Research Article

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MHD squeezed Darcy–Forchheimer nanofluid flow between two h -distance apart horizontal plates

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Abstract: This research is mainly concerned with the characteristics of magnetohydrodynamics and Darcy–Forchheimer medium in nanofluid flow between two horizontal plates. A uniformly induced magnetic impact is involved at the direction normal to the lower plate. Darcy–Forchheimer medium is considered between the plates that allow the flow along horizontal axis with additional effects of porosity and friction. The features of Brownian diffusive motion and thermophoresis are disclosed. Governing problems are transformed into non-linear ordinary problems using appropriate transformations. Numerical Runge–Kutta procedure is applied using MATLAB to solve the problems and acquire the data for velocity field, thermal distribution, and concentration distribution. Results have been plotted graphically. The outcomes indicate that higher viscosity results in decline in fluid flow. Thermal profile receives a decline for larger viscosity parameter; however, Brownian diffusion and thermophoresis appeared as enhancing factors for the said profile. Numerical data indicate that heat flux reduces for viscosity parameter. However, enhancement is observed in skin-friction for elevated values of porosity factor. Data of this paper are practically helpful in industrial and engineering applications of nanofluids.

Keywords: nanofluid, Darcy–Forchheimer medium, magnetohydrodynamics, squeezed flow

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1 Introduction

Heat and mass convection is a natural phenomenon in every physical situation involving viscous fluid materials. This phenomenon is quite natural in various structures such as between two parallel plates, over stretching surfaces, and within a cylindrical sphere. Therefore, one can define a fluid flowing between two surfaces as squeezing flow. Because of high significance and demand in industrial and other setups, squeezing flow has received special attention from the researchers working on fluid flow, heat, and mass transport analysis. In particular, the branches of fluid dynamics which belong to mechanical and biochemical engineering, food processing, chemical engineering, industrial processing, and many others are typical examples where the concept of squeezing flow is mostly used. In addition, we have seen this in gears, rolling elements, machine devices, lubrication, automotive engines, grease and oil setups, compression, injection, and shaping. The pioneer approach was reported by Stefan [1], which was highly appreciated by the research community. In this research, squeezing flow between two surfaces was disclosed for the first time with useful comments on the flow profiles. Later on, the squeezing flow has been discussed widely in many articles. For example, Rashidi et al. [2] discussed unsteady and axisymmetric squeezed flow of nanofluids for approximation of analytic solutions to the flow problems. Hayat et al. [3] disclosed the features of three-dimensional and squeezed flow using two parallel sheets because of mixed convection. Hayat et al. [4] reported squeezing flow in rotating frame between two disks. The problems were developed using second grade fluid. In another study, Hayat et al. [5] discussed the findings of flow bounded by porous squeezed enclosure disclosing the features of magnetic field effects. Shahmohamadi and Rashidi [6] reported some good findings on the squeezing flow of nanofluids subject to rotating channel. The lower plate was assumed to be porous. Some recent studies [7,8] are also referred for further understanding the above scenario.

