Research Article

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Stable walking of biped robot based on center of mass trajectory control

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Abstract: The effect of center of mass (CoM) trajectory on stable walking of biped robot was studied in this study. The method of predictive control by controlling CoM trajectory to generate stable walking patterns was discussed, which simulated the principle of adjusting center of gravity in advance during human walking to adapt to the change of road conditions. As for uncertainty of robot modeling and the influence of environment, adding variable zero moment point for variable predictive control system can achieve self-adaptive adjustment of CoM trajectory to generate stable walking patterns. The simulation experiment results indicated that walking stability of biped robot was sensitive to the change of CoM trajectory. As long as CoM trajectory was adjusted well, stable walking of biped robot can be controlled even in the presence of disturbances. It is proved that the method of predictive control by controlling CoM motion to generate stable walking pattern is consistent with reality.

Keywords: biped robot, variable zero moment point, center of mass (CoM) trajectory, predictive control, stable walking pattern

1 Introduction

Biped robot walking based on the zero moment point (ZMP) has been widely applied and achieved remarkable achievements [1–5]. Given dynamic degree of biped walking, according to the relation between projection of center of mass (CoM), ZMP, and supporting convex polygons, walking can be divided into static walking, quasi-dynamic walking, and dynamic walking. In static

walking, the projection of CoM of robot on the ground never exceeds the range of supporting polygons [6–9]. In dynamic walking, the projection of CoM can exceed the supporting polygon at some point, which requires online control of CoM to generate stable walking pattern [10–13].

Various methods have been proposed for control of biped robot walking, including the robust self-adaptive control [14] and self-disturbance rejection control [15], and these controls are effective for specific robots. Researches on human control for walking found the existence of predictive thoughts, and human used adjustment of center of gravity (CoG) to achieve stable walking. As human have high autonomy, the behavior of adjusting the CoG is done consciously or unconsciously. When walking on the flat road, the behavior of adjusting the CoG is unconscious. On the contrary, when human turn or encounter obstacles, the behavior may be conscious. Therefore, the control system of biped robot ASIMO presented by Honda in 2005 added control of predictive motion based on biped walking technology, which can predict the change of next motion in real time and move the CoG (CoM) of robot in advance [16,17]. Hence, the control of motion trajectory of CoG (CoM) of robot plays a key role in stable walking of robot [18,19].

Predictive control is mainly to control CoM motion to generate stable walking pattern and achieve tracking of expected ZMP trajectory by practical ZMP trajectory. The principle of controlling CoM motion to generate stable walking pattern is also consistent with the requirement of adjusting CoG during human walking. Predictive control is to use future target ZMP information, which is consistent with the principle that human walking uses future road information to achieve precise control of CoM motion. In ref. [20], predictive control was first applied to biped robot research, and the theory of walking pattern generation based on predictive control was proposed. It was further researched in ref. [13,21,22]. However, these papers did not study the effect of CoM in detail and analyze the influence of CoM trajectory on walking stability. Predictive control based on CoM trajectory adjustment is consistent with the principle of human walking control. Hence, the effect of predictive

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control for CoM trajectory adjustment on stable walking of robot was researched in this study. It is demonstrated that biped robot walking stability was sensitive to the change of CoM trajectory. As long as CoM trajectory was adjusted well, stable walking of biped robot can be controlled even in the presence of disturbances.

2 Predictive controller of com trajectory

As shown in Figure 1, the three-dimensional space robot was approximated as a three-dimensional linear inverted pendulum [13]. It was assumed that the Z-coordinate z_c of robot CoM was constant, which can greatly reduce calculation on the one hand, and on the other hand, the constant height of CoM was undoubtedly beneficial to reduce energy consumption from the perspective of energy saving. According to principles of robot walking stability, the robot was not stable until horizontal component of moment of ground force was zero. The point of action when they were zero was called zero moment point (ZMP) and was always within the supporting polygon.

