Research Article

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On-line detection algorithm of ore grade change in grinding grading system

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Abstract: In process industry control, process data is critical for both control and fault diagnosis. Timely detection of outliers and mutation point in process data can quickly adjust control parameters or discover potential failures throughout the system. Aiming at the shortcomings of the traditional mutation point detection method, such as the detection time delay and not being suitable for the process industrial data that mixed outliers, this paper proposes a mutation point and outliers detection method that is suitable for the grinding grading system using the wavelet method to analyze the “Efficient Scoring Vector.” In this algorithm, to distinguish between outliers and mutation points in the detection process, we propose a detection framework based on the relationship between Lipschitz index and wavelet coefficients. Under this framework, the detection algorithm proposed in this paper can detect outliers and mutation points simultaneously. The advantage of this is that the accuracy of the mutation point detection is not affected by the outliers. This means that the method can process grinding grading system process data containing outliers and mutation points under actual operating conditions and is more suitable for practical applications. Finally, the effectiveness and practicability of the proposed detection method are proved by simulation results.

Keywords: outlier, change-point, wavelet transform, Lipschitz exponent, grinding classification system, ore grade

1 Introduction

In process industry control, process data is critical for both control parameter adjustment and fault diagnosis [1].

Timely detection of outliers and trends in process data can quickly adjust control parameters or discover potential failures throughout the system [2]. For example, in the grinding grading system shown in Figure 1, factors such as changes in ore composition, changes in the production side, and wear of the steel ball can cause changes in the condition of the ball mill, resulting in a change in the control effect of the grinding grading process [3]. Whether the change of the ore composition or the wear of the steel ball will first be reflected in the migration of process data, so the rapid and accurate capture of small changes in process data is essential for the grinding and classification process control industry.

Since the characteristics of the mutation point and the abnormal point have similarity in a short time, it is difficult to distinguish the mutation point and the abnormal point in the initial stage of the process data change in the grinding classification system, so that the change in the ore grade or the potential cannot be found in time. It can be seen that for a process industry such as grinding grading system, it is necessary and important to detect and distinguish abnormal points and change points in a short time. From the current research situation, the detection of abnormal points and change points is almost independent of each other, but in the actual industry, these two kinds of data exist at the same time. In summary, in the research of the process control industry, it is necessary to simultaneously detect and distinguish these two types of data in one detection algorithm.

At present, there are relatively many researches on the problem of abnormal data detection, and some mature and practical methods have been formed. For example, statistical detection algorithms [4], batch-based detection algorithms based on distance or density [5,6], intelligent methods represented by neural networks [7], support vector methods suitable for small sample analysis [8], and cluster analysis method [9]. Since wavelet analysis has good local characteristics of the signal in both the frequency domain and the time domain, it is usually used for signal analysis [10]. In the field of anomaly data detection, the most representative one is the time series anomaly detection method based on the

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Figure 1: Configuration of typical SABC grinding and classification process.

wavelet transform modulus maxima principle proposed by Mallat and Hwang [11]. This method is suitable for extracting unstable changes from still signals. For unstable process signals, this method is no longer applicable because the modulus maximal principle method does not distinguish between the unstable state (i.e., the change point) and the abnormal change (i.e., the abnormality) of the signal.

In the field of research on mutation point detection, detection methods mainly focus on statistical theoretical methods. For example, in 1996, a method of mutation point detection using the idea of edge likelihood probability was proposed in ref. [12]. The method separates all the data from different positions in the middle and compares the maximum probability of the two parts before and after the segmentation to determine the location of the mutation point. Therefore, the calculation amount is large, which is not suitable for online mutation of a large number of time series in the process industry. Ref. [13] used the Gustafsson theory to determine the location of the mutation, which has the advantage of iterative calculation, so it is very suitable for the solution of the mutation point detection problem of the growth time series. However, the problem with this method is that its computational complexity is closely related to the frequency of occurrence of the abrupt point, which means that the amount of calculation of the method becomes abnormally large without a point of change for a long time. Therefore, in fact, it is only applicable to the problem of sequence data detection in which frequent mutations