Technological developments have led the fluid mechanics toward a more productive formulation that is known as nanofluids. This formulation generally contains nano-size metallic particles suspended in the base fluid for shorter period. However, the results of this shorter period suspension are highly effective because the suspension is more efficient and has more thermophysical properties such as electric and thermal conductivity and density. Thermal features and dynamic flexibility in the context of irreversibility, entropy, and many other respective features have shown some great improvements for nanofluids. The pioneer study was reported by Choi [9] disclosing the thermo-physical characteristics of base liquid with the addition of nanoparticles. The concept was highly appreciated by research community. Later, Parvin and Chamkha [10] reported free convection and entropy optimization of nanofluids flowing in odd-shaped cavity. Zaraki et al. [11] disclosed the properties of boundary layer convection considering the size, type, and shape of nanoparticles as well as the type of base fluid. Reddy and Chamkha [12] accounted the effect of Soret and Dufour on TiO_2 -water and Al_2O_3 -water type suspensions passing via stretching sheet. Chamkha et al. [13] disclosed the features of entropy optimization in Cu–O–water nanofluid using magnetic influence. Rasool et al. [14] discussed the effects of porosity and Darcy media in nanofluid flow via stretching surface. Ismael et al. [15] analyzed the entropy optimization in cavity filled with nanofluid via porous medium. Rasool et al. [16] reported flow of nanofluids bounded by a convective and vertically adjusted Riga plate. Many recent articles are typically based on the flow of nanofluids; however, some of them are listed here [17–39]. Fluid flow analysis via porous medium has many important applications in various mechanical and industrial procedures. For example, the underground water purification process, oil recovery and purification, outlining, pipe developments, and many other procedures are typical examples of fluid flow analysis through porous medium (known as Darcy–Forchheimer flow). The pioneer model was originally defined by Darcy for weak porosity conditions and smaller velocity. Later on, Forchheimer [40] remodeled it using nonlinear factor through velocity and the new name given to this model as Darcy–Forchheimer model. Muskat [41] presented homogeneous fluid flow through Darcy medium. Seddeek [42] disclosed the features of thermophoresis and dissipation in Darcy type fluid flow using the concept of mixed convection. Hayat et al. [43] analyzed the entropy optimization and heat and mass transport mechanism using bidirectional water-based nanofluid flow subject to convective conditions. Sadiq and Hayat [44] reported Darcy–Forchheimer Maxwell

type nanofluid flow via convectively heated stretching surface. Umavathi et al. [45] reported numerical investigation on Darcy type nanofluid flow bounded by vertically adjusted rectangular duct. Hayat et al. [46] performed framing of radiation effect and heat generation in Darcy type nanofluid flow using chemical reaction.

In this research, our motivation is based on three novel concepts: first, to involve two parallel plates having filled the gap with a porous medium that has never been reported yet; second, to involve magnetohydrodynamics (MHD) effect in this formulation; and finally, to see the impact of squeezing nature of the model on the fluid flow analysis. This article is organized in the following order. First, a viscous, MHD non-Newtonian nanofluid is considered between the h -distance apart two parallel plates via Darcy medium. The attributes of Brownian diffusive motion and thermophoresis are involved. Second, the so-formulated governing problems are transformed into nonlinear dimensionless problems using suitable transformations. Third, numerical Runge–Kutta method built in MATLAB is applied to solve the problems and acquire the data for velocity field, thermal distribution, concentration distribution, and Nusselt number. All the results have been plotted graphically. Finally, the discussion is provided on the results in detail.

2 Mathematical model

Here we consider a steady squeezed nanofluid flow contained between two horizontally adjusted h -distance apart plates. The location of plates is fixed at $x_2 = 0$ at one side and $x_2 = h$ at the other side in Cartesian coordinates. The bottom plate is stretched with at the rate of $u_1 = zx_1$, where z is a positive constant integer. A uniformly induced magnetic impact is involved at the direction normal to the lower plate. Darcy–Forchheimer medium is considered between the plates, which allows the flow along horizontal axis with additional effects of porosity and friction. The geometry of the problem can be seen in Figure 1. The governing equations (see for reference Sheikholeslami et al. [47]) are:

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} = 0, \quad (1)$$

$$\begin{aligned} u_1 \frac{\partial u_1}{\partial x_1} + u_2 \frac{\partial u_1}{\partial x_2} = & -\frac{1}{\rho_f} \frac{\partial p}{\partial x_1} + \nu \left(\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2} \right) \\ & - \frac{\sigma B_0^2}{\rho_f} u_1 - \frac{\nu}{K} u_1 - F u_1^2, \end{aligned} \quad (2)$$

Figure 1: Geometry.

