Research Article

Gilbert Makanda*, Vusi Mpendulo Magagula, Precious Sibanda, and Sandile Sydney Motsa

Numerical analysis of free convection from a spinning cone with variable wall temperature and pressure work effect using MD-BSQLM

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Abstract: The problem of the numerical analysis of natural convection from a spinning cone with variable wall temperature, viscous dissipation and pressure work effect is studied. The numerical method used is based on the spectral analysis. The method used to solve the system of partial differential equations is the multi-domain bivariate spectral quasi-linearization method (MD-BSQLM). The numerical method is compared with other methods in the literature, and the results show that the MD-BSQLM is robust and accurate. The method is also stable for large parameters. The numerical errors do not deteriorate with increasing iterations for different values of all parameters. The numerical error size is of the order of 10^{-10} . With the increase in the suction parameter ξ , fluid velocity, spin velocity and temperature profiles decrease.

Keywords: numerical analysis, spectral methods, multi-domain, error analysis

1 Introduction

The study of numerical methods in fluid flow has been limited to a very small number of methods used in the past several decades. The common methods that have been used

are the finite difference based methods such as Keller-box, finite element and finite volume. Other finite-based methods are commercially developed methods such as computational fluid dynamics (CFD) [1]. Other methods that are common are analytical methods, and these methods are only applied in simple cases arising in fluid flow models.

Analytical methods have been used in many studies, these include the work of Makinde [2], in which a simple model of entropy generation analysis was considered. Rees et al. [3] worked on an analytical solution for boundary layer flows for Bingham fluids. Other analytical solutions appear in the works involving unsteady fluid flow of third grade fluid [4] and in other non-Newtonian fluids over two-dimensional porous media [5]. Silva et al. [6] used analytical solution to study power law fluids in porous media. With the increase in complexity of fluid mathematical models, analytical solutions are becoming rarely used. The use of numerical methods has been on the increase.

Numerical methods have been used in many studies in fluid flow, which include the work of Cheng [7] in which the cubic spline method was used. Altun et al. [8] used the finite difference approach with discretization using the central difference scheme. The work of Ashari and Tafreshi [9] made use of FlexPDE, which is a commercial program based on finite element method. Other works in which finite difference methods were used include Ece [10] who used the Thomas algorithm on the flow around spinning cone, Haddout et al. [11] in fluid flow involving viscous dissipation and pressure work, Javaherdeh et al. [12] in free convection fluid flow on a moving vertical plate, Kefayati [13] used the finite difference lattice Boltzmann method, Rosali et al. [14] used the Keller-box method in the study of flow of micropolar fluids and Teymourash et al. [15] used the finite difference method in free convection in supercritical fluids. The homotopy analysis method was used in the study of fluid flow by Dinarvand et al. [16], Tyilmazoglu [17], Abd Elmaboud et al. [18] and others. Haque et al. [19] used the Nachtshim–Swigert iteration technique in the study of MHD fluid flow in micropolar fluid.

* **Corresponding author: Gilbert Makanda**, Department of Mathematical and Physical Sciences, Central University of Technology, Free State, Private Bag X20539, South Africa, e-mail: gilbertmakanda@yahoo.com

Vusi Mpendulo Magagula: School of Mathematics, Statistics and Computer Science, University of KwaZulu Natal, South Africa, e-mail: gutjwa@gmail.com

Precious Sibanda: School of Mathematics, Statistics and Computer Science, University of KwaZulu Natal, South Africa, e-mail: sibandap@ukzn.ac.za

Sandile Sydney Motsa: School of Mathematics, Statistics and Computer Science, University of KwaZulu Natal, South Africa, e-mail: sandilemotsa@gmail.com

Some studies in fluid flow are based on results from actual laboratory experiments. These studies include among others the work of Markicevic *et al.* [20] who studied capillary force driven flows in porous medium and Pinar *et al.* [21] who investigated flow structure around shallow waters.

