helpful and may have potential applications in the research of multipartite entanglement in physical systems.

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