Research Article

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Heat and mass transport investigation in radiative and chemically reacting fluid over a differentially heated surface and internal heating

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Abstract: The aim of this study is to investigate the heat and mass transport over a stretchable surface. The analysis is prolonged to the concept of thermal radiations and chemical reaction over a differentially heated surface with internal heating. This is significant from an engineering and industrial point of view. The nonlinear model is successfully attained by adopting the similarity transforms and then further computation is done via a hybrid Runge–Kutta algorithm coupled with shooting technique. The behavior of fluid velocity and heat and mass transport are then furnished graphically for feasible ranges of parameters. A comprehensive discussion of the results is provided against multiple parameters. Foremost, the local thermal performance and mass transport rate are explained via numerical computation. The major outcomes of the study are described in the end.

Keywords: convective flow, thermal radiations, heat transfer, stretchable surface, cross diffusion

1 Introduction

Thermal analysis of Newtonian fluids by considering the influence of external magnetic field, radiative heat flux, internal heating, cross-diffusion and chemical reaction over an unsteady convectively heated stretchable surface is a potential area of interest because of its potential applications in industrial and engineering areas. The applications include cooling electronic devices, fiber production, power generation and wire glass.

In 1970, Crane [1] reported flow over a nonporous stretchable surface. Afterward, Gupta and Gupta [2] studied thermal and mass transfer over a bilaterally stretchable surface. They investigated the influence of suction or injection parameters on the flow characteristics. Later on, Grubka and Booba [3] discussed thermal transportation in fluid over a surface which is capable of stretching continuously. Wang [4] extended the flow of carrier fluid in a rotating frame. He focused on the parameter λ and observed the fluid properties due to fluctuating λ . This parameter is the quotient of rotating surface and the rate of stretching sheet. Furthermore, he found the solution of the model by applying regular perturbation technique. Unsteady Newtonian flow model in a porous sheet was reported by Shafie et al. [5]. Combined effects of cross-diffusion on three-dimensional (3D) time-dependent flow over a stretching sheet were studied by Reddy et al. [6] in 2016. Influence of externally applied magnetic field, thermophoresis and internal heat source was part of their discussion.

Recently, Khan et al. [7] explored 3D squeezed flow of Newtonian fluid between two parallel plates. They considered a rotating system in which both plates and fluid rotate together in a counter clockwise pattern. They treated a nonlinear flow model numerically. The heat transfer investigation under the effects of externally imposed magnetic field over bi-latterly stretchable sheet in porous medium explored by Ahmad et al. [8]. For mathematical analysis of the model, they employed the homotopy analysis method. Heat transportation in magnetohydrodynamic flow over a porous stretching sheet

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which is capable of stretching in horizontal and vertical directions was discussed in ref. [9]. In 2017, Ullah et al. [10] explored dissipative flow of unsteady fluid (which is non-Newtonian in nature) over a linearly stretchable sheet. They reported the impacts of cross-diffusion and heat generation or absorption in the flow field. In 2016, Oyelakin et al. [11] discussed unsteady flow of the non-Newtonian flow model for a stretchable surface and encountered the impacts of thermal heat flux, convective and slip flow conditions. Hydromagnetic flow model for Casson fluid over an inclined stretchable plane by considering the influences of radiative heat flux and chemical reaction was reported by Reddy [12]. Shafie et al. [13] studied unsteady Falkner–Skan flow for a stretching sheet. The effects of cross-diffusion, thermal radiation and chemical reaction on a flow over a stretchable surface were reported in ref. [14] and [15], respectively. For further useful analysis of the various fluid mechanics problems, we can study the work presented in ref. [16–25].

Recently, Kumar et al. [26] conducted an analytical investigation of Cauchy reaction–diffusion equations. They successfully tackled the model by the said technique and explained comprehensively. The tumor and immune cell analysis in immunogenetic tumor model under the influence of nonsingular fraction derivative was carried out in ref. [27]. Moreover, significant investigation of various mathematical models under certain conditions by adopting multiple techniques was reported in ref. [28–44].