In all the aforementioned studies, none of them used spectral-based methods. There are some recently developed spectral-based methods such as the spectral quasi-linearization method (SQLM), BQLM, bivariate local linearization methods studies, among others [22–24]. These methods are based on approximating the derivative of functions at collocation points by the Lagrange or Chebyshev polynomials. The procedure is less rigorous than in the case of finite difference methods. In this study, the system of partial differential equations (PDEs) is solved using the multi-domain bivariate quasi-linearization method (MD-BSQLM).

This article is organized as follows: in Section 2, mathematical formulation of the problem is given. In Section 3, the method of solution in which the solution method is described is given. In Section 4, results and discussion are given, and finally in Section 5 the conclusion is given.

2 Mathematical formulation

A spinning cone in a Newtonian fluid is considered and maintained at a variable temperature $T_0 = T_\infty + x^n$ and

Figure 1: Physical model and coordinate system.

the ambient conditions are maintained at T_∞ . The velocity components u , v and w are in the directions of x , y and z , respectively, and the x -axis being inclined at an angle γ to the vertical. Ω is the angular velocity as shown in Figure 1. The effects of viscous dissipation and porous medium are considered.

The governing equations in this buoyant-driven flow are given as:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \quad (1)$$

$$\begin{aligned} u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \frac{w^2}{x} \\ = v \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)\cos\gamma - \Gamma u^2 - \frac{\nu}{K}u, \end{aligned} \quad (2)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} - \frac{uw}{x} = v \frac{\partial^2 w}{\partial y^2} - \Gamma w^2 - \frac{\nu}{K}w, \quad (3)$$

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \\ = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{T\beta}{\rho C_p} u \frac{\partial p}{\partial x} + \frac{\nu}{C_p} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right], \end{aligned} \quad (4)$$

where $r = x \sin \gamma$, g is the acceleration due to gravity, ν is the kinematic viscosity for the fluid, β is the coefficient of thermal expansion, α is the thermal diffusivity and C_p is the specific heat capacity at constant pressure. γ is the half vertex angle of the cone, K is the permeability of the porous medium and Γ is the Forchheimer parameter. The boundary conditions are as follows:

$$u = v = 0, \quad w = \Omega r, \quad T_0 = T_\infty + Ax^n \quad \text{at } y = 0, \quad (5)$$

$$\frac{\partial u}{\partial y} \rightarrow 0, \quad u \rightarrow 0, \quad T = T_\infty, \quad \text{as } y \rightarrow \infty, \quad (6)$$

where the subscripts 0 and ∞ refer to the surface and ambient conditions, respectively. The continuity equation is satisfied by the stream function:

$$ru = \partial \psi / \partial y, \quad rv = -\partial \psi / \partial x. \quad (7)$$

We introduce the non-dimensional variables

$$\left. \begin{aligned} \xi &= \frac{V_0 x}{\nu Gr^{\frac{1}{4}}}, \quad \psi = \nu r Gr^{\frac{1}{4}} \left[f + \frac{1}{2} \xi \right], \\ \eta &= \frac{y}{x} Gr^{\frac{1}{4}}, \quad g = \frac{x^2 w}{\nu} Gr^{-\frac{3}{4}}, \\ \theta &= \frac{T - T_\infty}{T_0 - T_\infty} \end{aligned} \right\}, \quad (8)$$

where $Gr = g\beta(T_0 - T_\infty)\cos\gamma/\nu^2$.

Substituting equations (7)–(8) in (1)–(4) gives the following equations:

$$f''' + \frac{n+7}{4}ff'' - \frac{n+1}{2}(f')^2 - \varepsilon^2 g^2 + \theta - k_p f' = \frac{1-n}{4}\xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right), \quad (9)$$

$$g'' + \frac{n+7}{4}fg' - \frac{1+3n}{4}f'g + \xi g' - k_p g = \frac{1-n}{4}\xi \left(f' \frac{\partial g}{\partial \xi} - g' \frac{\partial f}{\partial \xi} \right), \quad (10)$$

$$\frac{1}{\text{Pr}}\theta'' + \frac{n+7}{4}f\theta' - (n+\varepsilon_0)f'\theta + \xi\theta' + \text{Ec}[f''^2 + g'^2] = \frac{1-n}{4}\xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right), \quad (11)$$

