Research Article

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Stable solutions to the nonlinear RLC transmission line equation and the Sinh–Poisson equation arising in mathematical physics

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Abstract: The Sinh–Poisson equation and the RLC transmission line equation are important nonlinear model equations in the field of engineering and power transmission. The modified simple equation (MSE) procedure is a realistic, competent and efficient mathematical scheme to ascertain the analytic soliton solutions to nonlinear evolution equations (NLEEs). In the present article, the MSE approach is put forward and exploited to establish wave solutions to the previously referred NLEEs and accomplish analytical broad-ranging solutions associated with parameters. Whenever parameters are assigned definite values, diverse types of solitons originated from the general wave solutions. The solitons are explained by sketching three-dimensional and two-dimensional graphs, and their physical significance is clearly stated. The profiles of the attained solutions assimilate compacton, bell-shaped soliton, peakon, kink, singular periodic, periodic soliton and singular kink-type soliton. The outcomes assert that the MSE scheme is an advance, convincing and rigorous scheme to bring out soliton solutions. The solutions obtained may significantly contribute to the areas of science and engineering.

Keywords: nonlinear evolution equations, RLC transmission line, Sinh–Poisson equation, MSE method, analytic solutions.

1 Introduction

Modeling of most of the complex phenomena in nature is formulated through nonlinear evolution equations (NLEEs), and their execution in physical sciences, engineering and technology are broadly carried out and have drawn huge attention of researchers and professionals. Thus, NLEEs significantly contribute in managing the physical procedure. NLEEs are used in theoretical mechanics, biomechanics, optics, high-energy physics, condensed matter physics, chemical physics, solid-state physics, optical fiber, elastic media, elasticity, gas dynamics, plasma physics, hydrodynamics, reaction diffusion, ocean engineering, quantum engineering, electrodynamics, population dynamics, ecology etc. All of the topics are fundamentally governed by NLEEs. The soliton solutions are notable to understand the process of intricate nonlinear events. Numerous travelling wave equations involving nonlinear structures are renowned. Alternatively, there are only a few key methods in prepared laboratory research in which solutions can be realized directly and effectively. Nonlinear electric transmission lines are a decent example of such methods. The nonlinear transmission lines (NLTs) offer an appropriate technique for exploring the way in which the nonlinear excitations work intimately in nonlinear medium. It is dynamic to introduce that at the present-day NLTs have been suggested for universal usage by highly influencing wideband swing and signals, which is typically a matter of stress. Soliton fits in a wide family of restricted nonlinear travelling waves, the reputed solitary wave family. Since the 1960s, many investigators have detected the existence of solitons in NLTs for mathematical models and physical experiments. In mathematical physics, the numerical and analytical solutions to NLEEs significantly contribute to the soliton theory. Physicists, engineers and mathematicians have established precise solutions for the NLEEs that transform into a key task in the analysis of nonlinear physical incidents. Generally, there is no single approach to resolve all sorts of NLEEs, and a variety of scientific groups have

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successfully established many techniques, including the method of extended tanh-function [1], the method of tanh function [2,3], the method of sine-cosine [4], the method of modified extended tanh function [5], the method of $\exp(-\phi(\xi))$ -expansion [6], the method of exp-function [7], the method of Backlund transformation [8], the method of Miura transformation [9], the method of extended F-expansion [10], the method of improved F-expansion [11], the method of $(G'/G, 1/G)$ -expansion [18], the method of (G'/G) -expansion [12–15], the method of new auxiliary equation [16], the method of auxiliary equation [17], the method of Kudryashov [18–21], the method of modified Kudryashov [29], the method of extended trial function [31], the method of variational iteration [32], the method of modified simple equation (MSE) [22–32], extended modified direct algebraic method, extended mapping method and Seadawy techniques to find solutions for some nonlinear partial differential equations [33–38]. All of these methods have several advantages and disadvantages along with the problem considered, and up to now there is no unique technique that can be used to examine all sorts of NLEEs [39–43]. As a result, development of any specific process used to generate certain advanced solutions to the NLEEs is always accepted. Consequently, different ansatzes have been presented for looking up travelling wave solutions to NLEEs. The aim of this article is to extract stable and functional soliton solutions to the Sinh–Poisson equation and the nonlinear RLC transmission line (RLCTL) equation by making use of the MSE technique.

The outline of this article is as follows: in Section 2, the MSE scheme is analyzed. In Section 3, the nonlinear RLCTL equation and the Sinh–Poisson equation are examined through the introduced method. In Section 4, the symbolic significations and physical significance of the acquired solutions are presented, and in Section 5, conclusion is provided.

2 Methodology

In this section, the MSE method has been explained to analyze and formulate the stable soliton solutions to the nonlinear RLCTL and the Sinh–Poisson equation. The MSE scheme is an extremely functional computational strategy for extracting stable soliton solutions to NLEEs in mathematical physics, engineering and applied mathematics. NLEEs are often very intricate to study explicitly indeed. In reality, there is no single method to explore all kinds of NLEEs. Therefore, many researchers were irritated to search straightforward methods that can examine NLEEs categorically. But each method has some advantages and disadvantages. Consequently, establishment of

new methods is important to obtain exact fresh solutions to NLEEs.

