Research Article

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Edge effect of multi-degree-of-freedom oscillatory actuator driven by vector control

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Abstract: This article proposes a thrust equation model of two-degree-of-freedom oscillatory actuator considering its edge effect. The proposed thrust equation model clarifies that asymmetric permanent magnet flux linkage and inductance characteristics cause undesirable magnetic thrust and reluctance thrust, respectively.

Keywords: linear oscillatory actuator, vector control, edge effect, multi-degree-of-freedom, electromagnetic actuators

1 Introduction

Linear oscillatory actuators (LOAs) have been used in a wide range of applications such as an electric shaver and an air compressor [1,2]. Various kinds of multi-degree-of-freedom (DOF) resonant actuators have been developed for further improvement of LOAs [3]. The authors have developed a two-DOF oscillatory actuator [4] driven by three-phase current and verified its effectiveness through 3D finite element analysis (FEA) and measurement.

This actuator has linearly expanded structure of four-pole three-slot surface permanent magnet synchronous motors (SPMSMs). Therefore, this actuator exhibits a specific property called the edge effect because its stator and mover have finite lengths. It is well-known that linear motors also show the edge effect [5]. For example, in case of linear synchronous motors,

the air-gap flux distribution becomes unbalanced due to the edge effect [6]. To solve this problem, shape optimization or using auxiliary teeth has already been proposed [7,8]. On the other hand, we have not investigated how the edge effect influences thrust characteristics of the two-DOF actuator. Moreover, it has only three slots, which is much fewer than that of conventional linear motors. Therefore, the edge effect distorts sinusoidal distribution of the air-gap flux density from the four-pole permanent magnet. This fact indicates that the linearity of the thrust and the controllability of the two-DOF actuator decrease due to its edge effect.

In this article, we propose a thrust equation model of the actuator considering its edge effect. We clarify the thrust characteristics of the actuator under various conditions by using the proposed thrust equation model.

2 Two-DOF oscillatory actuator

2.1 Basic structure and operating principle

The basic structure of the two-DOF oscillatory actuator is shown in Figure 1. This actuator consists of a mover, a stator, and springs that support the mover. As mentioned previously, this actuator is similar to four-pole three-slot SPMSMs. Therefore, conventional vector control can be applied to operate this actuator. q -Axis current i_q and d -axis currents i_d correspond to the thrust in the x - and z -directions, respectively, as follows:

$$\begin{bmatrix} F_z \\ F_x \end{bmatrix} = \begin{bmatrix} F_d \\ F_q \end{bmatrix} = \phi \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (1)$$

Three-phase current is determined by inverse d - q transformation as follows:

$$\begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos \theta & -\sin \theta \\ \cos\left(\theta - \frac{2}{3}\pi\right) & -\sin\left(\theta - \frac{2}{3}\pi\right) \\ \cos\left(\theta + \frac{2}{3}\pi\right) & -\sin\left(\theta + \frac{2}{3}\pi\right) \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} \quad (2)$$

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Figure 1: Basic structure of two-DOF resonant actuator.

$$\phi = \frac{s_x}{l} \pi \quad (3)$$

Here s_x is the displacement in the x -direction and l is the pole pitch of the magnet mounted on the mover. This actuator is able to oscillate independently in x - and z -directions by AC inputs of i_d and i_q .

2.2 Influence of edge effect

The vector control described in the previous section assumes that flux density distribution in air gap is sinusoidal. Moreover, it assumes that three phase coils (U, V, and W) have the same magnetic flux properties. The two-DOF resonant actuator has linearly expanded structure of SPMSMs. Therefore, this actuator exhibits the edge effect because its stator and mover have finite lengths. In this section, we investigate the influence of the edge effect on the PM flux linkage and phase inductance characteristics. These characteristics are

Figure 2: 3D finite element mesh without air region.

Figure 3: PM flux linkage characteristics.

computed through 3D FEA. Figure 2 shows the 3D finite element mesh of our actuator, excluding the air region.

Figure 3 shows the analyzed PM flux linkage of U, V, and W phase coils. Solid lines in Figure 3 are approximated to sine waves with the same amplitude. The results show that the edge effect makes a difference between ideal PM flux linkage (solid line) and the 3D FEA results. Figures 4 and 5 show the analyzed self-inductance and mutual inductance of U, V, and W phase coils. Because of this actuator's structural symmetry, inductance characteristics of V and W phase coil are identical. Figure 4 reveals that U phase self-inductance L_u is slightly higher than V (W) phase self-inductance L_v (L_w). Additionally, mutual inductance between U and V phase M_{uv} is slightly higher than that between V and W phase M_{vw} . In order to discuss the difference, we compare the following two models. Figure 6 shows the analyzed flux density distribution of an enhanced model. The flux passing through V phase teeth is equally

Figure 4: U-phase inductance characteristics.

Figure 5: V(W)-phase inductance characteristics.

divided into the two adjacent teeth (U and W). Figure 7 shows the analyzed flux density distribution of the normal model. The flux passing through V phase teeth is not equally divided into the two teeth (U and W). This is because W phase teeth is far from V phase teeth.