with boundary conditions:

$$f(0) = f'(0) = 0, \quad g(0) = 1, \quad \theta(0) = 1, \quad f'(\infty) \rightarrow 0, \quad g(\infty) \rightarrow 0, \quad \theta(\infty) \rightarrow 0, \quad (12)$$

where n is the surface temperature exponent or temperature gradient parameter. $\varepsilon = ReR/XGr^{\frac{1}{2}}$ is the spin parameter, $k_p = 1/DaGr^{\frac{1}{2}}$ is the Darcian-drag coefficient, Pr is the Prandtl number, Ec is the Eckert number. $\varepsilon_0 = g\beta X/C_p$ is the pressure work parameter. $\xi = V_0 x/\nu Gr^{\frac{1}{2}}$ is the suction parameter. For the case $\xi = 0$ and $n = 1$, equations (9)–(12) become the equations of ref. [10]. X is the distance from the leading edge of the spinning cone. The velocities in the x and y directions may be expressed as:

$$v = -\frac{\nu Gr^{\frac{1}{2}}}{X} \left[\frac{n+7}{4}f + \frac{1-n}{4}\xi \frac{\partial f}{\partial \xi} - \frac{1-n}{4}\eta f' + \xi \right], \quad (13)$$

$$u = \frac{\nu Gr^{\frac{1}{2}}f'}{X}.$$

3 Method of solution

In this section, we describe the implementation of the MD-BSQLM. The method considers the use of several domains as compared to the traditional single domain system. In general, the system of non-linear PDEs is given as

$$\Gamma_1[F_1, F_2, \dots, F_n] = 0, \quad (14)$$

$$\Gamma_2[F_1, F_2, \dots, F_n] = 0, \quad (15)$$

$$\vdots \quad (16)$$

$$\Gamma_n[F_1, F_2, \dots, F_n] = 0. \quad (17)$$

The operators are of the form:

$$F_1 = \{f_1, f'_1, f''_1, \dots, f_1^p, f_{1\xi}, f'_{1\xi}\}, \quad (18)$$

$$F_2 = \{f_2, f'_2, f''_2, \dots, f_2^p, f_{2\xi}, f'_{2\xi}\}, \quad (19)$$

$$\dots \quad (20)$$

$$F_n = \{f_n, f'_n, f''_n, \dots, f_n^p, f_{n\xi}, f'_{n\xi}\}. \quad (21)$$

The primes refer to the derivative with respect to η and the subscript ξ refer to the derivative with respect to ξ . p is the order of derivative. The required solution is expressed as $f_k(\eta, \xi)$ and Γ_k for $k = 1, 2, \dots, n$ are the linear operators with all spatial derivatives of $f_k(\eta, \xi)$. The Gauss–Lobatto points and differentiation matrix are defined on $[-1, 1]$ and the spatial regions ξ -interval $\xi = [0, \eta_0]$ and the $\eta \in [a, b] \rightarrow [-1, 1]$.

To apply the MD-BSQLM, we decompose the interval for $\xi \in [0, \eta_0]$ in subintervals

$$\omega_l = [\xi_{l-1}, \xi_l] = \{[\xi_0, \xi_1], [\xi_1, \xi_2], \dots, [\xi_{q-1}, \xi_q]\},$$

each interval of ω_l is separated by Gauss–Lobatto points in which the PDE is solved with solutions $f_k^l(\eta, \xi)$ and the initial conditions $f_k^l(\eta, 0)$ for $k = 1, 2, \dots, n$. In each interval, the solution is approximated by the bivariate Lagrange interpolation polynomial:

$$f_k^{(l)}(\eta, \xi) = \sum_{i=0}^{N_\eta} \sum_{j=0}^{N_\xi} f_k^{(l)}(\eta_i, \xi_j) \mathcal{L}_i(\eta) \mathcal{L}_j(\xi) \quad (22)$$

for $k = 1, 2, \dots, n$, $l = 1, 2, \dots, p$.