In this sphere, we presume an NLEE of two distinct variables t and x to elucidate the MSE method:

$$\mathcal{F}(w, w_t, w_x, w_{tt}, w_{xx}, w_{xt} \dots) = 0, \quad (2.1)$$

where $w = w(t, x)$ is the wave function and $\mathcal{F}$ is the function of $w(t, x)$, subscripts are used for partial derivatives, where the greatest order linear and nonlinear terms are associated. The key points of this procedure are reported as follows:

First step: we integrate the basic variables x and t , starting with the composite variable ξ ,

$$w(x, t) = w(\xi), \quad \xi = k(x \pm \omega t) \quad (2.2)$$

where the wave transmission speed is ω , which assists to transmute equation (2.1) to the next ordinary differential equation (ODE):

$$\mathcal{M}(w, w', w'', w''', \dots) = 0, \quad (2.3)$$

where $\mathcal{M}$ is a polynomial in $w(\xi)$ and its derivatives are indicated by prime in regard to ξ .

Second step: We estimate the solution of (2.3) in pursuance of the MSE method provided as follows:

$$w(\xi) = \sum_{l=0}^N b_l \left(\frac{\psi'(\xi)}{\psi(\xi)} \right)^l, \quad (2.4)$$

where $b_l (l = 0, 1, 2, 3, \dots)$ are computable arbitrary constants wherein $b_N \neq 0$ and $\psi(\xi)$ is not the solution of any earlier prescribed equation nor a predetermined function, so that $\psi'(\xi)$ is not equal to zero. This method is noteworthy and remarkable because ψ is not a solution of any formerly specified equation nor ψ is a reputed function fixed in advance, whereas in the method of sine-cosine, the method of tanh function, the method of (G'/G) -expansion, the method of exp-function, the method of elliptic function etc. The result is suggested to be related to the functions which are the solutions of some simple equations. Thus, several newly found solutions may be conceivable by this scheme.

Third step: the value of the integral number N present in equation (2.4) can be ascertained by envisaging the maximum order derivative and nonlinear terms existing in equation (2.3).

Fourth step: we compute the needful differential coefficients $w', w'', w''', \dots$, and substitute equation (2.4) into (2.3) in order to estimate the obtainable function $\psi(\xi)$. In conclusion, we found a polynomial in $(\psi(\xi))^{-1}$ and its differential coefficients. Summing up the coefficients of analogous exponent of this polynomial to zero, the differential equation and a set of algebraic equations

can be found and unraveling these equations, we can establish the function $\psi(\xi)$ and $b_0, b_1, b_2, \dots$. Therefore, the estimation of the solution of equation (2.1) is completed.

3 Formulation of the solutions

To know the qualitative and quantitative characteristics of incident and processes precisely in diverse sphere of technical discipline, closed form solutions of mathematical models provide significant information. In Sections 3.1 and 3.2, we ascertain the analytical solutions to the nonlinear RLCTL equation and the Sinh–Poisson equation.

3.1 The nonlinear RLCTL equation

The nonlinear RLC electrical transmission line equation [44] is

$$l^2 \frac{\partial^2 w}{\partial x^2} - R_1 C_0 \frac{\partial w}{\partial t} + R_1 C_0 2bw \frac{\partial w}{\partial t} = 0, \quad (3.1.1)$$

where C is the capacitance, R is the resistance and L is the inductance.

To construct stable solitary wave solution to the nonlinear RLCTL equation by means of the MSE technique, we utilize travelling wave transformation with dimensionless wave variable

$$w(t, x) = w(\xi), \quad \xi = x - \omega t, \quad (3.1.2)$$

where ω is the dimensionless wave velocity. Using the transformation (3.1.2) into equation (3.1.1), we accomplish the following ODE:

$$l^2 \frac{d^2 w}{d\xi^2} + R_1 C_0 \omega \frac{dw}{d\xi} - 2R_1 C_0 \omega b w \frac{dw}{d\xi} = 0. \quad (3.1.3)$$

Integrating equation (3.1.3) gives

$$l^2 w' + R_1 C_0 \omega w - R_1 C_0 \omega b w^2 = 0. \quad (3.1.4)$$

The balancing theory between the nonlinear highest order term w^2 and the derivative term w' yields $N = 1$. Thus, the structure of solution of equation (3.1.4) is provided as follows:

$$w(\xi) = b_0 + b_1 \left(\frac{\psi'(\xi)}{\psi(\xi)} \right). \quad (3.1.5)$$

At this instance, it is simple to compute

$$w' = b_1 \left[-\frac{\psi'^2}{\psi} + \frac{\psi''}{\psi} \right]. \quad (3.1.6)$$

Using solution (3.1.5) and its derivative (3.1.6) into equation (3.1.4), we attain the following result:

$$R_1 \omega b_0 C_0 - R_1 b \omega b_0^2 C_0 + \frac{-l^2 a_1 \psi'^2 - R_1 b \omega b_1^2 C_0 \psi'^2}{\psi^2} + \frac{R_1 \omega b_1 C_0 \psi' - 2R_1 b \omega b_0 b_1 C_0 \psi' + l^2 b_1 \psi''}{\psi} = 0.$$

Equalizing the coefficient of ψ^0, ψ^{-1} and ψ^{-2} to zero, we acquire the following algebraic and differential equations:

$$R_1 \omega b_0 C_0 - R_1 b \omega b_0^2 C_0 = 0, \quad (3.1.7)$$

$$-l^2 b_1 \psi'^2 - R_1 b \omega b_1^2 C_0 \psi'^2 = 0, \quad (3.1.8)$$

$$R_1 \omega b_1 C_0 \psi' - 2R_1 b \omega b_0 b_1 C_0 \psi' + l^2 a_1 \psi'' = 0. \quad (3.1.9)$$

From equation (3.1.7), we attain

$$b_0 = 0 \text{ and } b_0 = \frac{1}{b}.$$

Equation (3.1.8) gives the value of b_1 as follows:

$$b_1 = -\frac{l^2}{(R_1 b \omega C_0)}, \quad \text{since } b_1 \neq 0.$$

The following context for the values of the unknowns b_0 and b_1 arises for establishing the solution of (3.1.4).