$$u_1 \frac{\partial u_2}{\partial x_1} + u_2 \frac{\partial u_2}{\partial x_2} = -\frac{1}{\rho_f} \frac{\partial p}{\partial x_2} + \nu \left(\frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial^2 u_2}{\partial x_2^2} \right), \quad (3)$$

$$u_1 \frac{\partial T}{\partial x_1} + u_2 \frac{\partial T}{\partial x_2} = \alpha \left(\frac{\partial^2 T}{\partial x_1^2} + \frac{\partial^2 T}{\partial x_2^2} \right) + \tau \left(D_B \left(\frac{\partial C}{\partial x_2} \frac{\partial T}{\partial x_2} + \frac{\partial C}{\partial x_1} \frac{\partial T}{\partial x_1} \right) + \frac{D_T}{T_h} \left(\frac{\partial T}{\partial x_1} \right)^2 + \left(\frac{\partial T}{\partial x_2} \right)^2 \right), \quad (4)$$

$$u_1 \frac{\partial C}{\partial x_1} + u_2 \frac{\partial C}{\partial x_2} = D_B \left(\frac{\partial^2 C}{\partial x_1^2} + \frac{\partial^2 C}{\partial x_2^2} \right) + \frac{D_T}{T_0} \left(\frac{\partial^2 T}{\partial x_1^2} + \frac{\partial^2 T}{\partial x_2^2} \right), \quad (5)$$

with BCs

$$u_1 = u_w = z x_1, \quad u_2 = 0, \quad C = C_h, \quad T = T_h, \quad \text{at } x_2 = 0, \quad (6)$$

$$u_1 = 0, \quad C = C_0, \quad T = T_0, \quad \text{at } x_2 = +h. \quad (7)$$

Differentiation of equation (2) w.r.t. x_2 and equation (3) w.r.t. x_1 and subtraction yield the following combined momentum equation:

$$\begin{aligned} & \frac{\partial u_1}{\partial x_1} \frac{\partial u_1}{\partial x_2} + u_1 \frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_2} + u_2 \frac{\partial^2 u_1}{\partial x_2^2} \\ & - u_1 \frac{\partial^2 u_2}{\partial x_1^2} - \frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_1} - u_2 \frac{\partial^2 u_2}{\partial x_1 \partial x_2} - \frac{\partial u_2}{\partial x_1} \frac{\partial u_2}{\partial x_2} \\ & = \nu \left(\frac{\partial^3 u_1}{\partial x_1^2 \partial x_2} + \frac{\partial^3 u_1}{\partial x_2^3} - \frac{\partial^3 u_2}{\partial x_1^3} - \frac{\partial^3 u_2}{\partial x_1 \partial x_2^2} \right) \\ & - \frac{\sigma B_0^2}{\rho_f} \frac{\partial u_1}{\partial x_2} - \frac{\nu}{K} \frac{\partial u_1}{\partial x_2} - 2F \frac{\partial u_1}{\partial x_2}. \end{aligned} \quad (8)$$

Define,

$$u_1 = z x_1 \frac{\partial f}{\partial \eta}, \quad u_2 = -z h f,$$

$$(T_0 - T_h) \theta(\eta) = (T - T_h),$$

$$(C_0 - C_h) \phi(\eta) = (C - C_h), \quad \eta = \frac{y}{h}.$$

Using (9) in (1), (4), (5), and (8), we have

$$f^{iv} - P(f'f'' - ff''') - Mf'' - \lambda f'' - 2F_f f'f'' = 0, \quad (10)$$

$$\theta'' + Nb\phi'\theta' + PPrf\theta' + Nt\theta'^2 = 0, \quad (11)$$

$$\phi'' + PScf\phi' + \frac{Nt}{Nb}\theta'' = 0, \quad (12)$$

$$\begin{aligned} f' &= 1, \quad f = 0, \quad \theta = 1 = \phi, \quad \text{at } \eta = 0, \\ f' &= 0, \quad f = 0, \quad \theta = 0 = \phi, \quad \text{at } \eta = 1, \end{aligned} \quad (13)$$

where $P = \frac{h^2 z}{\nu}$ is the viscosity parameter, $\lambda = \frac{h^2}{K}$ is the porosity, $F_f = \frac{F_z h x_1}{\nu}$ is the Forchheimer number such that $F = \frac{C_b}{\sqrt{K}}$ is the drag force coefficient and $M = \frac{\sigma B_0^2 h^2}{\rho_f \nu}$ is the magnetic parameter. In the energy equation, $Pr = \frac{\nu}{\alpha}$ is the Prandtl factor, $Nb = \frac{\tau D_B}{\nu} (C_0 - C_h)$ is the Brownian motion factor, $Nt = \frac{\tau D_T}{\nu T_h} (T_0 - T_h)$ is the thermophoresis factor, and $Sc = \frac{\nu}{D_B}$ is the Schmidt factor.