3 Voltage equation model considering edge effect

3.1 Modeling of flux linkage characteristic

In this section, we derive a flux linkage model considering the edge effect mentioned in the previous section. Two parameters i and k are used to compensate the difference between the ideal PM flux and 3D FEA results in Figure 3. The three-phase PM flux linkages Φ_u , Φ_v , and Φ_w are divided into a symmetric flux vector Φ_{sym} and an asymmetric flux vector Φ_{asym} as the following equation:

Figure 6: Flux density distribution of enhance d model.

Figure 7: Flux density distribution of normal model.

$$\begin{bmatrix} \Phi_u \\ \Phi_v \\ \Phi_w \end{bmatrix} = \Phi_{\text{sym}} + \Phi_{\text{asym}}$$

$$= \Phi_f \begin{bmatrix} \cos \theta \\ \cos \left(\theta - \frac{2}{3}\pi \right) \\ \cos \left(\theta + \frac{2}{3}\pi \right) \end{bmatrix} + \Phi_f \begin{bmatrix} 0 \\ k \cos \left(\theta - \frac{2}{3}\pi \right) - j\theta \\ k \cos \left(\theta + \frac{2}{3}\pi \right) + j\theta \end{bmatrix} \quad (4)$$

Here, Φ_f is amplitude of PM flux linkage. Two parameters L_c and M_c are used to compensate the bias shown in Figures 4 and 5. The inductance models are obtained as follows:

$$L_u = l_a + L_a + L_c \quad (5)$$

$$L_v = L_w = l_a + L_a \quad (6)$$

Figure 8: Magnetic flux vector diagram of two-DOF resonant actuator.

Figure 9: Voltage vector diagram of two-DOF resonant actuator.

$$M_{uv} = M_{vu} = M_{wu} = M_{uw} = -\frac{1}{2}L_a \quad (7)$$

$$M_{vw} = M_{wv} = -\frac{1}{2}L_a + M_c \quad (8)$$

Here, l_a and L_a are the leakage and effective inductance, respectively.

3.2 Voltage equation on dq reference frame

The voltage equation on UVW reference frame is expressed as follows:(9)

$$\begin{bmatrix} v_u \\ v_v \\ v_w \end{bmatrix} = \begin{bmatrix} R_a + pL_u & pM_{uv} & pM_{wu} \\ pM_{uv} & R_a + pL_v & pM_{vw} \\ pM_{wu} & pM_{vw} & R_a + pL_w \end{bmatrix} \begin{bmatrix} i_u \\ i_v \\ i_w \end{bmatrix} + p \begin{bmatrix} \Phi_u \\ \Phi_v \\ \Phi_w \end{bmatrix} \quad (9)$$

Here, R_a is the coil resistance of each phase and p is the differential operator. The Clarke transformation gives the voltage equation on $\alpha\beta\gamma$ reference frame.

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \begin{bmatrix} R_a + pL_0 & 0 \\ 0 & R_a + pL_1 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + p\Phi_a \begin{bmatrix} \left(1 + \frac{k}{3}\cos\theta\right) \\ \left(1 + k\right)\sin\theta - \frac{2j}{\sqrt{3}}\theta \end{bmatrix} \quad (10)$$

$$L_0 = l_a + \frac{3}{2}L_a + \frac{2}{3}L_c + \frac{1}{3}M_c \quad (11)$$

$$L_1 = l_a + \frac{3}{2}L_a - M_c \quad (12)$$

$$\Phi_a = \sqrt{\frac{3}{2}}\Phi_f \quad (13)$$

Figure 10: Vector diagram of conventional PMSMs.

Unlike the conventional SPMSMs, the diagonal element in equation (10) does not become equal because L_0 and L_1 contain the parameters L_c and M_c . The Park transformation gives the voltage equation on dq reference frame.

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + p\Phi_a \begin{bmatrix} \left(1 + \frac{2}{3}k\right) - \frac{k}{3}\cos 2\theta - \frac{2j}{\sqrt{3}}\theta \sin \theta \\ \frac{k}{3}\sin 2\theta - \frac{2j}{\sqrt{3}}\theta \cos \theta \end{bmatrix} \quad (14)$$

$$\begin{cases} A_{11} = R_a + p(L_p + L_m \cos 2\theta) \\ A_{12} = pL_m \sin 2\theta \\ A_{21} = -pL_m \sin 2\theta \\ A_{22} = R_a + p(L_p - L_m \cos 2\theta) \end{cases} \quad (15)$$

Table 1: Actuator's parameter

Symbol	Quantity (unit)	Value
Φ_f	Magnetic flux (μWb)	270
k	Coefficient for flux amplitude	0.115
j	Coefficient for flux slope	0.167
L_a	Effective inductance (μH)	34.1
l_a	Leakage inductance (μH)	12.2
L_c	Compensated self-inductance (μH)	4.6
M_c	Compensated mutual-inductance (μH)	7.8
L_0	α axis inductance (μH)	63.7
L_1	β axis inductance (μH)	55.4
L_p	Summed inductance (μH)	59.5
L_m	Subtracted inductance (μH)	4.17
P_n	Pole pairs	2
R_a	Coil resistance (Ω)	0.16

Figure 11: Thrust characteristics ($i_d = 0$, $i_q = 1$).

$$\begin{cases} B_{11} = -L_m \sin 2\theta \\ B_{12} = -L_p - L_m \cos 2\theta \\ B_{21} = L_p - L_m \cos 2\theta \\ B_{22} = L_m \sin 2\theta \end{cases} \quad (16)$$

$$L_p = \frac{L_0 + L_1}{2} \quad (17)$$

$$L_m = \frac{L_0 - L_1}{2} = \frac{2}{3}L_c + \frac{4}{3}M_c \quad (18)$$

We show the voltage equation model of conventional PMSMs to discuss equation (14).