Hence, following equations are given at the ZMP as shown in Figure 1:

$$\tau_x = mg(x - p_x) - m\ddot{x}z_c = 0$$

$$\tau_y = mg(y - p_y) - m\ddot{y}z_c = 0$$

The dynamic model of biped robot is obtained as the equation (1) by organizing the above equation,

$$\ddot{x} = \frac{g}{z_c}(x - p_x), \quad \ddot{y} = \frac{g}{z_c}(y - p_y) \quad (1)$$

where x and y are displacement of CoM in the horizontal plane, z_c is height of CoM, g is gravity acceleration, and p_x and p_y are the locations of ZMP in the horizontal plane.

Then, we analyzed the motion in the x -axis, especially using the x -axis as the transverse path and the y -axis as the longitudinal path.

Equation (1) was changed into a state-space model, and the differential of ZMP to time was defined as the input variable of the system, which was $v = \dot{p}_x = \dot{p}$, and the output P was the ZMP value in the x -axis. Then, the state-space model is expressed as:

$$\frac{d}{dt} \begin{bmatrix} x \\ \dot{x} \\ p \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{g}{z_c} & 0 & -\frac{g}{z_c} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v \quad (2a)$$

$$p = [0 \ 0 \ 1] \begin{bmatrix} x \\ \dot{x} \\ p \end{bmatrix} \quad (2b)$$

Using the sampling time to discretize the continuous state-space model (2), we obtain the equation (3):

$$x_{k+1} = A_1 x_k + B_1 v_k \quad (3a)$$

$$p_k = C_1 x_k \quad (3b)$$

where $x_k = [x(k\Delta t) \ \dot{x}(k\Delta t) \ p(k\Delta t)]^T$, $p_k = p(k\Delta t)$, $v_k = v(k\Delta t)$.

Predictive controller was designed to control CoM trajectory. To achieve stable walking of biped robot, the output p_k of the dynamic system was required to track the target ZMP p_k^{ref} as accurately as possible. According to the above dynamic model, the CoM trajectory x_k played a key role. As human walking adjusted their CoG in advance, the predictive controller was designed to adjust CoM motion of robots. Predictive control simulated the principle of human walking using the future

(a)

(b)

Figure 1: Biped robot model. (a) Prototype model. (b) Virtual model.

road information and applied information of future target ZMP to design a predictive controller to control CoM trajectory to achieve tracking of practical ZMP for target ZMP. Predictive controller minimized the following tracking performance indicator [23]:

$$J = \sum_{j=1}^{\infty} \{q(p_j^{\text{ref}} - p_j)^2 + rv_j^2\} \quad (4)$$

The meaning of performance indicator was that the tracking error was kept around zero at the expense of less control energy. q and r were positive weighting coefficients. According to the predictive control theory [23], the performance indicator was minimized using the target reference value of future N steps of ZMP. The predictive controller is designed as equation (5):

$$v_k = -Kx_k + [f_1, f_2, \dots, f_N] \begin{bmatrix} p_{k+1}^{\text{ref}} \\ \vdots \\ p_{k+N}^{\text{ref}} \end{bmatrix} \quad (5)$$

According to the predictive control theory in [23], where

$$K = (r + B_1^T P B_1)^{-1} B_1^T P A_1 \quad (6a)$$

$$f_j = (r + B_1^T P B_1)^{-1} B_1^T [(A_1 - B_1 K)^T]^{j-1} C_1^T q \quad (6b)$$

$$(j = 1, 2, \dots, N)$$

K is the state feedback coefficient and f_j is the predictive feedforward gain. The matrix P is the solution of the discrete algebraic Riccati equation (7).

$$P = A_1^T P A_1 + C_1^T q C_1 - A_1^T P B_1 (r + B_1^T P B_1)^{-1} B_1^T P A_1 \quad (7)$$