Case 1: when $b_0 = 0$ and $b_1 = -\frac{l^2}{(R_1 b \omega C_0)}$, from equation (3.1.9), we obtain

$$\psi = -\frac{e^{-\frac{R_1 \xi \omega C_0}{l^2}}}{R_1 \omega C_0} l^2 c_1 + c_2, \quad (3.1.10)$$

where c_2 and c_1 are arbitrary constants.

Now, substituting the values of b_0, b_1 and ψ into equation (3.1.5), we derive the exponential compact result of (3.1.4) as follows:

$$w(\xi) = -\frac{e^{-\frac{R_1 \xi \omega C_0}{l^2}}}{b \left(\frac{R_1 \omega C_0 c_2}{l^2 c_1} - e^{-\frac{R_1 \xi \omega C_0}{l^2}} \right)}. \quad (3.1.11)$$

In terms of primitive variable x and t , solution (3.1.11) turns into

$$w(x, t) = -\frac{e^{-\frac{R_1 \omega C_0}{l^2} (x - \omega t)}}{b \left(\frac{R_1 \omega C_0 c_2}{l^2 c_1} - e^{-\frac{R_1 \omega C_0}{l^2} (x - \omega t)} \right)}. \quad (3.1.12)$$

In order to achieve stable solutions concerning well-known functions, transforming the exponential function to hyperbolic function, solution (3.1.12) is converted into

$$w(x, t) = -\frac{\cosh\left(\frac{R_1\omega C_0}{2l^2}(x - \omega t)\right) - \sinh\left(\frac{R_1\omega C_0}{2l^2}(x - \omega t)\right)}{b\left[\left(\frac{R_1\omega C_0 c_2}{l^2 c_1} - 1\right)\cosh\left(\frac{R_1\omega C_0}{2l^2}(x - \omega t)\right) + \left(\frac{R_1\omega C_0 c_2}{l^2 c_1} + 1\right)\sinh\left(\frac{R_1\omega C_0}{2l^2}(x - \omega t)\right)\right]}. \quad (3.1.13)$$

In solution (3.1.13), c_1 and c_2 are integral constants; therefore, one might potentially accept their values. Based on this, if we accept $c_1 = 1$ and $c_2 = l^2/(R_1\omega C_0)$, from result (3.1.13) we acquire the ensuing particular solution:

$$w(x, t) = \frac{1}{2b} - \frac{1}{2b} \coth\left(\frac{R_1\omega C_0}{2l^2}(x - \omega t)\right). \quad (3.1.14)$$

Using hyperbolic function identities, (3.1.14) gives

$$w(x, t) = \frac{1}{2b} - \frac{i}{2b} \cot\left(\frac{iR_1\omega C_0}{2l^2}(x - \omega t)\right). \quad (3.1.15)$$

Furthermore, if we accept $c_1 = 1$ and $c_2 = -l^2/(R_1\omega C_0)$, from solution (3.1.13), we achieve the stable kink-shaped solution:

$$w(t, x) = \frac{1}{2b} - \frac{1}{2b} \tanh\left(\frac{R_1\omega C_0}{2l^2}(x - \omega t)\right). \quad (3.1.16)$$

The hyperbolic function identities provide the following solution:

$$w(x, t) = \frac{1}{2b} + \frac{i}{2b} \tan\left(\frac{iR_1\omega C_0}{2l^2}(x - \omega t)\right). \quad (3.1.17)$$

Case 2: when $b_0 = \frac{1}{b}$ and $b_1 = -\frac{l^2}{(R_1 b \omega C_0)}$, from equation (3.1.9), we obtain

$$\psi = \frac{e^{\frac{R_1 \xi \omega C_0}{l^2}}}{R_1 \omega C_0} l^2 c_3 + c_4, \quad (3.1.18)$$

where c_3 and c_4 are integral constants.