The physical quantities are given as:

$$\left(\frac{P x_1}{h} \right) C_f = f''(0), \quad Nu_x = \theta'(0). \quad (14)$$

3 Numerical solution

MATLAB-based numerical scheme is applied for the solutions of governing problems. The results are plotted graphically for the velocity field and thermal and solute profiles as well as for Nusselt factor and skin-frictional force. The governing equations, i.e., equations (10)–(12), are highly nonlinear problems that are numerically dealt with RK-45 technique accompanied with shooting method. The methods start with conversion of governing equations into first-order differential problems as follows:

$$f^{iv} = P(f'f'' + ff''') + Mf'' + \lambda f'' + 2F_f f'f'' = 0, \quad (15)$$

$$\begin{aligned} f &= \xi_1, f' = \xi_2, f'' = \xi_3, f''' = \xi_4, f^{iv} = \xi_4', \theta = \xi_5, \\ \theta' &= \xi_6, \theta'' = \xi_6', \phi = \xi_7, \phi' = \xi_8, \phi'' = \xi_8', \end{aligned} \quad (16)$$

$$\xi_4' = P(\xi_2 \xi_3 - \xi_1 \xi_4) + M(\xi_3) + \lambda \xi_3 + 2F_f \xi_2 \xi_3, \quad (17)$$

$$\xi_6' = -PPr \xi_1 \xi_6 - Nb \xi_8 \xi_6 - Nt \xi_6'^2, \quad (18)$$

$$\xi_8' = -PSc \xi_1 \xi_8 - \frac{Nt}{Nb} \xi_6', \quad (19)$$

(9) subject to:

$$\begin{aligned} \xi_1(0) &= 0, \xi_2(0) = 1, \xi_1(1) = 0 \quad \text{and} \quad \xi_2(1) = 1, \\ \xi_3(0) &= \alpha_1, \xi_5(0) = 1, \xi_5(1) = 0, \\ \xi_6(0) &= \alpha_2, \xi_7(0) = 1, \xi_7(1) = 0, \xi_8(0) = \alpha_3. \end{aligned} \quad (20)$$

The fifth-order RK45-numerical techniques efficiently solved this problem for various values of χ_{∞} , and subsequently the results are found accurate in region $0 \leq \chi \leq 1$ subject to suitable numerical values assigned to each fluid parameter.

4 Discussion

This section explores the graphical results and their physical justifications for the three main profiles, i.e., velocity, temperature, and concentration of nanoparticles. Figures 2–18 are plotted to disclose the impact of various fluid parameters involved in the present flow model. In

particular, Figures 2 and 3 are graphical display of the influence of Forchheimer parameter on the velocity field (f and f'), respectively. A closer look explores that velocity

Figure 2: Consequences of F_r on $f(\eta)$.

Figure 3: Consequences of F_r on $f'(\eta)$.

Figure 4: Consequences of M on $f(\eta)$.

Figure 5: Consequences of M on $f'(\eta)$.

Figure 6: Consequences of P on $f(\eta)$.

Figure 7: Consequences of P on $f'(\eta)$.

Figure 8: Consequences of P on $\theta(\eta)$.

Figure 9: Consequences of Nb on $\theta(\eta)$. Figure 10: Consequences of Nt on $\theta(\eta)$. Figure 11: Consequences of P on $\phi(\eta)$. Figure 12: Consequences of Nb on $\phi(\eta)$.

field shows reduction for larger Forchheimer number. Physically, the relation of Forchheimer number with drag force coefficient is responsible for this trend in velocity field. For larger Forchheimer number, an intensive

Figure 13: Consequences of Nt on $\phi(\eta)$. Figure 14: Variation in Nu for viscosity parameter P and thermophoresis Nt . Figure 15: Variation in Nu for viscosity parameter P and Brownian diffusion Nb . Figure 16: Variation in C_f for porosity factor λ .

drag force coefficient results in higher amount of friction offered to the fluid flow. Therefore, a decline is noticed in

Figure 17: Variation in C_f for magnetic parameter M .