To the best of authors' literature survey, no studies regarding bi-laterally stretchable surface in the existence of externally imposed magnetic field, thermal heat flux, resistive heating, thermo-diffusion, diffusion-thermo and chemical reactions have been noted so far. The flow model is formulated effectively by employing feasible self-similar variables. Section 3 contains mathematical analysis of the model. Then, influence of pertinent dimensionless parameters on the momentum, thermal and concentration profiles is highlighted in Section 4. Numerical computations are carried out for quantities of engineering interest such as shear stresses, local Nusselt and Sherwood numbers. Finally, key observations of the study are given in the end.

2 Description of the problem

Unsteady radiative and chemically reacting flow of Newtonian fluid in the existence of Lorentz forces, ohmic heating, cross-diffusion and convective flow condition is

taken over a surface which is extendable in horizontal and vertical directions. The surface meets at $z = 0$ and fluid flow in the region $z > 0$. The surface is capable of stretching bi-directionally with forces which are equal in magnitude. Furthermore, components of the velocity along are as follows:

along x -axis:

$$u_w(x, t) = ax(1 - at)^{-1},$$

along y -axis:

$$v_w(y, t) = by(1 - at)^{-1},$$

where a and b both are constants and greater than zero. The surface is thermally invariable. The auxiliary conditions at and far from the surface are mentioned in the flow geometry. Figure 1 elucidates the configuration of the model.

The set of equations that govern the streamline flow over a stretching surface by considering the influence of externally imposed Lorentz forces, radiative heat flux, cross thermal and concentration gradients and chemical reaction is defined as follows:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \quad (1)$$

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \frac{\mu}{\rho} \left(\frac{\partial^2 u^*}{\partial z^{*2}} \right) - \frac{\sigma B^2}{\rho} u^* \quad (2)$$

Figure 1: Interpretation of the model.

$$\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = \frac{\mu}{\rho} \left(\frac{\partial^2 v^*}{\partial z^{*2}} \right) - \frac{\sigma B^2}{\rho} v^* \quad (3)$$

$$\begin{aligned} \frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} \\ = \frac{k}{\rho c_p} \left(\frac{\partial^2 T^*}{\partial z^{*2}} \right) + \frac{DK_T}{\rho c_p C_s} \left(\frac{\partial^2 C^*}{\partial z^{*2}} \right) - \frac{16\sigma^* T_{\infty}^3}{3k^* \rho c_p} \left(\frac{\partial^2 T^*}{\partial z^{*2}} \right) \\ + \frac{\mu}{\rho c_p} \left\{ \left(\frac{\partial u^*}{\partial z^*} \right)^2 + \left(\frac{\partial v^*}{\partial z^*} \right)^2 \right\} + \frac{\sigma B^2}{\rho c_p} (u^{*2} + v^{*2}) \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{\partial C^*}{\partial t^*} + u^* \frac{\partial C^*}{\partial x^*} + v^* \frac{\partial C^*}{\partial y^*} + w^* \frac{\partial C^*}{\partial z^*} \\ = D \left(\frac{\partial^2 C^*}{\partial z^{*2}} \right) + \frac{DK_T}{T_m} \left(\frac{\partial^2 T^*}{\partial z^{*2}} \right) - k_1 (C^* - C_{\infty}^*). \end{aligned} \quad (5)$$

Equation (1) shows the mass conservation which gratifies automatically. Furthermore, equations of motion, thermal and concentration equations are described in equations (2)–(5) in the existence of externally imposed magnetic field, radiative heat flux, cross-diffusion and chemical reaction. The various physical quantities embedded in the aforementioned flow model are specific heat capacity (c_p), dynamic viscosity (μ), density (ρ), mass diffusivity (D), thermal conductivity (k), mean temperature (T_m), thermal diffusion (K_T), susceptibility of concentration (C_s), fluid concentration (C^*), mean absorption coefficient (k^*), fluid temperature (T^*), Stefan Boltzmann constant (σ^*) and chemical reaction (k_1). The components of the velocity along coordinate axes are denoted by u^* , v^* and w^* , respectively.