Introducing the estimation of the constants and the function ψ into (3.1.5), we attain the broad-ranging general solution as

$$w(\xi) = \frac{1}{b} - \frac{e^{\frac{R_1 \xi \omega C_0}{l^2}}}{b\left(\frac{R_1 \omega C_0 c_4}{l^2 c_3} - e^{\frac{R_1 \xi \omega C_0}{l^2}}\right)}. \quad (3.1.19)$$

Substituting the wave transmutation $\psi = x - \omega t$, we attain the next general solution:

$$w(x, t) = \frac{1}{b} - \frac{e^{\frac{R_1 \omega C_0}{l^2}(x - \omega t)}}{b\left(\frac{R_1 \omega C_0 c_4}{l^2 c_3} - e^{\frac{R_1 \omega C_0}{l^2}(x - \omega t)}\right)}. \quad (3.1.20)$$

Here c_3 and c_4 are free constants, thus if we put $c_3 = 1$ and $c_4 = \frac{l^2}{R_1 \omega C_0}$ therefore from solution (3.1.20), we found the following solution:

$$w(x, t) = \frac{1}{b} + \frac{1}{2b} \left(1 - \coth\left(\frac{R_1 \omega C_0}{2l^2}(x - \omega t)\right)\right). \quad (3.1.21)$$

By the use of the hyperbolic function consistencies, solution (3.1.21) converts into

$$w(x, t) = \frac{1}{b} + \frac{1}{2b} \left(1 - i \cot\left(\frac{iR_1 \omega C_0}{2l^2}(x - \omega t)\right)\right). \quad (3.1.22)$$

On the other hand, if we substitute $c_3 = 1$ and $c_4 = -\frac{l^2}{R_1 \omega C_0}$, from solution (3.1.20), we obtain the following soliton wave solution:

$$w(t, x) = \frac{1}{b} + \frac{1}{2b} \left(1 - \tanh\left(\frac{R_1 \omega C_0}{2l^2}(x - \omega t)\right)\right). \quad (3.1.23)$$

The hyperbolic function identities transform the result (3.1.23) into

$$w(t, x) = \frac{1}{b} + \frac{1}{2b} \left(1 + i \tan\left(\frac{iR_1 \omega C_0}{2l^2}(x - \omega t)\right)\right). \quad (3.1.24)$$

These solutions represent the singular kink, kink, steady plane, singular bell-shape, peakon, anti-peakon and other type solitons for the assorted assessment of the constraints which elucidate the electric flow in nonlinear RLCTL.

3.2 The Sinh–Poisson equation

Let us consider the Sinh–Poisson equation:

$$\phi_{xx} + \phi_{tt} = \beta^2 \sinh(\phi). \quad (3.2.1)$$

The Sinh–Poisson equation (3.2.1) can be transformed into ODE by the wave transformation $\xi = x - \omega t$ as

$$(1 + \omega^2)\phi'' = \beta^2 \sinh(\phi), \quad (3.2.2)$$

where ω is the travelling wave speed. The variable transformation

$$w = e^\phi, \quad (3.2.3)$$

and the identity

$$\sinh(\phi) = \frac{e^\phi - e^{-\phi}}{2}, \quad (3.2.4)$$

refine equation (3.2.2) into an ODE as follows:

$$-2(1 + \omega^2)(w')^2 + 2(1 + \omega^2)ww'' - \beta^2(w^3 - w) = 0. \quad (3.2.5)$$

Balancing the nonlinear term w^3 and the highest order derivative ww'' yields $N = 2$.

Thus, the solution of (3.2.5) can be written as:

$$w = b_0 + b_1 \frac{\psi'}{\psi} + b_2 \left(\frac{\psi'}{\psi} \right)^2, \quad (3.2.6)$$

where b_0 , b_1 and b_2 are unknown constraints to make out wherein $b_2 \neq 0$, $\psi(\phi)$ is an undetermined function to be determined.

Substituting solution (3.2.6) into (3.2.5) generates a polynomial equation and setting the coefficient of ψ^{-i} , $i = 0, 1, 2, 3$, to zero, we attain the following algebraic and differential equations:

$$\beta^2 b_0 - \beta^2 b_0^3 = 0. \quad (3.2.7)$$

$$\beta^2 b_1 \psi' - 3\beta^2 b_0^2 b_1 \psi' + 2(1 + \omega^2) b_0 b_1 \psi''' = 0. \quad (3.2.8)$$

$$\begin{aligned} & -3\beta^2 b_0 b_1^2 \psi'^2 + \beta^2 b_2 \psi'^2 - 3\beta^2 b_0^2 b_2 \psi'^2 \\ & -6(1 + \omega^2) b_0 b_1 \psi' \psi'' - 2(1 + \omega^2) b_1^2 \psi''^2 \\ & + 4(1 + \omega^2) b_0 b_2 \psi''^2 + 2(1 + \omega^2) b_1^2 \psi' \psi''' \\ & + 4(1 + \omega^2) b_0 b_2 \psi' \psi''' = 0. \end{aligned} \quad (3.2.9)$$

$$w = \left[\frac{(c_1 \sqrt{1 + \omega^2} + \beta c_2) \cosh\left(\frac{\beta \xi}{2\sqrt{1 + \omega^2}}\right) - (c_1 \sqrt{1 + \omega^2} - \beta c_2) \sinh\left(\frac{\beta \xi}{2\sqrt{1 + \omega^2}}\right)}{(c_1 \sqrt{1 + \omega^2} - \beta c_2) \cosh\left(\frac{\beta \xi}{2\sqrt{1 + \omega^2}}\right) - (c_1 \sqrt{1 + \omega^2} + \beta c_2) \sinh\left(\frac{\beta \xi}{2\sqrt{1 + \omega^2}}\right)} \right]^2. \quad (3.2.16)$$