The full description of this method is described in the study by Magagula et al. [22]. Applying the method with $p = 3$ and $n = 3$.

$$\sum_{v=1}^3 \left[A_{1,v}^{(i,l)} F_{v,i}^{(l)} + \beta_{v,r}^{(l,1)} \sum_{j=0}^{N_\xi} d_{i,j} F_{v,j}^l + \gamma_{v,r}^{(l,1)} \sum_{j=0}^{N_\xi} d_{i,j} D F_{v,j}^l \right] = R_{1,i}^l, \quad (23)$$

$$\sum_{v=1}^3 \left[A_{2,v}^{(i,l)} F_{v,i}^{(l)} + \beta_{v,r}^{(l,2)} \sum_{j=0}^{N_\xi} d_{i,j} F_{v,j}^l + \gamma_{v,r}^{(l,2)} \sum_{j=0}^{N_\xi} d_{i,j} D F_{v,j}^l \right] = R_{2,i}^l, \quad (24)$$

$$\sum_{v=1}^3 \left[A_{3,v}^{(i,l)} F_{v,i}^{(l)} + \beta_{v,r}^{(l,3)} \sum_{j=0}^{N_\xi} d_{i,j} F_{v,j}^l + \gamma_{v,r}^{(l,3)} \sum_{j=0}^{N_\xi} d_{i,j} D F_{v,j}^l \right] = R_{3,i}^l, \quad (25)$$

where

$$A_{1,1}^{(i,l)} = \sum_{s=0}^3 \alpha_{1,s,r}^{(l,1)} \mathbf{D}^{(s)} = \alpha_{1,3,r}^{(l,1)} \mathbf{D}^{(3)} + \alpha_{1,2,r}^{(l,1)} \mathbf{D}^{(2)} + \alpha_{1,1,r}^{(l,1)} \mathbf{D}^{(1)} + \alpha_{1,0,r}^{(l,1)} \mathbf{I}, \quad (26)$$

$$A_{1,2}^{(i,l)} = \sum_{s=0}^3 \alpha_{2,s,r}^{(l,1)} \mathbf{D}^{(s)} = \alpha_{2,3,r}^{(l,1)} \mathbf{D}^{(3)} + \alpha_{2,2,r}^{(l,1)} \mathbf{D}^{(2)} + \alpha_{2,1,r}^{(l,1)} \mathbf{D}^{(1)} + \alpha_{2,0,r}^{(l,1)} \mathbf{I}, \quad (27)$$

$$A_{1,3}^{(i,l)} = \sum_{s=0}^3 \alpha_{3,s,r}^{(l,1)} \mathbf{D}^{(s)} \quad (28)$$

$$= \alpha_{3,3,r}^{(l,1)} \mathbf{D}^{(3)} + \alpha_{3,2,r}^{(l,1)} \mathbf{D}^{(2)} + \alpha_{3,1,r}^{(l,1)} \mathbf{D}^{(1)} + \alpha_{3,0,r}^{(l,1)} \mathbf{I},$$

$$A_{2,1}^{(i,l)} = \sum_{s=0}^3 \alpha_{1,s,r}^{(l,2)} \mathbf{D}^{(s)} \quad (29)$$

$$= \alpha_{1,3,r}^{(l,2)} \mathbf{D}^{(3)} + \alpha_{1,2,r}^{(l,2)} \mathbf{D}^{(2)} + \alpha_{1,1,r}^{(l,2)} \mathbf{D}^{(1)} + \alpha_{1,0,r}^{(l,2)} \mathbf{I},$$

$$A_{2,2}^{(i,l)} = \sum_{s=0}^3 \alpha_{2,s,r}^{(l,2)} \mathbf{D}^{(s)} \quad (30)$$

$$= \alpha_{2,3,r}^{(l,2)} \mathbf{D}^{(3)} + \alpha_{2,2,r}^{(l,2)} \mathbf{D}^{(2)} + \alpha_{2,1,r}^{(l,2)} \mathbf{D}^{(1)} + \alpha_{2,0,r}^{(l,2)} \mathbf{I},$$

$$A_{2,3}^{(i,l)} = \sum_{s=0}^3 \alpha_{3,s,r}^{(l,2)} \mathbf{D}^{(s)} \quad (31)$$