Using the predictive controller (5), the control law (5) is substituted into the original system (3), and the original kinetic system (3) is transformed as follows:

$$x_{k+1} = (A_1 - B_1 K)x_k + B_1 \sum_{j=1}^N f_j p_{k+j}^{\text{ref}} \quad (8a)$$

$$p_k = C_1 x_k \quad (8b)$$

Assuming $A = A_1 - B_1 K$, $B = B_1$, $C = C_1$, the original system is transformed as equation (9):

$$x_{k+1} = Ax_k + Bu_k \quad (9a)$$

$$p_k = Cx_k \quad (9b)$$

The control input is expressed as equation (10):

$$u_k = \sum_{j=1}^N f_j p_{k+j}^{\text{ref}} \quad (10)$$

The q and r in the performance indicator (4) are determined experimentally, and the authors have discussed them in ref. [24,25]. According to the discussion

of predictive controller parameters in these studies [24,25], the following robot structure parameters and predictive controller parameters are applied to calculate and test.

3 Walking pattern generated by CoM trajectory

The method of CoM motion generation based on predictive control is described by block diagram as shown in Figure 2. Future target ZMP reference value was stored in the buffer of Figure 2, and its output value was used as the current reference value. Predictive controller used ZMP reference value of buffer and state feedback of the robot to calculate the control input and can obtain CoM motion that satisfied tracking of output ZMP for target ZMP. Predictive control is to generate walking pattern by CoM trajectory. The online walking pattern of robot was planned by using the above predictive control system. Figure 3 demonstrates the walking pattern generation strategy based on predictive control. First, the CoM trajectory of foot was planned according to step length, step height, walking cycle, and practical structure parameters of robot. ZMP trajectory of walking can be planned by the foot trajectory. During the 1-ft support period, ZMP fell within the support range of the single foot. During the 2-ft support period, the polynomial fitting method can be used to smoothly move the ZMP from the previous support foot to the later support foot.

It was necessary to ensure that the ZMP had sufficient stability margin. Meanwhile, because the relative location between the center of ankle joint and foot CoM remained unchanged, the trajectory of ankle joint was obtained by CoM trajectory of foot. Then, the motion trajectory of CoM can be solved by predictive control system (see Figure 2) according to planned

Figure 2: CoM trajectory generation based on predictive control.

Figure 3: Walking pattern generation strategy based on predictive control.

trajectory of ZMP that was expected ZMP trajectory. It was assumed that the relative position of CoM at the waist remained unchanged. As a result, the trajectory of waist can be solved by the trajectory of CoM. Once the motion of the waist and ankle joint was determined, each joint angle of the legs can be solved according to inverse kinematics, that is, the walking pattern of biped robot was generated.

Figure 4 illustrates walking pattern generation based on predictive control. First, reference ZMP trajectory shown in Figure 4(a) was planned according to foot trajectory. We set the robot step length, each step time, single-foot support time, and double-feet support time. During the single-foot support period, ZMP fell within the support range of the single foot. During the double-feet support period, the polynomial fitting method can be used to smoothly move the ZMP from the previous support foot to the later support foot. It is necessary to ensure that the ZMP had sufficient stability margin. Then, Control System Toolbox in Matlab was used to solve Riccati equation (7) to obtain numerical solutions of P , K , and f_j . The input u_k can be calculated by convolution equation (10) according to planned reference ZMP and predictive gain $[f_1, f_2, \dots, f_N]$ generated by equation (6b) (see Figure 4(b)). The system input is

shown in Figure 4(c). Finally, the input signal was transmitted to the system (9) to obtain a suitable CoM trajectory X , as shown in Figure 4(d). By using the CoM trajectory to generate time series of joint angles by inverse kinematics, that is, walking mode, the tracking of target ZMP trajectory by practical ZMP trajectory can be achieved.