$$\begin{aligned} & 4(1 + \omega^2) b_0 b_1 \psi'^3 - \beta^2 b_1^3 \psi'^3 - 6\beta^2 b_0 b_1 b_2 \psi'^3 \\ & - 2(1 + \omega^2) b_1^2 \psi'^2 \psi'' - 20(1 + \omega^2) b_0 b_2 \psi'^2 \psi'' \\ & - 4(1 + \omega^2) b_1 b_2 \psi' \psi''^2 + 6(1 + \omega^2) b_1 b_2 \psi'^2 \psi''' = 0. \end{aligned} \quad (3.2.10)$$

$$\begin{aligned} & 2(1 + \omega^2) b_1^2 \psi'^4 + 12(1 + \omega^2) b_0 b_2 \psi'^4 - 3\beta^2 b_1^2 b_2 \psi'^4 \\ & - 3\beta^2 b_0 b_2^2 \psi'^4 - 10(1 + \omega^2) b_1 b_2 \psi'^3 \psi'' \\ & - 4(1 + \omega^2) b_2^2 \psi'^2 \psi''^2 + 4(1 + \omega^2) b_2^2 \psi'^3 \psi''' = 0. \end{aligned} \quad (3.2.11)$$

$$8(1 + \omega^2) b_1 b_2 \psi'^5 - 3\beta^2 b_1 b_2^2 \psi'^5 - 4(1 + \omega^2) b_2^2 \psi'^4 \psi'' = 0. \quad (3.2.12)$$

$$4(1 + \omega^2) b_2^2 - \beta^2 b_2^3 = 0. \quad (3.2.13)$$

Solving equations (3.2.7), (3.2.13) and (3.2.12), we obtain the following results:

$$b_0 = 0, \pm 1, b_2 = \frac{4(1 + \omega^2)}{\beta^2}$$

and

$$\psi = -\frac{4c_1(1 + \omega^2)e^{-\frac{\beta^2 a_1}{4(1 + \omega^2)\xi}}}{\beta^2 a_1} + c_2, \quad (3.2.14)$$

where c_2 and c_1 are integral constants.

Case 1: When $b_0 = 0$, solving the differential equation by substituting the value of ψ , we attain $b_1 = 0$. For the value of $b_0 = 0$ and $b_1 = 0$, we obtain

$$\frac{\psi'}{\psi} = 0, \text{ which is not acceptable.}$$

Case 2: When $b_0 = 1$, $b_2 = \frac{4(1 + \omega^2)}{\beta^2}$ and $\psi = -\frac{4c_1(1 + \omega^2)e^{-\frac{\beta^2 b_1}{4(1 + \omega^2)\xi}}}{\beta^2 b_1} + c_2$, from equation (3.2.9), we attain

$$b_1 = \pm \frac{4\sqrt{1 + \omega^2}}{\beta}.$$

Substituting these values of b_0 , b_2 , ψ and $b_1 = \pm \frac{4\sqrt{1 + \omega^2}}{\beta}$ into equation (3.2.6), we achieve the following comprehensive solution:

$$w = \frac{\left((1 + \omega^2) c_1^2 + 2\sqrt{1 + \omega^2} e^{\frac{\beta \xi}{1 + \omega^2}} \beta c_1 c_2 + e^{\frac{2\beta \xi}{\sqrt{1 + \omega^2}}} \beta^2 c_2^2 \right)}{\left(\sqrt{1 + \omega^2} c_1 - e^{\frac{\beta \xi}{1 + \omega^2}} \beta c_2 \right)^2}, \quad (3.2.15)$$

where c_1 and c_2 are constants on integration.

Altering the exponential function to the hyperbolic function, we obtain

Since c_1 and c_2 are integral constants, we may choose the value of these constants as $c_1 = 1$ and $c_2 = \frac{\sqrt{1 + \omega^2}}{\beta}$. Therefore, we accomplish the solution

$$w(\xi) = \coth^2 \left(\frac{\beta \xi}{2\sqrt{1 + \omega^2}} \right). \quad (3.2.17)$$

Alternatively, if we choose $c_1 = 1$ and $c_2 = \frac{\sqrt{1 + \omega^2}}{\beta}$, then solution (3.2.16) is simplified as

$$w(\xi) = \tanh^2 \left(\frac{\beta \xi}{2\sqrt{1 + \omega^2}} \right). \quad (3.2.18)$$

Now, if we choose the value $b_1 = -\frac{4\sqrt{1 + \omega^2}}{\beta}$, we achieve solutions similar to solutions (3.2.17) and (3.2.18) and thus have not been recorded.

In order to obtain the results of the Sinh–Poisson equation (3.2.1), the transformation (3.2.3) has to be re-processed. Therefore, solutions (3.2.17) and (3.2.18), respectively, become

$$\phi(x, t) = \ln \left(\coth^2 \left(\frac{\beta(x - \omega t)}{2\sqrt{1 + \omega^2}} \right) \right) \quad (3.2.19)$$

and

$$\phi(x, t) = \ln \left(\tanh^2 \left(\frac{\beta(x - \omega t)}{2\sqrt{1 + \omega^2}} \right) \right). \quad (3.2.20)$$

Case 3: When $b_0 = -1$, substituting the values of b_0 , b_2 and ψ into (3.2.9), we attain $b_1 = \pm \frac{4\sqrt{-1-\omega^2}}{\beta}$.