occur, and when the sequence data to be detected is stable for a long time, the method will have a catastrophic calculation amount. Ref. [14] proposed a change point detection method based on a wavelet footprint. The advantage of this method is that it has good detection accuracy and performance, but it is not suitable for the process industry because it is also a batch detection method. Ref. [15] proposes a method for detecting a sudden change point by comparing the difference between the reference window and the data in the sliding window. The advantage of this method is to achieve online detection of the point of mutation. The disadvantages are as follows: first, as the data in the reference window increases, the characterization value of the mutation is also increased, which means that if there is no sudden occurrence for a long time, the characterization value will increase indefinitely. This indicates that the selection of the mutation point characterization value in the algorithm is not ideal. Second, the accuracy of the detection method is proportional to the length of the sliding window, so that when the mutation amplitude is small, the detection algorithm has a large detection delay. For the grinding grading process system, this detection delay is very undesirable.

Considering the two shortcomings of two kinds of window structure detection methods and the necessity of detecting anomalies and change points at the same time, in this paper, we propose a method which can run on-line and detect abnormal points and change-points in time and at the same time – an algorithm for detecting abnormal

points and change-points based on an effective fractional vector of wavelet analysis. The advantages and innovations of this method are summarized as follows: (1) in a framework, it can simultaneously detect the abnormal data and change-points, more in line with the actual application needs. (2) The detection algorithm proposed in this paper can realize the on-line real-time monitoring of the change-point and outliers, not the batch method. (3) This method does not have a large delay problem for the detection of the change-point, and the statistics in the detection process do not grow with time.

2 The ESV (effective score vector) detected method [16]

2.1 The efficient score vector

The ESV expression is defined as follows [17]:

$$V_k(\boldsymbol{\theta}, \boldsymbol{\eta}) = \sum_{i=1}^k \nabla_{\boldsymbol{\xi}} \log f(x_i; \boldsymbol{\theta}, \boldsymbol{\eta}), \quad \boldsymbol{\xi} = (\boldsymbol{\theta}, \boldsymbol{\eta}), \quad (1)$$

where $f(x; \boldsymbol{\theta}, \boldsymbol{\eta})$ is the density function, $\boldsymbol{\theta} \in \Omega_1 \subset \mathbb{R}^d$, $d \geq 1$ denotes the parameter variable of “interested” in the algorithm; $\boldsymbol{\eta} \in \Omega_2 \subset \mathbb{R}^p$, $p \geq 0$ denotes the parameter variable “not interested” in the algorithm, also called the redundant parameter; $\Omega = \Omega_1 \times \Omega_2$ is the space in which the parameter variable is located; and $\hat{\boldsymbol{\eta}}_k$ is the estimated value of the redundant parameter $\boldsymbol{\eta}$.

Then, ESV can be expressed as follows:

$$\begin{aligned} V_k(\boldsymbol{\theta}_0, \hat{\boldsymbol{\eta}}_k) &= \sum_{i=1}^k \nabla_{\boldsymbol{\xi}} \log f(x_i; \boldsymbol{\theta}_0, \hat{\boldsymbol{\eta}}_k) \\ &= \sum_{i=1}^k \nabla_{\boldsymbol{\theta}} \log f(x_i; \boldsymbol{\theta}_0, \hat{\boldsymbol{\eta}}_k). \end{aligned} \quad (2)$$

Here, we give two assumptions [16]:

$$\begin{aligned} H_0 : \boldsymbol{\theta} &= \boldsymbol{\theta}_0, \boldsymbol{\eta} \text{ unknown, for all observations;} \\ H_A : x_1, \dots, x_{\tau-1} &\text{ have } f(x; \boldsymbol{\theta}_0, \boldsymbol{\eta}), \boldsymbol{\eta} \text{ unknown,} \\ x_{\tau}, x_{\tau+1}, \dots &\text{ have } f(x; \boldsymbol{\theta}_A, \boldsymbol{\eta}), \boldsymbol{\eta}, \boldsymbol{\theta}_A \text{ unknown.} \end{aligned} \quad (3)$$

In which, $\boldsymbol{\theta}_0$ is the parameter in the distribution of the data before the occurrence of the change-point, $\boldsymbol{\theta}_A$ is the parameter in the distribution of the data after the occurrence of the change-point, and τ is the moment of the change-point. It can be seen from the assumption that the redundant parameters are unknown.

