Research Article

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Development of a generic framework for lumped parameter modeling

<https://doi.org/10.1515/phys-2020-0168>

received January 30, 2020; accepted June 3, 2020

Abstract: The purpose of this article is to present a generic reluctance network modeling tool dedicated to the modeling of electrical machines. This tool is used for the study of permanent magnet machines. The focus will be on the modeling methodology and software implementation. More precisely, the aspects related to genericity will be discussed. In order to validate the developed tool, the simulation of a 12 slot/10 pole flux-switching permanent magnet machine is conducted, and the results obtained from this generic framework are compared to the corresponding finite element analysis.

Keywords: equivalent circuit modeling, reluctance network, generic framework, permanent magnet machines, flux-switching

1 Introduction

The lumped parameter method (LPM) presents a good compromise between analytical methods, which are known for their reduced computation burden, and numerical methods, mainly the finite element method (FEM), which are known for their relatively good agreement with experimental measurements, when done with the minimum simplifying assumptions. The LPM helps reduce the computation time, while not increasing the number of simplifying assumptions. When used for the modeling of electromagnetic devices, the LPM is known as the magnetic equivalent circuit or

the reluctance network (RN) [1,2]. In this contribution, efforts toward the development of a generic framework for the RN approach are presented. As finite element analysis (FEA), the first step in this framework is to mesh the studied domain using reluctance elements and then generate the corresponding algebraic equation system. The following steps are the solving of the generated equation system and exploitation of the obtained results for the analysis of the studied device.

Flux-switching permanent magnet (FSPM) machines are gaining in popularity due to their robustness, wide speed range, high torque, and high power density [3–5]. It is shown that FSPM machines have a higher air gap flux density due to the flux focusing effect and fewer mechanical issues in the rotor compared to surface permanent magnet (PM) machines [6]. However, the dual-salient structure results in higher harmonic orders of flux density in the air gap, which require a more refined mesh in FEA. Several studies [7–9] have presented a good agreement and a better performance of the mesh-based RN method used in FSPM machine computation, but they only focused on a limited aspect of global quantities, and the saturation is frequently ignored, which is a dominating phenomenon in FSPM machines. In this work, a generic framework for a mesh-based and interpolation coupled RN method is proposed, including a more programming oriented mathematical model and an extensible tool. Then, the global quantities of a 12 slot/10 pole FSPM machine are calculated using the tool. Compared to the method used in ref. [10], a mesh-based pre-processing and the application of the interpolation method on the motion interface avoid the specific knowledge of the investigated machines and increase the genericity of the RN method. Moreover, by using the connectivity matrix, a more programming-suitable formulation is obtained.

In the next sections, the framework will be more thoroughly described. The focus will be on the modeling methodology and software implementation. More precisely, the aspects related to genericity will be discussed. In order to validate the developed tool, the results obtained from this framework are compared to the corresponding FEA.

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2 Framework presentation

Aspects related to the mathematical modeling and software implementation of the developed tool are discussed in Sections 2.1 and 2.2 in detail. Compared to previous studies on the subject, where the focus was on mathematical aspects and validation, efforts toward the building of a generic computational model are highlighted.

2.1 Mathematical model

The geometry is discretized in a number of bidirectional elementary reluctance blocks as shown in Figure 1. R_{x1} , R_{x2} , and R_{y1} , R_{y2} are the reluctances on the x - and y -axis branches of the block element, respectively. Compared to previous studies, elements are allowed to be attached with two kinds of materials, which enables establishing a more precise model for sloping sides in machines.

$$\begin{cases} R_{x1} = \frac{1}{\mu_i} T_x & R_{x2} = \frac{1}{\mu_j} T_x \\ R_{y1} = \frac{1}{\mu_i} T_y & R_{y2} = \frac{1}{\mu_j} T_y, \end{cases} \quad (1)$$

where T_x and T_y are shape functions in the x - and y -direction, determined by the shape and size of elements. μ_i and μ_j are different permeabilities assigned to the element.