Substituting the values of b_0 , $b_1 = \frac{4\sqrt{-1-\omega^2}}{\beta}$, b_2 and ψ into solution (3.2.6) yields the following general solution:

$$w = -\frac{2e^{\frac{\beta\xi}{\sqrt{-1-\omega^2}}}\sqrt{-1-\omega^2}(1+\omega^2)\beta c_3 c_4 + (1+\omega^2)^2 e^{\frac{2\beta\xi}{\sqrt{-1-\omega^2}}} c_3^2 + (-1-\omega^2)\beta^2 c_4^2}{\left((1+\omega^2)e^{\frac{\beta\xi}{\sqrt{-1-\omega^2}}} c_3 - \sqrt{-1-\omega^2}\beta c_4\right)^2}, \quad (3.2.21)$$

where c_3 and c_4 are arbitrary constants. Transforming the exponential function to hyperbolic function, solution (3.2.21) becomes

$$w = -\frac{\left((1+\omega^2)c_3 + \beta c_4 \sqrt{-1-\omega^2}\right) \cosh\left(\frac{\beta\xi}{2\sqrt{-1-\omega^2}}\right) + \left((1+\omega^2)c_3 - \beta c_4 \sqrt{-1-\omega^2}\right) \sinh\left(\frac{\beta\xi}{2\sqrt{-1-\omega^2}}\right)}{\left((1+\omega^2)c_3 - \beta c_4 \sqrt{-1-\omega^2}\right) \cosh\left(\frac{\beta\xi}{2\sqrt{-1-\omega^2}}\right) + \left((1+\omega^2)c_3 + \beta c_4 \sqrt{-1-\omega^2}\right) \sinh\left(\frac{\beta\xi}{2\sqrt{-1-\omega^2}}\right)}^2. \quad (3.2.22)$$

Since c_3 and c_4 are integral constants, we can choose the values $c_3 = 1$ and $c_4 = \frac{\sqrt{-1-\omega^2}}{\beta}$. Therefore, solution (3.2.22) turns out to the following form:

$$w(\xi) = -\tanh^2\left(\frac{\beta\xi}{2\sqrt{-1-\omega^2}}\right). \quad (3.2.23)$$

Again, if we choose the values $c_3 = 1$ and $c_4 = -\frac{\sqrt{-1-\omega^2}}{\beta}$, solution (3.2.22) becomes

$$w(\xi) = -\coth^2\left(\frac{\beta\xi}{2\sqrt{-1-\omega^2}}\right). \quad (3.2.24)$$

Now, if we choose the value of $b_1 = -\frac{4\sqrt{-1-\omega^2}}{\beta}$, we achieve the same result as equations (3.2.21) and (3.2.22).

By using the transformation $w = e^\psi$ and $\xi = x - \omega t$, solutions (3.2.23) and (3.2.24) become

$$\phi(x, t) = \ln\left(-\tanh^2\left(\frac{\beta(x - \omega t)}{2\sqrt{-1-\omega^2}}\right)\right) \quad (3.2.25)$$

and

$$\phi(x, t) = \ln\left(-\coth^2\left(\frac{\beta(x - \omega t)}{2\sqrt{-1-\omega^2}}\right)\right). \quad (3.2.26)$$

These solutions represent the periodic bell-shape, singular bell-shape, bell shape, compacton and other sorts of solutions for atypical values of the constants, which better narrate the Sinh–Poisson equation.

4 Graphical representations

The graphical depictions of analytic solutions of NLEEs exhibit and allow us to look on the variation of internal structure of numerous advanced tangible phenomena,

such as spatial localization of transfer processes, existence of peaking regimes, multiplicity or absence of steady states below varied conditions and many others. However,

general solutions are usually used as peculiar examples to demonstrate the basic principles of the theory, which recognizes the mathematical structure [45]. In this section, we portray different types of solitons originated for different values of parameters. The sorts of the solitons are peakon, compacton, cuspon, periodic solitons, bell-shape soliton, kink waves and others.

4.1 Graphical description of the solution: the nonlinear RLCTL equation

In this section, for certain estimation of the constraints, we interpret the graphical depiction of the attained results to the nonlinear RLCTL equation. The three-dimensional (3D) and two-dimensional (2D) figures of the obtained results to the nonlinear RLCTL are illustrated as follows.

For the specific values of the parameters $b = 1$, $\omega = 1.7$, $l = 2$, $C_0 = 1.2$ and $R_1 = 1.9$, the nature of the result (3.1.14) is the singular kink shape within the limit $-10 \leq x \leq 10$ and $-10 \leq t \leq 10$. The 2D figure of this result for $t = 0$ is illustrated in Figure 1.

The profile of solution (3.1.15) is the steady plane shape for the fixed values $b = -0.37$, $\omega = 1.64$, $l = -0.01$, $C_0 = 1.74$ and $R_1 = 1.91$ of the constraints within the limit $-8 \leq x \leq 8$ and $-8 \leq t \leq 8$. The 2D figure of this result for $t = 0$ is illustrated in Figure 2.

Figure 1: 3D graph of the result (3.1.14) with $b = 1$, $\omega = 1.7$, $l = 2$, $C_0 = 1.2$ and $R_1 = 1.9$.

Figure 2: 3D and 2D graphs of the result (3.1.15) with $b = -0.37$, $\omega = 1.64$, $l = -0.01$, $C_0 = 1.74$ and $R_1 = 1.91$.