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} R_a + pL_d & 0 \\ 0 & R_a + pL_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega \begin{bmatrix} 0 & -L_q \\ +L_d & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + p \begin{bmatrix} \Phi_a \\ 0 \end{bmatrix} \quad (19)$$

Here, ω is the rotational speed.

First, we focus on the first terms on the right side in equations (14) and (19). The diagonal elements in equation (14) depend on the electrical angle of the mover θ . On the other hand, the diagonal elements in equation (19) do not depend. This is because dq transformation means the conversion of AC components to DC ones. Moreover, the first matrix in equation (14) has nonzero elements, which correspond to interference between d - and q -axes.

Finally, we visualize the voltage equation model considering the edge effect. Figures 8 and 9 show the magnetic flux and voltage vector diagrams of the two-DOF resonant actuator. Vector diagram of conventional PMSMs is shown in Figure 10 for comparison. From Figure 8, PM flux linkage vector Φ_a is dependent on the mover position θ , which indicates that the direction of Φ_a does not always become the same as the direction of d -axis. The other flux vector except Φ_a also depend on θ . Therefore, the total flux

vector Φ_o has complicated behavior when the actuators are operated to oscillate.

3.3 Thrust equation model considering edge effect

In this section, we derive thrust equation model from the voltage equation model considering the edge effect. The thrust of the actuator F_x is calculated from the cross product of a current vector $i_a = (i_d, i_q)$ and the flux vector Φ_{dq} , as follows:

$$\begin{aligned} F_x &= -P_n i_a \times \Phi_{dq} \\ &= F_1 + F_2 + F_3 + F_4 \\ &= P_n \Phi_a i_q \\ &\quad + P_n \Phi_a \left(\frac{2k}{3} + \frac{k}{3} \cos 2\theta + \frac{2j}{\sqrt{3}} \theta \sin \theta \right) i_q \\ &\quad + P_n \Phi_a \left(\frac{k}{3} \sin 2\theta - \frac{2j}{\sqrt{3}} \theta \cos \theta \right) i_d \\ &\quad + P_n ((L_m \sin 2\theta)(i_d^2 - i_q^2) + 2L_m \cos 2\theta i_d i_q) \end{aligned} \quad (20)$$

Here, P_n is the pole pairs. The first term on the right side corresponds to the magnetic thrust caused by the symmetric PM flux linkage Φ_{sym} . The second and third terms correspond to the magnetic thrust caused by the asymmetric PM flux linkage Φ_{asym} . Especially, the third term interferes the x -axis thrust Φ_x by d -axis current i_d . The fourth term is the reluctance thrust caused by asymmetric inductance characteristics. Surprisingly, this actuator can generate reluctance thrust though the mover does not have saliency.

This paper clarifies the thrust performances of the actuator on the basis of equation (20). The two parameters L_c and M_c are determined from the following equations:

Figure 12: Thrust characteristics ($i_d = 0$, $i_q = 10$).

Figure 13: Thrust characteristics ($i_d = 1$, $i_q = 1$).

$$L_c = L_u - L_v \quad (21)$$

$$M_c = M_{vw} - M_{uv} \quad (22)$$

Table 1 shows the actuator's parameter. These values are determined from the 3D FEA results. As expected, L_m is much smaller than L_p because the edge effect is not dominant. Figures 11, 12, and 13 show each thrust component when the vector control is applied under various conditions. These results are normalized by the value of the first term Φ_1 . From Figure 12, the thrust cannot be kept constant when i_q is comparatively large. It is because Φ_4 is proportional to the square of i_q . From comparison between Figures 11 and 13, the d -axis current i_d interferes with the thrust characteristic because of F_2 and F_3 . These results suggest that the thrust characteristics are successfully investigated on the basis of the proposed thrust equation model.

4 Conclusion

This paper proposed a thrust equation model of two-DOF oscillatory actuator considering its edge effect. 3D FEA

revealed that PM flux linkage and phase inductance characteristics were different from the ideal one due to the edge effect. The proposed thrust equation model clarified that asymmetric PM flux linkage and inductance characteristics caused undesirable magnetic thrust and reluctance thrust, respectively.

From the opposite point of view, this actuator has a potential to be driven as a kind of linear reluctance actuator although the mover has no saliency. In the near future, we will design a new linear reluctance actuator without saliency of the mover, which utilize the reluctance thrust caused by the edge effect effectively.

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