It can be seen from the ESV expression $\{V_k, k > 1\}$ in equation (2): when the assumption of H_0 is true, the mean value of the sequence value $\{V_k, k > 1\}$ is zero, When H_A is to be true, the value of the sequence value $\{V_k, k > 1\}$ of ESV will gradually increase, and its increase will increase linearly with the increase in the data after the change-points.

2.2 Change-point detection algorithm-based ESV [19]

2.2.1 For finite data

Let the total amount of data be n , first give two functions as follows:

$$\begin{aligned} a(x) &= (2 \log x)^{1/2}, \\ b(x) &= 2 \log x + \frac{1}{2} \log \log x - \frac{1}{2} \log \pi \end{aligned} \quad (4)$$

According to equation (4) and the Lemmas 1 and 2 in ref. [18], the principle of unilateral detection and the principle of bilateral detection are as follows:

- (1) Unilateral detection: as the number of data k continues to increase

$$k^{-1/2} \mathbf{W}_k \geq \mathbf{C}_1(\boldsymbol{\alpha}), \quad (5)$$

then the data sequence has a change-point near k , where $\mathbf{W}_k$ is an expression that contains ESV:

$$\mathbf{W}_k = \Gamma^{-1/2}(\boldsymbol{\theta}_0, \boldsymbol{\eta}) V_k(\boldsymbol{\theta}_0, \hat{\boldsymbol{\eta}}_k). \quad (6)$$

In which, the expression for $\Gamma(\boldsymbol{\theta}, \boldsymbol{\eta})$ is

$$\Gamma(\boldsymbol{\theta}, \boldsymbol{\eta}) = \mathbf{I}_{11} - \mathbf{I}_{12} \mathbf{I}_{22}^{-1} \mathbf{I}_{21}, \quad (7)$$

where $\mathbf{I} = \begin{pmatrix} \mathbf{I}_{11} & \mathbf{I}_{12} \\ \mathbf{I}_{21} & \mathbf{I}_{22} \end{pmatrix}$ is the information matrix and $\boldsymbol{\alpha} = [\alpha^1, \dots, \alpha^i, \dots, \alpha^d]$ is the confidence vector.

- (2) Bilateral detection: as the number of data k continues to increase

$$k^{-1/2} |\mathbf{W}_k| \geq \mathbf{C}_1^*(\boldsymbol{\alpha}), \quad (8)$$

then the data sequence has a change-point near k . Likewise, $\boldsymbol{\alpha}$ is the confidence vector.

2.2.2 For infinite data

Here, we introduce a method based on bilateral tests only for infinite data.

In bilateral testing, as the number of data t increases

$$|\mathbf{W}(t)| \geq \mathbf{C}_2^*(\boldsymbol{\alpha}), \quad (9)$$

then the data sequence has a change-point near t . Likewise, $\boldsymbol{\alpha}$ is the confidence vector. And each component in $\mathbf{C}_2^*(\boldsymbol{\alpha})$ can be calculated according to equation (19):

$$C_2^{*i}(\alpha) = [-2 \log \alpha^i + \log(t+1)]^{1/2} [t+1]^{1/2} \quad (10)$$

3 Improved outlier and change-point detection algorithm

3.1 Solutions for data distribution problems

In the ESV algorithm, it is assumed that the detected data sequence obeys the distribution f . For the grinding analysis system process data, if we want to detect the ore grade value, we can think that it obeys the Gaussian distribution, $N(\mu, \sigma^2)$.

In this case, we can see that the variance of the Gaussian distribution has changed. Based on the above analysis, we re-give the following assumptions:

$$\begin{aligned} H_0 : \sigma^2 &= \sigma_0^2, \mu \text{ unknown, for all observations.} \\ H_A : e_1, \dots, e_{\tau-1} &\text{ have } N(e; \mu, \sigma_0^2), \mu \text{ unknown,} \\ e_\tau, e_{\tau+1}, \dots &\text{ have } N(e; \mu, \sigma_A^2), \mu, \sigma_A^2 \text{ unknown.} \end{aligned} \quad (11)$$

3.2 Solution for detecting delay problems

For the traditional ESV-based detection algorithm, the disadvantage is that there is a change-point detection delay problem: the smaller the amplitude of the data sequence changed, the more serious the detection delay. The above description can be seen from the simulation (Figure 2).