For our flow problem, boundary conditions are the following:

At $z = 0$:

$$u^* = u_w, \quad v^* = v_w, \quad w^* = 0, \quad k \frac{\partial T^*}{\partial z^*} = h(T^* - T_w^*), \quad C^* = C_w^*. \quad (6)$$

At $z \rightarrow \infty$:

$$u^* \rightarrow 0, \quad v^* \rightarrow 0, \quad T^* \rightarrow T_{\infty}^*, \quad C^* \rightarrow C_{\infty}^*. \quad (7)$$

Feasible similarity variables which help to transform dimensional flow model into self-similar form are the following [45]:

$$\begin{aligned} u^* &= ax(1 - at)^{-1}F'(\eta), \\ v^* &= by(1 - at)^{-1}G'(\eta), \\ w^* &= -(av(1 - at))^{-1}[F(\eta) + G(\eta)], \\ \beta(\eta) &= \frac{T^* - T_{\infty}^*}{T_w^* - T_{\infty}^*}, \\ \phi(\eta) &= \frac{C^* - C_{\infty}^*}{C_w^* - C_{\infty}^*}, \\ \eta &= (a(v(1 - at))^{-1})^{\frac{1}{2}}. \end{aligned} \quad (8)$$

By means of self-similar transformations embedded in equation (8), we arrive with the following dimensionless nonlinear flow model:

$$F''' + F''[F + G] - F'^2 - S \left[F' + \frac{\eta}{2} F'' \right] - M^2 F' = 0, \quad (9)$$

$$G''' + G''[F + G] - G'^2 - S \left[G' + \frac{\eta}{2} G'' \right] - M^2 G' = 0, \quad (10)$$

$$\begin{aligned} (1 + \text{Rd})\beta'' + \text{Pr} \left[-S \frac{\eta}{2} \beta' + \beta'(F + G) + D_f \phi'' + \text{Ec}_x F''^2 \right. \\ \left. + \text{Ec}_y G''^2 + M^2 (\text{Ec}_x F'^2 + \text{Ec}_y G'^2) \right] = 0, \end{aligned} \quad (11)$$

$$\begin{aligned} \phi'' + \text{Sc} \left[-S \frac{\eta}{2} \phi' + \phi'(F + G) \right] + \text{SrSc}\beta'' - \gamma \text{Sc}\phi \\ = 0. \end{aligned} \quad (12)$$

Self-similar boundary conditions for particular flow model are the following:

$$\begin{aligned} F(\eta) \downarrow_{\eta=0} = 0, \quad F'(\eta) \downarrow_{\eta=0} = 1, \quad G(\eta) \downarrow_{\eta=0} = 0, \\ G'(\eta) \downarrow_{\eta=0} = c, \quad \beta'(\eta) \downarrow_{\eta=0} = B_i(\beta(\eta) - 1), \\ \phi(\eta) \downarrow_{\eta=0} = 1 \end{aligned} \quad (13)$$

$$\begin{aligned} F'(\eta) \downarrow_{\eta \rightarrow \infty} \rightarrow 0, \quad G'(\eta) \downarrow_{\eta \rightarrow \infty} \rightarrow 0, \\ \beta(\eta) \downarrow_{\eta \rightarrow \infty} \rightarrow 0, \quad \phi(\eta) \downarrow_{\eta \rightarrow \infty} \rightarrow 0. \end{aligned} \quad (14)$$

In equations (9)–(14), nondimensional quantities Hartmann parameter ($M^2 = \frac{\sigma B_0^2}{\rho a}$), time-dependent parameter S which is defined as $\frac{\alpha}{a}$, radiation number ($\text{Rd} = \frac{16\sigma^* T_{\infty}^3}{3kk^*}$), Prandtl number ($\text{Pr} = \frac{\nu \rho c_p}{k}$), Dufour parameter ($D_f = \frac{DK_T(C_w^* - C_{\infty}^*)}{C_s c_p \nu (T_w^* - T_{\infty}^*)}$), Eckert numbers Ec_x and Ec_y along horizontal and vertical directions ($\text{Ec}_x = \frac{u_w^2}{c_p (T_w^* - T_{\infty}^*)}$, $\text{Ec}_y = \frac{v_w^2}{c_p (T_w^* - T_{\infty}^*)}$), Schmidt number ($\text{Sc} = \frac{\nu}{D}$), Soret parameter ($\text{Sr} = \frac{D_m k_T (T_w^* - T_{\infty}^*)}{\nu T_m (C_w^* - C_{\infty}^*)}$), chemical reaction number ($\gamma = \frac{k_1(1 - at)}{a}$) and Biot's parameter ($B_i = \frac{h}{k} \left(\frac{\nu}{a} \right)^{-\frac{1}{2}}$).