$$= \alpha_{3,3,r}^{(l,2)} \mathbf{D}^{(3)} + \alpha_{3,2,r}^{(l,2)} \mathbf{D}^{(2)} + \alpha_{3,1,r}^{(l,2)} \mathbf{D}^{(1)} + \alpha_{3,0,r}^{(l,2)} \mathbf{I},$$

$$A_{3,1}^{(i,l)} = \sum_{s=0}^3 \alpha_{1,s,r}^{(l,3)} \mathbf{D}^{(s)} \quad (32)$$

$$= \alpha_{1,3,r}^{(l,3)} \mathbf{D}^{(3)} + \alpha_{1,2,r}^{(l,3)} \mathbf{D}^{(2)} + \alpha_{1,1,r}^{(l,3)} \mathbf{D}^{(1)} + \alpha_{1,0,r}^{(l,3)} \mathbf{I},$$

$$A_{3,2}^{(i,l)} = \sum_{s=0}^3 \alpha_{2,s,r}^{(l,3)} \mathbf{D}^{(s)} \quad (33)$$

$$= \alpha_{2,3,r}^{(l,3)} \mathbf{D}^{(3)} + \alpha_{2,2,r}^{(l,3)} \mathbf{D}^{(2)} + \alpha_{2,1,r}^{(l,3)} \mathbf{D}^{(1)} + \alpha_{2,0,r}^{(l,3)} \mathbf{I},$$

$$A_{3,3}^{(i,l)} = \sum_{s=0}^3 \alpha_{3,s,r}^{(l,3)} \mathbf{D}^{(s)} \quad (34)$$

$$= \alpha_{3,3,r}^{(l,3)} \mathbf{D}^{(3)} + \alpha_{3,2,r}^{(l,3)} \mathbf{D}^{(2)} + \alpha_{3,1,r}^{(l,3)} \mathbf{D}^{(1)} + \alpha_{3,0,r}^{(l,3)} \mathbf{I}.$$

The system of PDEs (9)–(12) is written in the form:

$$\Gamma_1 = f''' + \frac{n+7}{4} f f'' - \frac{n+1}{2} (f')^2 - \varepsilon^2 g^2 \quad (35)$$

$$+ \theta - k_p f' - \frac{1-n}{4} \xi \left(f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right),$$

$$\Gamma_2 = g'' + \frac{n+7}{4} f g' - \frac{1+3n}{4} f' g + \xi g' - k_p g \quad (36)$$

$$- \frac{1-n}{4} \xi \left(f' \frac{\partial g}{\partial \xi} - g' \frac{\partial f}{\partial \xi} \right),$$

$$\Gamma_3 = \frac{1}{\text{Pr}} \theta'' + \frac{n+7}{4} f \theta' - (n + \varepsilon_0) f' \theta + \xi \theta' \quad (37)$$

$$+ \text{Ec} [f''^2 + g'^2] - \frac{1-n}{4} \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right),$$

where the coefficients are

$$\alpha_{1,3,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{1,r}^{(3,l)}} = 1, \quad (38)$$

$$\alpha_{1,2,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{1,r}^{(2,l)}} = \left(\frac{n+7}{4} \right) f_{1,r}^{(0,l)} + \left(\frac{1-n}{4} \right) \xi \frac{\partial f_{1,r}^{(0,l)}}{\partial \xi}, \quad (39)$$

$$\alpha_{1,1,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{1,r}^{(1,l)}} \quad (40)$$

$$= -(n+1) f_{1,r}^{(1,l)} - K_p - \left(\frac{1-n}{4} \right) \xi \frac{\partial f_{1,r}^{(1,l)}}{\partial \xi},$$

$$\alpha_{1,0,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{1,r}^{(0,l)}} = - \left(\frac{n+7}{4} \right) f_{1,r}^{(2,l)}, \quad (41)$$

$$\beta_{1,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial [f_{1,r}^{(0,l)}]_\xi} = \left(\frac{1-n}{4} \right) \xi f_{1,r}^{(2,l)}, \quad (42)$$