In this paper, the wavelet transform theory is introduced to improve this shortcoming in the ESV algorithm. The detailed contents are the following paragraphs.

In Figure 2, it can be seen that the W_k value before the change-point is almost zero, and the wavelet coefficient obtained by the decomposition of the data sequence should be near zero. From the point of the change, the W_k value becomes the slope function form, the wavelet coefficients at this change-point will appear to be a

modulus maximum. According to the above analysis, we can determine the position of the mutation point by using the wavelet decomposition W_k value curve according to the relationship between the Lipschitz index of the function and the maximum value of the wavelet transform modulus [11].

Considering a large amount of data in the data sequence of the grinding classification system, the online recursive wavelet decomposition method proposed in ref. [20] is used to analyze the W_k value by online wavelet analysis.

3.3 Change-point, anomaly uniform detection algorithm

3.3.1 ESV representation of abnormal point

First, we use the zero-mean white noise sequence as an example to illustrate the different representations of ESV statistics for change points and outliers. Add an outlier at step 200 of this set of data and start data changes at step 500. The image is shown in Figure 3.

As can be seen from Figure 3, the W_k value of the anomaly point and the change-point is different, the W_k curve at the anomaly point is the step function form, and the W_k curve of the change-point is expressed as the slope function form.

3.3.2 Change-point and anomaly point detection algorithm based on wavelet analysis

Because of the different characteristics of the ESV statistic of the outliers and the mutation points under the wavelet transform, we propose to use the characterization relationship between the modulus maxima principle of the wavelet transform and the Lipschitz exponent [21] to detect and distinguish the anomaly points and change-points:

When $\nu > 0$, the wavelet coefficients increase with an increase in the wavelet scale.

When $\nu = 0$, the wavelet coefficients are independent of the scale.

Use this relationship to detect and distinguish between outliers and change-point. The specific algorithm is as follows:

Step 1. Wavelet decomposition of the fitting residual $E(t)$ is carried out at two wavelet scales.

Figure 2: The chart of W_k for change-point.

Step 2. Calculate the modulus of the wavelet decomposition coefficients at two scales and make the difference to get the value of E_k .

Step 3. According to the following principles for outlier and change-point detection and differentiation:

Case 1. In step 1, there is no modulus max point, and the value of E_k in step 2 does not change, indicating that the value of W_k curve is always kept near zero, and there is no change, indicating that there is no abnormal point and no change-point here.

Case 2. In step 1, the modulus maxima point appears, and in step 2, E_k has no modulus max point, indicating that the wavelet coefficients are the same at the two scales here, and should be where the anomaly is.

Case 3. In step 1 and step 2, there are modulus maxima points, which indicate that the wavelet coefficients are different at two scales and should be the point where the change-points are.

4 Simulation verification

4.1 Verification

To verify the effectiveness of the algorithm, we perform the mean shift processing of the ore grade data in the grinding classification system, which turns it into a set of zero-mean data sequences. As can be seen from the previous analysis, when the ore grade remains unchanged, the grade data processed by the mean shifting should obey the zero-mean normal distribution [22,23]. When the ore product changes, it no longer obeys the zero-mean Gaussian distribution. To simulate the presence of outliers in the data, we randomly add extremum data to the ore grade data. The following two sets of algorithm validation data are obtained:

- (1) At 500 steps, the ore grade changes slightly and an abnormal value is added in step 200 to form the first set of data.

Figure 3: The values of W_k for outlier and change-point.

Figure 4: Two groups of data for detection. (a) White noise data, (b) white noise data.

Figure 5: Detection results for the first group of data. (a) The chart of the statistics W_k , (b) wavelet decomposition for the statistics W_k , (c) the chart of the statistics E_k .

Figure 6: Detection results for the second group of data. (a) The chart of the statistics W_k , (b) wavelet decomposition for the statistics W_k , (c) the chart of the statistics E_k .