The physical parameters of interest of Newtonian fluid are shear stresses, local heat and mass transfer. The formulae for these quantities are defined in the following manner:

$$C_{Fx} = \frac{\tau_{wx}}{\rho u^2}, \quad C_{Fy} = \frac{\tau_{wy}}{\rho v^2} \quad \text{and} \quad Nu_x = \frac{q_{wx}}{k(T_w - T_{\infty})} + q_r \quad (15)$$

The shear stresses τ_{wy} , τ_{wx} and the heat flux q_w are as follows:

$$\tau_{wx} = \mu \left(\frac{\partial u}{\partial z} \right), \quad \tau_{wy} = \mu \left(\frac{\partial v}{\partial z} \right) \quad \text{and} \quad q_w = -k \left(\frac{\partial T}{\partial z} \right). \quad (16)$$

By means of similarity variables, we arrive with equation (17):

$$\begin{aligned} C_{Fx} \sqrt{\text{Re}_x} &= F''(0), \\ C_{Fy} \sqrt{\text{Re}_y} &= G''(0), \\ \text{Nu}(\text{Re}_x)^{-\frac{1}{2}} &= -(1 + \text{Rd})\beta'(0), \\ \text{Sh}(\text{Re}_x)^{-\frac{1}{2}} &= -\phi'(0), \end{aligned} \quad (17)$$

where local Reynolds number is defined as $\text{Re}_x = \frac{u_w x}{\nu}$.

3 Mathematical analysis

The particular flow model is coupled and nonlinear in nature. For such variety of nonlinear models, closed form solutions are very rare (under certain conditions) or even does not exist. Therefore, for the said flow model numerical treatment is very suitable. For numerical treatment, Runge–Kutta method [46] is adopted with addition of shooting techniques. To initiate the numerical calculation, we have the following suitable substitution:

$$\begin{aligned} y_1^* &= F, \quad y_2^* = F', \quad y_3^* = F'', \quad y_4^* = G, \quad y_5^* = G', \quad y_6^* = G'' \\ y_7^* &= \beta, \quad y_8^* = \beta', \quad y_9^* = \phi, \quad y_{10}^* = \phi'. \end{aligned} \quad (18)$$

Dimensionless forms of particular models defined in equations (9)–(12) are as follows:

$$F''' = -F''[F + G] + F'^2 + S \left[F' + \frac{\eta}{2} F'' \right] + M^2 F', \quad (19)$$

$$G''' = -G''[F + G] + G'^2 + S \left[G' + \frac{\eta}{2} G'' \right] + M^2 G', \quad (20)$$

$$\begin{aligned} \beta'' &= \frac{1}{(1 + \text{Rd})} \left[-\text{Pr} \left[-S \frac{\eta}{2} \beta' + \beta'(F + G) - D_f \phi'' \right] \right. \\ &\quad \left. + \text{Ec}_x F''^2 + \text{Ec}_y G''^2 + M^2 (\text{Ec}_x F'^2 + \text{Ec}_y G'^2) \right], \end{aligned} \quad (21)$$

$$\phi'' = -\text{Sc} \left[-S \frac{\eta}{2} \phi' + \phi'(F + G) \right] - \text{SrSc}\beta'' + \gamma \text{Sc}\phi. \quad (22)$$