Again, solution (3.1.15) provides the spike-like soliton for the particular values $b = -0.23$, $\omega = -1.88$, $l = 0.05$, $C_0 = 0.77$ and $R_1 = 0.47$ of the constraints into the interim $-8 \leq x \leq 8$ and $-8 \leq t \leq 8$. The 2D figure of this result for $t = 0$ is illustrated in Figure 3.

The 2D and 3D patterns of the result (3.1.16) are obtained for the definite values $b = 1.83$, $\omega = 1.85$, $l = 0.26$, $C_0 = -0.31$ and $R_1 = 0.11$. The 3D profile of this solution is kink-shaped soliton, which is descending from one asymptotic line to another asymptotic line. The 2D graph of this solution is traced at $t = 0$ (Figure 4).

Alternatively, we achieve 2D and 3D patterns of the result (3.1.16) for the definite values $b = 1.83$, $\omega = 1.62$, $l = 1.04$, $C_0 = 1.36$ and $R_1 = 1.68$ of the constraints within the limit $-10 \leq t \leq 10$ and $-10 \leq x \leq 10$. The 3D plot of this result is kink soliton which drops from one asymptotic position to other. The 2D figure is traced at $t = 0$ (Figure 5).

The result (3.1.17) gives soliton solution which is peakon type for the values $b = -1.03$, $\omega = -1.03$, $l = 1.59$, $C_0 = 1.64$ and $R_1 = 1.81$ of the parameters. The 2D profile of this result is depicted for the value of $t = 0$ (Figure 6).

Furthermore, solution (3.1.17) results the anti-peakon soliton for the specific values $b = 0.47$, $\omega = -1.03$, $l = 1.59$, $C_0 = 1.64$ and $R_1 = 1.81$ of the parameters. The 2D plot of this result is delineated at $t = 0$ (Figure 7).

The 3D shape of solution (3.1.21) for certain values $b = 1.78$, $\omega = 1.85$, $l = -1.9$, $C_0 = 2$ and $R_1 = 1.7$ of the parameters is the singular-kink-type solution within the limit $-10 \leq t \leq 10$ and $-10 \leq x \leq 10$. The 2D graph is plotted at $t = 0$ (Figure 8).

The 3D shape of the result (3.1.22) gives the soliton solution which is a singular solution for the specific values of the parameters $b = 1.13$, $\omega = 1.13$, $l = 1.3$, $C_0 = 0.37$ and $R_1 = 1.53$ with the interval $-8 \leq t \leq 8$ and $-8 \leq x \leq 8$. The 2D plot is depicted for the value of $t = 0$ (Figure 9).

Figure 3: 3D and 2D graphs of the result (3.1.15) with $b = -0.23$, $\omega = -1.88$, $l = 0.05$, $C_0 = 0.77$ and $R_1 = 0.47$

Figure 4: 3D and 2D graphs of the result (3.1.16) with $b = 1.83$, $\omega = 1.85$, $l = 0.26$, $C_0 = -0.31$ and $R_1 = 0.11$.

Figure 5: 3D and 2D graphs of the result (3.1.16) with $b = 1.83$, $\omega = 1.62$, $l = 1.04$, $C_0 = 1.36$ and $R_1 = 1.68$.

The result (3.1.23) of the RLCTL is the soliton solution which is kink-type solution for the values of the parameters $b = 1.66$, $\omega = -1.07$, $l = 0.11$, $C_0 = -0.12$ and $R_1 = 0.2$. The kink soliton descends from the left asymptotic

position to the right position. The 2D plot of the result is plotted at $t = 0$ (Figure 10).

The result (3.1.24) of the RLCTL gives the soliton solution, which is kink-type for the parameters $b = -0.35$,

Figure 6: 2D and 3D graphs of the result (3.1.17) with $b = -1.03$, $\omega = -1.03$, $l = 1.59$, $C_0 = 1.64$ and $R_1 = 1.81$.

Figure 7: 3D and 2D graphs of the result (3.1.17) with $b = 0.47$, $\omega = -1.03$, $l = 1.59$, $C_0 = 1.64$ and $R_1 = 1.81$.

Figure 8: 2D and 3D graphs of the result (3.1.21) with $b = 1.78$, $\omega = 1.85$, $l = -1.9$, $C_0 = 2$ and $R_1 = 1.7$.

Figure 9: 3D and 2D graphs of the result (3.1.22) with $b = 1.13$, $\omega = 1.13$, $l = 1.3$, $C_0 = 0.37$ and $R_1 = 1.53$.

Figure 10: 3D and 2D graphs of the result (3.1.23) with $b = 1.66$, $\omega = -1.07$, $l = 0.11$, $C_0 = -0.12$ and $R_1 = 0.2$.

$\omega = 1.04$, $l = 1.72$, $C_0 = -0.14$ and $R_1 = -0.25$. The 2D plot of this result is obtained at $t = 0$ (Figure 11).

It is observed that the results of nonlinear RLCTL equation give the singular kink, singular bell-shape, anti-kink, kink, steady plane, peakon, anti-peakon and other types of solitons for the diverse values of the parameters.

4.2 Graphical description of the solution: the Sinh–Poisson equation

In this section, we portray the pictorial representations of the established solutions for various assessments of the constraints to the Sinh–Poisson equation. The forms of these solutions are singular bell-shape soliton, compacton, bell-shape and periodic singular-type solitons.