$$\gamma_{1,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial [f_{1,r}^{(1,l)}]_\xi} = - \left(\frac{1-n}{4} \right) \xi f_{1,r}^{(1,l)}, \quad (43)$$

$$\alpha_{2,3,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{2,r}^{(3,l)}} = 0, \quad \alpha_{2,2,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{2,r}^{(2,l)}} = 0, \quad (44)$$

$$\alpha_{2,1,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{2,r}^{(1,l)}} = 0, \quad (45)$$

$$\alpha_{2,0,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{2,r}^{(0,l)}} = -2\varepsilon^2 f_{2,r}^{(0,l)}, \quad (46)$$

$$\alpha_{3,3,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{3,r}^{(3,l)}} = 0, \quad \alpha_{3,2,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{3,r}^{(2,l)}} = 0, \quad (47)$$

$$\alpha_{3,1,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{3,r}^{(1,l)}} = 0, \quad \alpha_{3,0,r}^{(l,1)} = \frac{\partial \Gamma_1}{\partial f_{3,r}^{(0,l)}} = 1. \quad (48)$$

The equations in (9)–(12) are written as follows:

$$\alpha_{1,3,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{1,r}^{(3,l)}} = 0, \quad \alpha_{1,2,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{1,r}^{(2,l)}} = 0, \quad (49)$$

$$\alpha_{1,1,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{1,r}^{(1,l)}} = - \left(\frac{1+3n}{4} \right) f_{2,r}^{(0,l)} - \left(\frac{1-n}{4} \right) \xi \frac{\partial f_{2,r}^{(0,l)}}{\partial \xi}, \quad (50)$$

$$\alpha_{1,0,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{1,r}^{(0,l)}} = \left(\frac{n+7}{4} \right) f_{2,r}^{(1,l)}, \quad (51)$$

$$\beta_{1,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial [f_{2,r}^{(0,l)}]_\xi} = \left(\frac{1-n}{4} \right) \xi f_{2,r}^{(1,l)}, \quad (52)$$

$$\gamma_{1,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial [f_{1,r}^{(1,l)}]_\xi} = - \left(\frac{1-n}{4} \right) \xi f_{1,r}^{(1,l)}, \quad (53)$$

$$\alpha_{2,3,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{2,r}^{(3,l)}} = 0, \quad \alpha_{2,2,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{2,r}^{(2,l)}} = 1, \quad (54)$$

$$\alpha_{2,1,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{2,r}^{(1,l)}} = \xi + \left(\frac{n+7}{4} \right) f_{1,r}^{(0,l)} + \left(\frac{1-n}{4} \right) \xi \frac{\partial f_{1,r}^{(0,l)}}{\partial \xi}, \quad (55)$$

Table 1: Comparison of the skin friction and heat transfer coefficients for the results of Ece [10], SQLM and MD-SQLM for $\xi = \text{Ec} = \varepsilon = \varepsilon_0 = Kp = 0$, $n = 1$, $\text{Pr} = 1$

q	Ece [10]		Makanda [26] (SQLM)		MD-BSQLM	
	$f''(\eta, \xi)$	$-\theta(\eta, \xi)$	$f''(\eta, \xi)$	$-\theta(\eta, \xi)$	$f''(\eta, \xi)$	$-\theta(\eta, \xi)$
1	0.681483	0.638855	0.68148334	0.63885473	0.68148022	0.63885087
10					0.68148334	0.63885473

$$\alpha_{2,0,r}^{(l,1)} = \frac{\partial \Gamma_2}{\partial f_{2,r}^{(0,l)}} = -\left(\frac{1+3n}{4}\right)f_{1,r}^{(1,l)} - Kp, \quad (56)$$

$$\alpha_{3,3,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{3,r}^{(3,l)}} = 0, \quad \alpha_{3,2,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{3,r}^{(2,l)}} = 0, \quad (57)$$

$$\alpha_{3,1,r}^{(l,2)} = \frac{\partial \Gamma_2}{\partial f_{3,r}^{(1,l)}} = 0, \quad \alpha_{3,0,r}^{(l,1)} = \frac{\partial \Gamma_2}{\partial f_{3,r}^{(0,l)}} = 0, \quad (58)$$