By means of substitution defined in equation (18), we arrive with the following first-order coupled initial value problem:

$$\begin{bmatrix} y_1^{*'} \\ y_2^{*'} \\ y_3^{*'} \\ y_4^{*'} \\ y_5^{*'} \\ y_6^{*'} \\ y_7^{*'} \\ y_8^{*'} \\ y_9^{*'} \\ y_{10}^{*'} \end{bmatrix} = \begin{bmatrix} y_2^* \\ y_3^* \\ -y_3^*(y_1^* + y_4^*) + y_2^{*2} + S \left(y_2^* + \frac{\eta}{2} y_3^* \right) + M^2 y_2^* \\ y_5^* \\ y_6^* \\ -y_6^*(y_1^* + y_4^*) + y_5^{*2} + S \left(y_5^* + \frac{\eta}{2} y_6^* \right) + M^2 y_5^* \\ y_8 \\ \frac{-\text{Pr}}{(1 + \text{Rd})} \left[\frac{-\eta S y_8^*}{2} + \frac{y_8^*}{(y_1^* + y_4^*)^{-1}} \right. \\ \left. + D_f y_{10}^{*'} + \text{Ec}_x y_3^{*2} + \text{Ec}_y y_6^{*2} \right. \\ \left. + M^2 (\text{Ec}_x y_2^{*2} + \text{Ec}_y y_5^{*2}) \right] \\ y_{10} \\ -\text{Sc} \left[\frac{-\eta S}{2} y_{10}^* + y_{10}^*(y_1^* + y_4^*) \right] - \text{SrSc} y_8^{*'} + \gamma \text{Sc} y_9^* \end{bmatrix}. \quad (23)$$

Equation (24) represents the feasible initial conditions:

$$\begin{bmatrix} y_1^* \\ y_2^* \\ y_3^* \\ y_4^* \\ y_5^* \\ y_6^* \\ y_7^* \\ y_8^* \\ y_9^* \\ y_{10}^* \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ l_1 \\ 0 \\ c \\ l_2 \\ B_i(l_3 - 1) \\ l_3 \\ 1 \\ l_4 \end{bmatrix}, \quad (24)$$

where l_i (for $i = 1, 2, 3, 4$) are unknown. Finally, by using Mathematica 10.0, for certain values of η , the solutions obtained for the particular model are mentioned in Table 1.

4 Graphical results

This section emphasizes on the behavior of certain physical quantities comprised in model in the flow regimes.

Table 1: Solutions of the model

η	$F'(\eta)$	$G'(\eta)$	$\beta(\eta)$	$\phi(\eta)$
0.0	1.0000000000	0.5000000000	1.0916010490	1.0000000000
0.5	0.5118143680	0.2688586210	1.0279142480	0.7147326187
1.0	0.2651520682	0.1430345519	0.8874642876	0.5168891804
1.5	0.1408798173	0.0771050749	0.7392255557	0.3774248984
2.0	0.0771803163	0.0425830984	0.6026484287	0.2769116916
2.5	0.0435734988	0.0241503982	0.4806747267	0.2025531418
3.0	0.0251736075	0.0139879162	0.3713942978	0.1457967415
3.5	0.0146094661	0.0081289752	0.2717459280	0.1007993763
4.0	0.0081157527	0.0045187005	0.1785994338	0.0634365579
4.5	0.0036573808	0.0020368268	0.0889803279	0.0306495517
5.0	-1.0191×10^{-7}	-7.1764×10^{-8}	-4.3337×10^{-9}	-2.7332×10^{-8}

These quantities are time-dependent parameter S (also known as unsteady parameter), Hartmann parameter M , Radiation number Rd , Dufour parameter D_f , Eckert number in x direction Ec_x , Eckert number in y direction Ec_y , Soret number Sr , Schmidt number Sc , chemical reaction γ and Biot's number B_i . Influences of the

aforementioned parameters in flow behavior are presented in Figures 2–11.