Compacton is another type of soliton with conservative spatial help to such an extent that each compacton is a soliton restricted to a limited center. Compactons are characterized by solitary waves with the noteworthy soliton property that subsequent to crashing into different compactons, they reappear with the equivalent cognizant shape. This molecule like waves displays flexible impact that is like the soliton crash. It was found that a compacton is a single wave with a reduced help where the nonlinear dispersion limits it to a limited center, and along these lines the exponential wings evaporate. The 2D and 3D graphs of the results of the Sinh–Poisson equation are illustrated as follows.

The solution (3.2.19) provides the soliton solution which is the singular bell shape for the definite values of the parameters $\beta = 0.39$ and $\omega = 1.55$. The 2D plot of this result is depicted within the limit $-10 \leq t \leq 10$ and $-10 \leq x \leq 10$ at $t = 0$ (Figure 12).

Figure 11: 2D and 3D graphs of the result (3.1.24) with $b = -0.35$, $\omega = 1.04$, $l = 1.72$, $C_0 = -0.14$ and $R_1 = -0.25$.

Figure 12: 3D and 2D graphs of the result (3.2.19) with $\beta = 0.39$, $\omega = 1.55$.

Again, the result (3.2.19) provides the singular-type solution, which is a singular bell-shaped solution for another value of parameter $\beta = 2$ with travelling speed $\omega = 2$. The plane shape of this result is attained at $t = 0$ (Figure 13).

The 3D shape of the result (3.2.20) is the soliton solution which is the bell shape for the value $\beta = 1.15$ with wave velocity $\omega = 0.41$. The 2D plot is sketched at $t = 0$ within the limit $-8 \leq (x, t) \leq 8$ (Figure 14).

Once again, the structure of the figure of the result (3.2.20) is a soliton-type solution, which is an anti-bell shaped solution. The 3D figure is sketched for the particular value $\beta = 3.02$ with travelling speed $\omega = 2$ within the limit $-8 \leq t \leq 8$ and $-8 \leq x \leq 8$. The 2D figure is drawn at $t = 0$ (Figure 15).

The solution (3.2.25) provides the compacton for the fixed value $\omega = 1.23$ and $\beta = 0.07$ of the constraints and portrayed within the limit $-10 \leq t \leq 10$ and $-10 \leq x \leq 10$. The 2D figure is drawn at $t = 0$ (Figure 16).

The solution (3.2.25) provides the soliton solution for the certain value of $\beta = -0.24$ and $\omega = 0.82$ within the limit $-8 \leq t \leq 8$ and $-8 \leq x \leq 8$. The 2D plot of this result is constructed at $t = 0$ (Figure 17).

The solution (3.2.25) accepts the soliton solution for the certain value $\beta = -2.14$ and $\omega = -4$ of the constraint and sketched into the limit $-8 \leq t \leq 8$ and $-8 \leq x \leq 8$. The 2D plot of this result is attained at $t = 0$ (Figure 18).

The result (3.2.26) of the Sinh–Poisson equation represents the soliton solution for the value of $\beta = 0.57$ and wave speed $\omega = 0.74$. The 2D plot of the result with the interval $-8 \leq (x, t) \leq 8$ is attained at $t = 0$ (Figure 19).

The result (3.2.26) of the Sinh–Poisson equation represents the soliton solution for the value of $\beta = 0.14$ and wave speed $\omega = 0.91$. The 2D plot of this result with the interval $-8 \leq t \leq 8$ and $-8 \leq x \leq 8$ is attained at $t = 0$ (Figure 20).

Finally, we observe that the Sinh–Poisson equation provides the bell shape, bell-shape compacton, singular

Figure 13: 3D and 2D graphs of the result (3.2.19) with $\beta = 2$, $\omega = 2$.

Figure 14: 3D and 2D graphs of the result (3.2.20) for $\beta = 1.15$, $\omega = 0.41$.

Figure 15: 3D and 2D graphs of the result (3.2.20) for $\beta = 3.02$, $\omega = 2$.

Figure 16: 2D and 3D graphs of the result (3.2.25) for $\beta = 0.07$, $\omega = 1.23$.

Figure 17: 2D and 3D graphs of the result (3.2.25) for $\beta = -0.24$, $\omega = 0.82$.

Figure 18: 3D and 2D graphs of the result (4.4.25) for $\beta = -2.14$, $\omega = -4$.

Figure 19: 2D and 3D graphs of the result (3.2.26) for $\beta = 0.57$, $\omega = 0.74$.

Figure 20: 2D and 3D graphs of the result (3.2.26) for $\beta = 0.14$, $\omega = 0.91$.

soliton, periodic and several types of solitons for different values of the parameters.

5 Conclusion

The Sinh–Poisson equation and the nonlinear RLCTL equation have been considered in this article, and the MSE method is put forth to establish stable soliton solutions in terms of hyperbolic function and trigonometric function. Choosing arbitrary values of the free parameters, diverse types of known solitary wave solutions, videlicet, the kink-shaped soliton, singular kink solutions, bell-shape soliton steady plane, peakon, compacton, singular periodic wave solutions and several types are ascertained. It is noteworthy to notice that through the MSE method the values of the free parameters are determined not using computer

algebra software, such as Mathematica or Maple. It can be figured out that the MSE method is unified, more powerful and can touch many other NLEEs of physics, applied mathematics and engineering without the aid of any auxiliary equation. This study affirms that the MSE method is capable of extracting compatible and stable soliton solutions to other NLEEs.

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