4 Self-adaptive control system of CoM trajectory

4.1 Design of self-adaptive control system

In Section 2, predictive control was used to control the centroid trajectory to generate the walking mode. The input of predictive controller planned future target ZMP information in advance. The information can also be considered as people walking along a paved road and human walking will use the future road information, predictive control exactly simulated the principle of human walking using future road information. Environmental disturbances such as rugged ground may exist on the roads of human walking. People usually keep stable walking by adjusting the CoG of the body. By studying human walking, it is demonstrated that people adjusted their CoG of body bypass obstacles without hesitation or to recover balance when they encountered obvious obstacles or lost balance during walking. While walking was in the normal range, people were under the unconscious or conscious experience control so that the body fluctuations were as small as possible. Of course, this kind of experience control also included the principle of using future road information and the predictive thoughts existed. The result of this kind of conscious treatment was that the physical activity ran smoothly in the normal range, and the CoG was adjusted in time when sudden events happened in time to make sure the balance of the body while walking and stable walking with disturbances of environment. This kind of CoG adjusting strategy is significant for the stable walking control of robots.

As can be seen from the previous section, predictive control designed predictive controller by the feedback of servo system and future target ZMP information to control CoM motion of robots to achieve tracking of practical ZMP for target ZMP. Nevertheless, target ZMP of predictive controller was planned offline in advance without considering model error and environment

Figure 4: The walking pattern generation process. (a) Reference ZMP trajectory. (b) Predictive gain. (c) System input. (d) CoM trajectory and output ZMP.

disturbances when designing the controller to adjust CoM trajectory, resulting in the decrease of walking stability and even failure of walking. Therefore, to make walking pattern have self-adaptivity like human, it is necessary to consider the real-time change of ZMP, and expected ZMP was defined as the sum of the planned reference ZMP and the real-time variable ZMP. Variable ZMP had the function of eliminating external interference, adapting to uneven ground and attitude control [22]. The effect can be achieved by the inverse system of the predictive control system. They worked together to adjust the CoM trajectory to make walking pattern has self-adaptivity and expected ZMP is given the self-adaptive expression as equation (11):

$$p^{exp} = p^{ref} + p^{var} + \varepsilon \quad (11)$$

where the left side of equation (11) indicates the expected ZMP. The first item on the right is reference ZMP that is offline planned based on inert parameters

such as stride, walking speed, and walking cycle, representing the global stability of the robot and remaining unchanged during walking of robot. The second item is real-time ZMP, that is, variable ZMP. The third term is random disturbances. According to the research [24,25], it can be eliminated by selecting the parameters of the predictive controller.

Figure 5 shows the established predictive control system with variable ZMP. Variable ZMP was added by the inverse system. In this figure, future target ZMP reference value is restored in the buffer, and its output is used as a current reference value. The whole system used ZMP reference value in the buffer and variable ZMP as control input. Variable ZMP system was achieved by inverse system of predictive control system represented by state-space model (9). The design of the inverse system is as follows.

Using robot parameters and predictive control parameters shown in Table 1, the pulse transfer function

Figure 5: Predictive control system with variable ZMP.

Table 1: Structure parameters and predictive controller parameters

$g/m\ s^{-2}$	z_c/m	$\Delta t/s$	N	$N\Delta t/s$	r	q
9.8	0.85	0.005	360	1.8	1×10^{-8}	1

of predictive control system represented by state-space model (9) is defined as equation (12):

$$G(z) = C(zI - A)^{-1}B = \frac{0.005(z - 1.017)(z - 0.9832)}{(z - 0.9832)^2(z - 0.0003997)} \quad (12)$$

To achieve variable ZMP subsystem shown in Figure 5, we define equation (13):

$$G(z) \cdot G^{\text{inv}}(z) = z^{-1} \quad (13)$$

Variable subsystem is considered an inverse system of the original predictive control system. The pulse transfer function of the inverse system is given by equation (12):

$$G^{\text{inv}}(z) = \frac{(z - 0.9832)^2(z - 0.0003997)}{0.005z(z - 1.017)(z - 0.9832)} \quad (14)$$

Hence, in the discrete time domain, the transfer function of the inverse system can be obtained. The state-space model of the variable inverse system is given below.