The circuit solver is developed based on Kirchhoff's laws [11]. It can be established for a network containing N_n nodes and N_b branches as follows:

$$C\Phi_B = -C\Phi_B^s, \quad (2)$$

where Φ_B is the vector of the fluxes on branches, Φ_B^s is the vector of the fluxes generated by current sources, such as a PM and current. C is a connectivity matrix of $N_n \times N_b$ size, and the item C_{ij} is defined as follows: (3)

$$C_{ij} = \begin{cases} 0 & \text{if node } i \text{ and branch } j \text{ are not connected} \\ 1 & \text{if branch } j \text{ flows out of node } i \\ -1 & \text{if branch } j \text{ flows into node } i. \end{cases} \quad (3)$$

According to Ohm's law, Φ_B and Φ_B^s can be written as:

$$\begin{cases} \Phi_B = PU \\ \Phi_B^s = PF, \end{cases} \quad (4)$$

where U , P , and F are the voltage vector, the diagonal matrix of permeance, and the magneto-motive force (MMF)

Figure 1: Reluctance block elements.

source vector, respectively. And the conversion between the voltage U and the node potential V can be expressed as:

$$U = C^T V. \quad (5)$$

Consequently, equation (2) is transformed into

$$CPC^T V = -CPF. \quad (6)$$

MMF sources due to the PM need to be placed on the branches of the magnetization direction in the elements through all layers of PM areas, where each source is calculated as shown in equation (7) [12]. H_c is the PM magnetic coercivity, and l_{mag} is the length along the magnetization direction for each element (Figure 2).

$$F_{pm,e} = H_c l_{mag,e}. \quad (7)$$

To obey Ampere's circuital law, MMF sources due to current need to model the distribution of MMF in the air gap first, which is a function of spatial position according to the winding distribution, denoted as $F_g(x, t)$. $\varphi_A(x)$, $\varphi_B(x)$, and $\varphi_C(x)$ are defined as the functions of MMF distribution in the air gap, when the unit current is provided for each phase. x is the x -axis coordinate of each element. Combining the spatial position function with the three-phase input current, we have

Figure 2: Modeling of the PM unit.

Figure 3: MMF distribution in the air gap.

$$\begin{cases} F_A(x, t) = I_m \cos(\omega t + \theta) \varphi_A(x) \\ F_B(x, t) = I_m \cos(\omega t + \theta - 120) \varphi_B(x) \\ F_C(x, t) = I_m \cos(\omega t + \theta + 120) \varphi_C(x), \end{cases} \quad (8)$$

where F_A , F_B , and F_C are the functions of MMF distribution changing with time. By summing them up, the expression of $F_g(x, t)$ is

$$F_g(x, t) = F_A(x, t) + F_B(x, t) + F_C(x, t). \quad (9)$$

Figure 3 shows the MMF distribution in the air gap, when setting $t = 0$ s, $\theta = 0^\circ$. Then, the MMF value contributed by each element should be calculated using the proportion to the whole current area at the corresponding point.

$$F_{l,e} = \frac{l_{l,e}}{h_w} F_g(x_{c,e}, t), \quad (10)$$

where h_w is the height of the winding area and $l_{l,e}$ is the length along the magnetization direction for each element.

The main principle of motion handling is to avoid regenerating the mesh in the studied domain with no or little compromise in precision. An interpolation method is employed here, based on the continuity of the scalar magnetic potential and the continuity of the flux density normal component at the sliding interface (Figure 4).