- (2) The data sequence at 500 steps, the ore grade changes slightly, and the abnormal value is increased in 200 steps to form a second set of data.

The above two sets of data are shown in Figure 4, and the detected results are shown in Figures 5 and 6.

In Figures 5(a) and 6(a), the curve for the statistics W_k is marked as the outliers. Since the step of the W_k curve at the outlier is not obvious, we will enlarge the step here and see it in the small window in Figures 5(a) and 6(a). In conjunction with Figures 5(a) and 6(a), we can see that the W_k curve does exhibit a step change at the outliers, and after the change-point at 500 steps, the W_k curve exhibits a slope curve whose slope is proportional to the degree of changing in the variance of the data sequence.

Figures 5(b) and 6(b) show the wavelet decomposition coefficient images of the statistical W_k at two scales, where the dotted line is the wavelet coefficient at $f = 13$ and the solid line is the wavelet coefficient under $f = 15$. From Figures 5(b) and 6(b), it can be seen that the wavelet coefficients exhibit a modulus maxima at both the outliers and the change-points.

In addition, we calculate the wavelet decomposition coefficient difference of the statistical curves of the two scales, and the results are shown in Figures 5(c) and 6(c). It can be seen from the figure that the wavelet coefficients of the two scales where the outliers are located are almost the same, and the difference is close to zero, indicating that the wavelet modulus maxima are independent of the wavelet scale, that is, the Lipschitz exponent of the step function is zero. Near the 500 steps, the difference between the wavelet coefficients of the two-scale curves increases, that is, for the slope function, the wavelet coefficients will increase with the wavelet scale as they increase [24,25]. It can be seen from the above simulation results that the outlier and change point detection algorithms proposed in this paper have certain effectiveness.

4.2 Algorithm comparison

To prove the advantage of the proposed mutation detection algorithm in detecting the delay, we use the ESV

Figure 7: Detection results by the conventional ESV method. (a) The detection result by ESV method for the data in Figure 3a, (b) the detection result by ESV method for the data in Figure 3b.

algorithm described in Section 3 to perform the mutation detection to facilitate comparison. Because the traditional ESV algorithm can only detect the abrupt change point and cannot detect the abnormal value, we adjust the two sets of data to be detected, that is, remove the abnormal value [26]. The results of detecting the mutation points of the above two sets of data based on the conventional ESV algorithm are shown in Figure 7.

Figure 7(a and b) shows the results of the two sets of data. In Figure 7(a) and (b), the slope curve after 500 steps is a statistical w curve; the broken line is a detection threshold curve calculated by equation (10). The other solid line (always zero) is the zero reference line, indicating that the statistical value before the change point is almost zero.

It can be seen from Figure 7 that the traditional ESV-based change point detection algorithm judges the position of the change point by finding the intersection position of the detection threshold curve and the statistical w curve, but

the time for detecting the sudden change point according to this detection mode is far delayed. At the location where the mutation occurred (500 steps). Moreover, this delay is inversely proportional to the magnitude of the mutation, i.e., the smaller the magnitude of the mutation, the later the detected time. From the detection results obtained by the detection method proposed in this paper (see Figures 5 and 6), the detection delay is smaller than the delay of the conventional ESV detection method regardless of the magnitude of the change point. The results show that the proposed detection method has advantages over the traditional ESV algorithm in detecting the change point.

5 Conclusion

In this paper, the grinding grading system is used as the application background. For the characteristics of

process industrial process data similar to the grinding grading system and the real-time testing requirements, a method for detecting and distinguishing outliers and sudden points is proposed. The main ideas of the method are as follows: first, the wavelet value is used to analyze the ESV statistical value, which overcomes the shortcoming of the traditional ESV algorithm based on the large delay in detecting the sudden change point. Second, using the difference between the ESV values of the mutation points and outliers and the relationship between the wavelet modulus maxima and the Lipschitz exponent, a method of wavelet decomposition of the ESV curve is proposed to detect and distinguish outliers and mutations point. This makes it possible to detect and distinguish outliers and abrupt points within the same framework and greatly reduces the detection delay of the point of mutation. Simulation results show that the method has certain effectiveness and practicability.

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