Equation (9b) is moved forward one sampling moment and we can get equation (15):

$$p_{k+1} = Cx_{k+1} = CAx_k + CBu_k \quad (15)$$

Because $CB \neq 0$, we obtain equation (16)

$$u_k = -(CB)^{-1}CAx_k + (CB)^{-1}p_{k+1} = C_2x_k + D_2p_{k+1} \quad (16)$$

where $C_2 = -(CB)^{-1}CA$ and $D_2 = (CB)^{-1}$.

The state equation can be obtained from equations (9a) and (16):

$$x_{k+1} = (A + BC_2)x_k + BD_2p_{k+1} \quad (17)$$

Assuming $A_2 = A + BC_2$, $B_2 = BD_2$, the state-space model of variable ZMP subsystem is obtained from equations (16) and (17):

$$x_{k+1}^{\text{inv}} = A_2x_k^{\text{inv}} + B_2p_k^{\text{var}} \quad (18a)$$

$$u_k^{\text{var}} = C_2x_k^{\text{inv}} + D_2p_k^{\text{var}} \quad (18b)$$

where x_k^{inv} is the state variable of the variable inverse system. The pulse transfer function of the inverse system (18) can be calculated as:

$$G^{\text{inv}}(z) = C_2(zI - A_2)^{-1}B_2 + D_2 = \frac{(z - 0.9832)^2(z - 0.0003997)}{0.005z(z - 1.017)(z - 0.9832)} \quad (19)$$

By comparing equations (14) and (19), it is indicated that the inverse system was established by the predictive control system (9) and the structure of the inverse system principle is shown in Figure 5.

4.2 Stability improvement of control system

As can be seen from transfer function (19) of inverse system, not all the poles fell within the unit circle, resulting in unstable inverse system. Hence, pole configuration required to be performed on inverse system. As observed in the experiment in the next section, if stability configuration is not performed on poles, the robot will adjust the CoM trajectory without limit to ensure stable walking when it encounters disturbances. However, it is impossible. Just like, it is difficult for people with faults in the cerebellum to maintain their body balance by adjusting their CoG. As a result, stability improvement of the predictive control system with the inverse system is necessary. According to the controllability theory of the linear system, the inverse system established by the state-space equation (18) was judged to be fully controllable. The poles can be arbitrarily configured by state feedback and the new inverse system is shown in Figure 6.

Figure 6: Predictive control system with stable inverse system.

Because inverse system (18) was fully controllable, according to pole configuration theory, the poles can be arbitrarily configured as long as the configured poles fell in the unit circle and ensure the stability of inverse system. Due to the transfer matrix of system (18), $D_2 \neq 0$, this system is a non-strict real system, the pole configuration will affect the zero point of the inverse system. If the pole is configured to 0.3, 0.5, and 0.7, then K^{inv} is expressed as:

$$K^{\text{inv}} = [-365.4571 \quad -10.4996 \quad 0.1801]$$

The pulse transfer function of the inverse system is transformed into a stable equation:

$$\begin{aligned} G^{\text{inv}}(z) &= C_2(zI - A_2 + B_2K^{\text{inv}})^{-1}B_2 + D_2 \\ &= \frac{(z - 0.752)(z^2 - 0.7144z + 0.1401)}{0.005(z - 0.3)(z - 0.5)(z - 0.7)} \quad (20) \end{aligned}$$

Although the zero point changed, it was still in the unit circle. The whole predictive control system with stable inverse system was established. As can be seen from the above argument, the system simulated the

self-adaptive adjustment strategy for CoG during human walking. Therefore, using the system to control the CoM trajectory can generate a stable walking pattern to make the biped robot achieve stable walking.

5 Simulation experiment and analysis

We demonstrated the method of stable walking based on predictive control of CoM trajectory, and walking pattern generation system can simulate self-adaptive adjusting strategy for CoG during human walking. Then, we validated that the CoM adjustment of robot walking was consistent with practical adjustment principle during human walking. From the above demonstration, variable ZMP had the effect of adjusting stability during stable walking of robot. The global stability of robot was controlled by the reference ZMP planned according to the walking parameters such as the step length, the step

Figure 7: CoM trajectory of predictive control system tracking expected ZMP. (a) Predictive system CoM trajectory. (b) Unstable variable predictive system CoM trajectory. (c) Stable variable predictive system CoM trajectory.