Figure 2a and b elucidates the influence of parameter c (stretching ration parameter) and time-dependent parameter (S) on axial velocity $F'(\eta)$. The decreasing effects of these parameters are noted for $F'(\eta)$. Declines in the fluid velocity are noted for growing c and S . The velocity decrement is rapid for higher c and quite slow decrement is observed for unsteady parameter S . Furthermore, far from the sheet, fluid velocity vanishes asymptotically. The effects of Lorentz forces on $F'(\eta)$ are presented in Figure 3. It is investigated that for stronger Lorentz forces, $F'(\eta)$ varies prominently. For varying magnetic number, fluid velocity decreases as stronger magnetic field opposes the flow, therefore, decrement in the velocity field occurs. For externally imposed magnetic field, rapid declines in the velocity are noted as compared to that of c and S , which is elucidated in Figure 2a and b, respectively.

The impacts of parameter c , unsteady number S and M on transverse velocity component are depicted in

Figure 2: Impact of (a) c and (b) S on $F'(\eta)$. **Figure 3:** Impact of M on $F'(\eta)$.

Figure 4: Impact of (a) c and (b) S on $G'(\eta)$.

Figures 4–6. Fascinating behavior of parameter c on the transverse velocity $G'(\eta)$ is noted. This behavior is shown in Figure 4a. Arising parameter c causes the rapid increment in the fluid velocity in the vicinity of the surface. For more stretching surface fluid velocity increases rapidly and vanishes asymptotically beyond $\eta = 3$. More stretching surface favors the velocity of the fluid. On the contrary, variations in the transverse velocity profile due to arising unsteady parameter S are portrayed in Figure 4b. For more unsteady fluid, the transverse velocity profile starts decreasing. In the region $0.5 \leq \eta \leq 1.5$, decreasing behavior of the velocity is rapid. Beyond this area, asymptotic behavior of the transverse velocity can be observed. Three-dimensional scenario of the velocity profile for varying stretching parameter c and unsteady parameter S is depicted in Figure 5a and b, respectively. Figure 6 represents the impact of magnetic field on $G'(\eta)$. The transverse velocity $G'(\eta)$ is decreasing function of magnetic field. As magnetic field becomes stronger, transverse velocity profile decreases rapidly. For less magneto-fluid decrease in the velocity $G'(\eta)$ gradually slows down.

Figure 5: 3D scenario of $G'(\eta)$ for varying (a) c and (b) S .

Figure 6: Influence of M on $G'(\eta)$.

In order to analyze thermal profile $\beta(\eta)$ for variables B_i , D_f , Pr , Rd and Eckert number Ec_x , Figures 7–9 are portrayed. Here, it is important to mention that current flow model reduces to steady case for $S = 0$ and represents two-dimensional flow model for $S = 0$, $c = 0$. Thermal behavior of the fluid is also depicted for the aforementioned

Figure 7: Impact of (a) B_i and (b) D_f on $\beta(\eta)$.

cases. Figure 7a elucidates the temperature variations for increasing Biot's number B_i . Physically Biot's number B_i is the quotient of the thermal transportation resistances present inside and at the body surface. The values of Biot's number much smaller than one correspond to uniform thermal field inside the body and such sort of problems are thermally easy to handle. The higher values of Biot's number (much larger than one) indicate that the temperature inside the body is non-uniform and such type of flow problems are more tedious. The variations in the fluid temperature against Biots number are plotted in Figure 7a and observed that increasing B_i favors the fluid temperature. In the case of unsteady flow, thermal profile rises rapidly comparative to that of steady and two-dimensional case. The influence of Dufour parameter on dimensionless thermal profile $\beta(\eta)$ is portrayed in Figure 7b. The fluid temperature increases prominently for increasing Dufour number. At the surface of the sheet, temperature varies rapidly. Thermal profile rises rapidly for unsteady case. However, for steady and two-dimensional cases, these effects are quite slow.

The influences of Prandtl number, radiation and Eckert numbers on thermal profile are portrayed in

Figure 8: Impact of (a) Pr and (b) Rd on $\beta(\eta)$.

Figures 8 and 9. Hence, Prandtl number is the quotient of momentum to thermal diffusivities. Therefore, smaller values of the Prandtl parameter lead to larger Prandtl number and smaller Prandtl number corresponds to larger thermal diffusivity. This behavior of the Prandtl number is shown in Figure 8a. It is noted that for larger Prandtl values thermal profile decreases rapidly. For two-dimensional and steady cases decrement in the

Figure 9: Impact of Ec_x on $\beta(\eta)$.