Figure 4: Interface between the rotor and the stator.

$$\begin{cases} V_{si} = V'_{ri} = \frac{x_{si} - x_{ri}}{x_{r(i+1)} - x_{ri}} V_{ri} + \frac{x_{si} - x_{r(i+1)}}{x_{ri} - x_{r(i+1)}} V_{r(i+1)} \\ B_{ri} = B'_{si} = \frac{x_{ri} - x_{si}}{x_{s(i-1)} - x_{si}} B_{si} + \frac{x_{ri} - x_{s(i-1)}}{x_{si} - x_{s(i-1)}} B_{s(i-1)}. \end{cases} \quad (11)$$

The subscripts s and r are used to denote the values from the stator side and the rotor side. Equation (11) shows a linear interpolation on the interface, and for the interpolation of k orders, the formulation is as follows:

$$\begin{cases} V_{si} = V'_{ri} = \sum_{j=0}^k \left(\prod_{t=0, t \neq j}^k \frac{x_{si} - x_{rt}}{x_{rj} - x_{rt}} \right) V_{rj} \\ B_{ri} = B'_{si} = \sum_{j=0}^k \left(\prod_{t=0, t \neq j}^k \frac{x_{ri} - x_{st}}{x_{sj} - x_{st}} \right) B_{sj}. \end{cases} \quad (12)$$

By using the projection against a specific direction, we can obtain the normal component of the flux density of interface nodes from the connecting branch components.

Apart from the sliding interface, the RN is also defined by the Dirichlet boundary condition and the periodic or anti-periodic condition (Figure 5). For node n_{Dk} , we have

$$V_{n_{Dk}} = 0. \quad (13)$$

For node i and node j , no matter what kind of boundary condition is assigned, they are viewed as adjacent elements, so they are connected by a branch, denoted as b . When using the periodic condition, we create the connectivity matrix as per the definition in equation (3). When using the anti-periodic condition, element C_{jb} in the connectivity matrix is set to 1 rather than -1 .

The Newton-Raphson method is a technique used for solving nonlinear equations numerically [13].

Figure 5: Boundary condition.

		0.043633	0.043633	0.043633	0.043633	0.043633	0.043633	0.043633	0.043633
0	6.05E-02	2	2	2	2	2	2	2	2
1.00E-02	4	4	4	4	4	4	4	4	4
1.00E-02	4	4	4	1	1	1	1	4	4
1.00E-02	4	4	4	1	1	1	1	4	4
5.00E-04	1	1	1	1	1	1	1	1	1
Conductor				-1	-1	-1	-1		
High	1	2							
WindingA	1								
WindingB	2								
WindingC	3								

Figure 6: Format of describing the structure of the machine in EXCEL.

This method uses an iterative process to approach one knowing that root of a function, and this root depends on the initial value. For equation (2), we can write

$$f(V_k) = CP(V_k)C^TV_k + CP(V_k)F \quad (15)$$

$$V_{k+1} = V_k - J^{-1}f(V_k) \quad (14) \quad J = \frac{\partial f}{\partial V} = C \left(C^T \circ \frac{\partial P}{\partial V} \right) V + C(C^T \circ P) + C \left(F \circ \frac{\partial P}{\partial V} \right), \quad (16)$$

Figure 7: Implementation of the tool.

where operator $\circ$ is the element-wise product.

$$\frac{\partial P}{\partial V} = \begin{bmatrix} \frac{\partial p_1}{\partial v_1} & \dots & \frac{\partial p_1}{\partial v_{N_n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial p_{N_b}}{\partial v_1} & \dots & \frac{\partial p_{N_b}}{\partial v_{N_n}} \end{bmatrix}. \quad (17)$$

Branch b connects node i and node j , and by considering each part in two adjacent elements of a unique branch separately, the derivative of permeance with respect to node potentials is obtained from the corresponding components based on the expression of permeance calculation.