Figure 8: The adjustment condition of unstable variable predictive system CoM trajectory. (a) The adjustment range of the CoM trajectory in unstable system is very large. (b) The adjustment amplitude of the CoM trajectory of the unstable system will cause the robot to lose control or fall.

speed, and the walking cycle, and was not easily changed throughout the walking control. When the robot encountered disturbances such as external forces, uneven ground, and posture imbalance, variable ZMP was added for CoM adjustment control. The variable ZMP has many different forms. Here, the commonly used waveform variable ZMP was selected to validate the predictive control system with stable inverse system shown in Figure 6, $p^{\text{var}} = 0.03 \sin(50t)$ and $p^{\text{var}} = 0.05 \sin(50t)$ were added to validate the system in the experiment.

The experiment result is shown in Figure 7. Figure 7(a) indicates that the original predictive control system can track expected ZMP well, and CoM trajectory adjustment is consistent with reality. Figure 7(b) demonstrates the unstable predictive control system with variable ZMP. To achieve tracking of practical ZMP for expected ZMP, the change of CoM trajectory adjustment was large, resulting in the fall of the robot and the failure of walking. It is difficult for people with faults in cerebellum to remain stable walking by adjusting their CoG. Figure 7(c) demonstrates that for variable ZMP with high-frequency

Figure 9: The adjustment condition of stable variable predictive system CoM trajectory. (a) The stability system's CoM trajectory adjustment is very small. (b) The microadjustment of the trajectory of the CoM of the stability system realizes the stable walking.

variation, the predictive control system with improved stability can achieve the tracking of practical ZMP for expected ZMP by CoM adjustment.

The 3D simulation experiments are carried out through the 3D simulation platform and algorithm developed by the authors of references [26–28]. Figure 8 shows the adjustment condition of CoM of Figure 7(b) relative to Figure 7(a). The adjusted error margin further indicated that CoM adjustment of an unstable system is difficult. Figure 8(b) 3D simulation shows that the adjustment amplitude of the CoM trajectory of the unstable system will cause the robot to lose control or fall. Figure 9 shows the adjustment condition of CoM of Figure 7(c) relative to Figure 7(a). The corresponding CoM trajectory of two figures did not seem to change. However, their calculation values changed slightly from Figure 9. The walking pattern generated by the small change of the CoM trajectory obtained different ZMP tracking effects. One was the conventional ZMP trajectory and the other was the variable ZMP trajectory considering the environmental change, but all achieved stable walking. Figure 9(b), 3D simulation, shows that the CoM adjustment of the stabilization system achieves stable walking, which indicated that the walking pattern of biped robot was sensitive to the change of CoM trajectory [29–32]. Hence, predictive control can generate stable walking pattern by controlling the CoM motion, which is consistent with the real-time slight adjustment of CoG trajectory during human walking to adapt the change of road condition to achieve stable walking.

6 Conclusion

The method of predictive control by controlling CoM trajectory to generate stable walking pattern was discussed. Walking pattern generation system based on predictive control simulated the principle of adjusting CoG in advance to adapt to the change of road condition during human walking. For designed variable predictive control system, adding variable ZMP can achieve self-adaptive adjustment of CoM trajectory to generate stable walking pattern. It is demonstrated that as long as CoM trajectory was adjusted well, stable walking of biped robot can be controlled even in the presence of disturbance. The experimental analysis indicated that biped robot walking stability was sensitive to the change of CoM trajectory. It is proved that predictive control can generate stable walking pattern by controlling the CoM motion, which is consistent with the reality, and the

real-time slight adjustment of CoG trajectory during human walking to adapt to the change of road condition to achieve stable walking.

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