$$\alpha_{1,3,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{1,r}^{(3,l)}} = 0, \quad (59)$$

$$\alpha_{1,2,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{1,r}^{(2,l)}} = 2\text{Ec}f_{1,r}^{(2,l)}, \quad (60)$$

$$\alpha_{1,1,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{1,r}^{(1,l)}} = -(n + \varepsilon_0)f_{3,r}^{(0,l)} - \left(\frac{1-n}{4}\right)\xi \frac{\partial f_{3,r}^{(0,l)}}{\partial \xi}, \quad (61)$$

$$\alpha_{1,0,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{1,r}^{(0,l)}} = \left(\frac{n+7}{4}\right)f_{3,r}^{1,l}, \quad (62)$$

$$\beta_{1,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial [f_{1,r}^{(0,l)}]_\xi} = \left(\frac{1-n}{4}\right)\xi f_{3,r}^{(1,l)}, \quad (63)$$

$$\gamma_{1,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial [f_{3,r}^{(0,l)}]_\xi} = -\left(\frac{1-n}{4}\right)\xi f_{1,r}^{(1,l)}, \quad (64)$$

$$\alpha_{2,3,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{2,r}^{(3,l)}} = 0, \quad \alpha_{2,2,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{2,r}^{(2,l)}} = 0, \quad (65)$$

$$\alpha_{2,1,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{2,r}^{(1,l)}} = 2\text{Ec}f_{2,r}^{(1,l)}, \quad (66)$$

$$\alpha_{2,0,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{2,r}^{(0,l)}} = 0, \quad (67)$$

$$\alpha_{3,3,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{3,r}^{(3,l)}} = 0, \quad \alpha_{3,2,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{3,r}^{(2,l)}} = \frac{1}{\text{Pr}}, \quad (68)$$

$$\alpha_{3,1,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{3,r}^{(1,l)}} = \left(\frac{n+7}{4}\right)f_{1,r}^{(0,l)} + \left(\frac{1-n}{4}\right)\xi \frac{\partial f_{1,r}^{(0,l)}}{\partial \xi}, \quad (69)$$

$$\alpha_{3,0,r}^{(l,3)} = \frac{\partial \Gamma_3}{\partial f_{3,r}^{(0,l)}} = -(n + \varepsilon_0)f_{1,r}^{(1,l)}. \quad (70)$$

Using the matrix system described in Magagula et al. [22], the solution to the systems (9)–(12) is obtained.

4 Results and discussion

The problem considered in this article is a coupled system of PDEs (9)–(12) describing fluid flow around a spinning cone. In this section, the effect of changing temperature exponent n , the spin parameter ε , the pressure work effect ε_0 , the Prandtl number Pr , Darcian-drag force term Kp and the Eckert number Ec on spin velocity, fluid velocity and temperature profiles is studied. The cases $\xi = 0$, $n = 1$, $Kp = 0$, $\text{Ec} = \varepsilon = \varepsilon_0 = 0$, $\text{Pr} = 0.72$ reduce the system of equations (9)–(12) to those of ref. [10]. The MD-BSQLM is applied to solve the system of PDEs, and the method is validated by comparison to other methods in the literature as shown in Table 1. The results show that the method used is in agreement with the results of Ece [10] who used the Thomas algorithm and Makanda et al. [26] who used the successive linerization method. q is the number of domains.

In Table 1, it is shown that using more than one domain resulted in obtaining accurate results.

Figure 2 depicts the effect of increasing ξ on the velocity profiles. Increasing the suction parameter ξ results in the decrease in fluid velocity. The MD-BSQLM admits the

Figure 2: Effect of varying ξ on velocity profiles for $n = 1$, $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $\text{Pr} = \text{Ec} = 1$.

Figure 3: Effect of varying ξ on spin velocity profiles for $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = n = 1$.

use of large parameters for the values of ξ . The result is in excellent agreement with those obtained in ref. [7].