Figure 9: Studied structure.

$$\frac{\partial p_b}{\partial v_n} = \begin{cases} \frac{1}{l_b R_b^2} \left(\frac{T_i}{\mu_i^2} \dot{\mu}_i + \frac{T_j}{\mu_j^2} \dot{\mu}_j \right) \frac{\partial |V_i - V_j|}{\partial v_n} & n = i, j \\ 0 & n \neq i, j, \end{cases} \quad (18)$$

where l_b is the length of branch b . Note that the Dirichlet boundary condition and interpolation formulation at the sliding interface modify some values in the coefficient matrix of linear equations, but they have no effect on the calculation of the Jacobian matrix. Accordingly, these items are set to zero after getting the Jacobian matrix using equation (13).

Figure 8: Process of the circuit solver layer.

2.2 Tool structure

The framework is developed in four layers to provide genericity, flexibility, and extensibility (Figure 7). The frontal layer is a designing part serviced by widely used mesh tools. Apart from commercial software like ANSYS® and FLUXTM, GMSH is an open-source toolkit with a powerful and fast mesh function. In this study, an Excel worksheet constructed in a specific format that describes the structure of the machine is used as an input file (Figure 6).

Table 1: Machine parameters

Mechanical air-gap (mm)	0.35
r_{so} , r_{si} , r_{ri} (mm)	128, 70.4, 22
θ_1 , θ_2 , θ_3 , θ_4 (°)	7.5, 15, 7.5, 7.5
l_1 (mm)	4.6
Magnetization type	Circumferential magnetization
PM magnetic remanence (T)	1.12
Axial length (mm)	75
Speed (rpm)	480
Peak current (A)	10
Coil turns	70

Figure 10: Air-gap magnetic flux density component comparison, $I_m = 0$ A. (a) Normal component and (b) tangential component.

Then, the framework reads the generated file into the circuit generator layer using an adaptor, which decodes the mesh information according to the pre-setting rules. The Excel worksheet in Figure 6 contains dimensions and confined mesh instructs for ExcelAdaptor to decide the concrete mesh scheme. DomainManager helps to manage the geometry properties of the studied domain and generate the size data for each element. Coupled with object Winding and object MaterialManager, DomainManager is processed by object CircuitGenerator, which provides the circuit descriptor encapsulating detailed information of branches that are required in object Assembler during iterations. MaterialManager offers the function of maintaining properties defined by users and helps to manage different types of materials especially nonlinear materials. Also, an interface is opened to accept the customer defined B–H curve fitting function. By default, the program uses an analytic saturation curve with knee adjustment.

Figure 11: Flux linkage and EMF comparison, $I_m = 0$ A. (a) Flux linkage and (b) induced voltage.

$$B(H) = \mu_0 H + \frac{J_s}{2(1 - a)[H_a + 1 - \sqrt{(H_a + 1)^2 - 4H_a(1 - a)}]}, \quad (19)$$

where

$$H_a = \frac{\mu_0(\mu_r - 1)H}{J_s}. \quad (20)$$

The implementation of object Winding depends on different configurations of winding, which means that distributed windings and concentrated windings ought to be modeled in two ways. CircuitGenerator calls the unified interface to get the function $F_g(x, t)$ in the air gap when producing an MMF distribution model in the initial state to accelerate the solving process.

Figure 8 shows the main process of simulation at each time step in the circuit solver layer, when users launch a new analysis. Due to the nonlinear properties of

(a)

(b)

Figure 12: Air-gap magnetic flux density component comparison, $I_m = 10$ A. (a) Normal component and (b) tangential component.

(a)

(b)

Figure 13: Flux linkage and EMF comparison, $I_m = 10$ A. (a) Flux linkage and (b) Induced voltage.

materials, the circuit solver layer is developed in two types of iterative methods: fixed-point method and Newton–Raphson method. The Newton–Raphson method approaches a sufficiently precise value much faster than the fixed-point method because of using the tangent of the graph, but sometimes it is unable to reach a convergence as stable as the others. So in the program, both methods are provided. The construction of the Jacobian matrix has been described in the last section. The core solvers of fields are divided in single modules so that users are capable of building their own solver on these stable functions.