Figure 10: Influences of Sr on $\phi(\eta)$.

temperature field is rapid. In the vicinity of the sheet, these effects are very prominent and rapid. On the other side, thermal radiation and Eckert numbers favor the thermal profile. In the case of Eckert number accelerating behavior of thermal field is rapid as comparative to that of thermal radiation parameter. For more radiative fluid temperature $\beta(\eta)$ increases rapidly. Furthermore,

Figure 11: Influences of (a) γ and (b) Sc on $\phi(\eta)$.

Table 2: Numerical computation for skin friction coefficient

c	S	M	$C_{Fx}\sqrt{Re_x}$	$C_{Fy}\sqrt{Re_y}$
0.3	0.3	0.3	-1.19008	-0.29178
0.5			-1.17605	-0.85713
0.7			-1.20345	-1.13528
0.3	0.5		-1.24917	-0.31276
	0.7		-1.30684	-0.33306
	0.9		-1.36293	-0.35259
	0.3	0.5	-1.21000	-0.64673
		0.7	-1.34748	-0.34808
		0.9	-1.46135	-0.38729

favorable temperature behavior is investigated for more dissipative fluid.

The pertinent nondimensional physical parameters play a significant role in the concentration field. Soret effects on the concentration profile $\phi(\eta)$ are elucidated in Figure 10. Impacts of the Soret parameter on concentration field $\phi(\eta)$ are almost negligible for time dependent, time independent and 2D cases. Although these influences are inconsequential, for unsteady case the effects are quite dominant as compared to that of other two cases. The concentration of the fluid drops against higher values of γ and Schmidt number. For stronger Schmidt number, concentration profile decreases rapidly comparative to that of increasing chemical reaction parameter. For stronger chemical parameter, concentration field decreases slowly and away from the sheet, these influences vanish asymptotically.

The shear stresses, local Nusselt and Sherwood numbers are treated numerically and values for various parameters are given in Tables 2 and 3, respectively. It is noted that for more chemically reacting fluid rate of mass transfer increases.

4 Conclusions

The inspection of radiative and chemical reacting fluids over a differentially heated surface with internal heating is conducted. The results against multiple quantities are furnished and explained broadly. It is noted that the fluid motion drops against stronger Lorentz forces. The surface provides extra heat to the surface due to convective condition which leads to escalations in the temperature $\beta(\eta)$. Similarly, increase in the temperature is examined against stronger thermal radiations, Dufour and dissipative effects. Moreover, the mass transport improved due to Soret

Table 3: Computation of Nusselt and Sherwood numbers

Rd	Pr	D_f	B_i	Ec_x	Ec_y	Sc	Sr	γ	$Nu(Re_x)^{-\frac{1}{2}}$	$Sh(Re_x)^{-\frac{1}{2}}$
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.075490	0.423523
0.5									0.084750	0.422480
0.7									0.092123	0.421651
0.3	0.5								0.095328	0.421451
	0.7								0.113529	0.419554
	0.9								0.129881	0.417852
	0.3	0.5							0.068077	0.424311
		0.7							0.060614	0.425104
		0.9							0.053099	0.425909
		0.3	0.5						0.093141	0.423233
			0.7						0.103523	0.423063
			0.9						0.110353	0.422951
			0.3	0.5					0.014551	0.430250
				0.7					−0.046381	0.436977
				0.9					−0.107327	0.443703
				0.3	0.5				0.070661	0.424053
					0.7				0.065832	0.424583
					0.9				0.061002	0.425112
					0.3	0.5				0.554987
						0.7				0.675027
						0.9				0.785614
						0.3	0.5			0.429561
							0.7			0.435641
							0.9			0.441762
							0.3	0.5		0.491546
								0.7		0.552005
								0.9		0.606718

effects and drops for Schmidt parameter. The local thermal performance rate increases for higher Rd and it opposes the mass transport rate.

Future recommendations: In the future, conducted analysis can be prolonged to the concept of cross-diffusion gradients, porosity factor, nonlinear thermal radiations, thermal and velocity slip conditions.

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