Figure 3 shows the effect of increasing ξ on the spin velocity profiles. The increase in the suction parameter results in the decrease in spin velocity profiles. Large parameter values are still admitted in this case.

Figure 4 depicts the effect of increasing ξ on the temperature profiles, which results in the decrease in temperature profiles. The result is in excellent agreement with those obtained in ref. [7]. The solutions for the temperature profiles do not diverge to large values of ξ .

In this study, it is of interest to investigate the effect of varying the suction parameter ξ on the errors of the

Figure 4: Effect of varying ξ on temperature profiles for $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = n = 1$.

Figure 5: Effect of varying ξ on E_f for $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = n = 1$.

solutions for the $f(\xi, \eta)$, $g(\xi, \eta)$ and $\theta(\xi, \eta)$. The results are shown in Figures 5–7.

In Figures 5–7, it is shown that increasing ξ does not degenerate the errors of $f(\xi, \eta)$, $g(\xi, \eta)$ and $\theta(\xi, \eta)$. Large parameter values for ξ show the stability of the MD-BSQLM.

In this study, the effect of varying the temperature exponent n and the spin parameter ε on the errors for $f(\xi, \eta)$, $g(\xi, \eta)$ and $\theta(\xi, \eta)$ is pertinent. The results are shown in the form of graphs in Figures 8–13.

Figures 8 and 9 show that the errors for $f(\xi, \eta)$ do not deteriorate for the values of $n = 0, 0.5, 0.8, 1$, and the values of $n > 1$ leads to the divergence of the numerical method used, this is consistent with the values used in

Figure 6: Effect of varying ξ on E_g for $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = n = 1$.

Figure 7: Effect of varying ξ on E_θ for $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = n = 1$.

Figure 10: Effect of varying n on E_g for $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = 1$.

Figure 8: Effect of varying n on E_f for $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = 1$.

Figure 9: Effect of varying ε on E_f for $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = 1$.

ref. [25]. The values of ε are also admitted up to 0.9, which is consistent with the values used in ref. [10]. Values more than this would mean a large spinning velocity resulting in the divergence of solutions of $f(\xi, \eta)$.

Figures 10 and 11 depict that the errors for $g(\xi, \eta)$ do not fluctuate for the values of $n = 0, 0.5, 0.8, 1$, and the values of $n > 1$ leads to the divergence of the numerical method used. The values of ε are also admitted up to 0.9. Increasing ε results in less fluctuations in the errors with increasing iterations for the solutions of $g(\xi, \eta)$.

In Figures 12–13, the error graphs for $\theta(\eta, \xi)$ show that the error does not change much with the increase in the temperature exponent parameter n and the spin parameter ε as iterations increase. This shows the stability and accuracy of the MD-BSQLM.

Figure 11: Effect of varying ε on E_g for $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = n = 1$.

Figure 12: Effect of varying n on E_θ for $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = 1$.

Figure 13: Effect of varying n on E_θ for $\varepsilon = 0.5$, $\varepsilon_0 = 0.5$, $Kp = 0.7$, $Pr = Ec = 1$.

5 Conclusion

A spinning cone with variable surface temperature, suction and pressure work effect was considered. A system of PDEs describing the fluid flow around a spinning cone was solved using the MD-BSQLM. The numerical method was shown to be robust and accurate by comparison with other results obtained in the literature. Considering more domains makes this method unique and accurate. The method can be preferred in solving PDEs arising in fluid flow with large parameters.

The effects of increasing the suction parameter involving large values resulted in the decrease of velocity, spin velocity and temperature profiles. The errors for all the

solutions for $f(\xi, \eta)$, $g(\xi, \eta)$ and $\theta(\xi, \eta)$ did not deteriorate with increasing iterations for larger values of the suction parameter ξ . Increasing the surface temperature exponent and the spin parameter showed that the errors for the solutions of $f(\xi, \eta)$, $g(\xi, \eta)$ and $\theta(\xi, \eta)$ did not change much with increasing iterations. Using more than one domain results in accurate results. This shows that the MD-BSQLM is robust and accurate. This method can be recommended for use in PDEs arising in fluid flow with large parameters.

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