The circuit solver layer inquires the upper layer to obtain the updated magnetic state through the specified interface at each step, which ensures the decoupling between two layers so that users are capable of designing their own modules by utilizing the well-encapsulated

modules. Overloading of the default motion and boundary processing function is supported in the program, so a more effective motion handling method like the hybrid analytical method, which performs better in calculating the magnetic field in the air gap, is obtained.

All fields output by the circuit solver layer are registered in the DomainManager and exported to the CSV format, a type of file supported by many programs for post-processing.

3 Validation of the developed tool

In this section, a concrete validation study is described to highlight the good operation and correctness of the developed tool.

Figure 14: Torque and iron loss versus input current. (a) Torque versus current amplitude. (b) Iron loss versus current amplitude.

3.1 Structure presentation

The structure in Figure 9 is a classical 12/10 FSPM machine with concentrated windings, which has been analyzed in many previous cases. Although the dimensions of the machine were chosen to simplify the modeling study, the values can be set as any number in actual application. Table 1 gives the main machine characteristics.

3.2 Validation study

The comparisons shown in Figures 10–14 are between the FEM model meshed with 18,756 nodes and the RN model meshed with 4,800 nodes. Local quantities, such as the tangential and normal component of air gap flux density (Figures 10 and 12); global quantities, such as

Table 2: Errors of the proposed method compared to FEA

I_m (A)	Flux (error %)	Voltage (error %)	Torque (error %)	Iron loss (error %)
0	1.22	1.25	—	1.03
10	1.56	1.78	2.48	2.25

flux linkage and induced voltage versus time (Figures 11 and 13); torque; and iron loss (Figure 14) are evaluated by the classical method used in FEM. Flux linkage is the sum of fluxes (in the r -axis) passing through all block elements located at slot mid-height spanning one coil pitch. The electromotive force is obtained by the differentiation of the phase flux. The Maxwell stress tensor method is used for the computation of torque. For iron loss estimation, the experimentally verified iron loss model is used here [14]:

$$P_{\text{core}} = P_h + P_c + P_a, \quad (21)$$

$$\begin{cases} P_h = k_h f B_m^\beta \\ P_c = k_c f^2 B_m^2 \\ P_a = k_a f^{1.5} B_m^{1.5}, \end{cases} \quad (22)$$

where P_h , P_c , and P_e are, respectively, static hysteresis loss, classical eddy current loss, and excess loss; k_h , k_c , and k_e are the coefficients of the corresponding loss component; f and B_m are the frequency and amplitude of the fundamental flux density; and β is an empirical parameter obtained from experimental measurement.

As the FEA model is used as a reference, it is indicated that the RN model achieves a very good agreement with fewer nodes and faster computation speed. The errors of the flux linkages and the voltage waveforms could be obtained by comparing their RMS values obtained from both approaches, and the maximum error is lower than a few percent. For the torque and the core loss, the error is obtained by directly comparing the values of these quantities, and again the error is lower than a few percent (Table 2).

4 Conclusion

In this article, a generic calculating framework for lumped parameter modeling has been presented. The universal mathematical model has been put forward in the matrix format, based on which the structure of a flexible and extensible solver was discussed in detail. The adaptive mesh-based discretization and MMF source

distribution techniques allow easy handling of different electromagnetic structures with different properties.

The interpolation method employed in modeling motion components avoids re-meshing at each time-step. Nonlinearities are considered in the solver, by combining the fixed-point method and the Newton–Raphson method. A fairly good agreement between the proposed tool and the commercial FEM software, in terms of flux density, flux linkage, induced voltage, torque, and loss, is obtained. It is a good step toward a powerful and easy-to-use RN tool in the pre-design stage.

Acknowledgments: The authors would like to thank the CNRS (GdR SEEDS 2994 du CNRS), National Natural Science Foundation of China under Grant no. 51825701, Key R&D Program of Jiangsu Province under Grant no. BE2019073, and Université Le Havre Normandie (GREAH) for the funding of this work.

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