Figure 18: Consequences of Pr on $\theta(\eta)$.

velocity profile. Figures 4 and 5 show the impact of magnetic field on fluid flow through Darcy medium. The impact of magnetic field is inversely related to the fluid flow. Physically, a surface normal implementation of magnetic field creates certain bumps in the direction of fluid flow. Therefore, a decline is noticed in both f and f' . Impact of viscosity parameter P on both f and f' is shown in Figures 6 and 7, respectively. Physically, for larger values of P , the inverse relation of kinematic viscosity confirms the enhancement in dynamic viscosity, and therefore, a decline in velocity field is justified for larger values of viscosity parameter. Figure 8 shows the impact of viscosity parameter on thermal distribution. The relevant boundary layer shows declining trend for augmented values of viscosity parameter. Impact of Brownian diffusive motion and thermophoresis on thermal distribution is shown in Figures 9 and 10. The non-predictive movement of nanoparticles because of the Brownian motion rises for stronger thermophoretic force, resulting in a more rapid transport from hot region to the colder region. Therefore, a rise in thermal distribution

is noticed for both the parameters. For elevated values of viscosity parameter, one can see an enhancement in the concentration distribution shown in Figure 11, which equally confirms the mathematical expression of viscosity parameter and its physical significance in fluid flow. For elevated values of Brownian diffusive motion parameter, concentration profile shows reduction. Physically, the random motion reduces for larger Brownian; however, the case is opposite in case of thermophoresis. At $Nb = 0.01$, the impact of Brownian diffusion is quite obvious; however, the impact becomes slighter for further improvement in the Brownian diffusion, whereas a linear enhancement is seen in concentration distribution for $Nt = 0.01, 0.1, 0.2, 0.3$, and so on. The variation in Nusselt number is shown in Figures 14 and 15. In Figure 14, the cross effect of viscosity parameter and thermophoresis is involved, whereas in Figure 15, thermophoresis is replaced with Brownian diffusion. The rate of heat flux reduces in both cases. Figures 16 and 17 are plotted to see the fluctuation in skin-friction for porosity factor and magnetic parameter, respectively. The larger friction produced by Forchheimer medium and retardation offered by magnetic impact result in enhancement of skin-friction. The impact of Prandtl number on thermal profile is shown in Figure 18. A decline is noticed for larger values of Pr .

5 Conclusions

Here the impact of MHD and Darcy medium on nanofluid flow contained between two h -distance apart plates is taken into account. A viscous nanofluid saturates the given porous medium. The characteristics of Brownian diffusion and thermophoresis are disclosed. Numerical scheme using MATLAB is applied to solve the problems and acquire the data for velocity field, temperature, and concentration distributions. Salient features of the study are listed below:

- Velocity field shows reduction for larger Forchheimer number. The drag force coefficient is responsible for this trend.
- The impact of magnetic field is inversely related to the fluid flow. A decline is noticed in velocity profile.
- For larger values of P , the inverse relation of kinematic viscosity confirms a decline in velocity field.
- The non-predictive movement of nanoparticles because of the Brownian diffusion rises for stronger thermophoretic force, resulting in a more rapid transport from hot region to the colder region.

- For elevated values of viscosity parameter, an enhancement in the concentration distribution is noticed.
- The rate of heat flux reduces for Brownian diffusion and thermophoresis.
- Skin-friction receives enhancement for elevated porosity factor and magnetic parameter.

Nomenclature

B_0	magnetic number
C_f	skin-friction
D_B	Brownian diffusion in $\text{m}^2 \text{ s}^{-1}$
D_T	thermophoretic diffusion in $\text{m}^2 \text{ s}^{-1}$
h	distance between plates in m
M	non-dimensional magnetic number
MHD	magnetohydrodynamics
Nb	Brownian diffusion parameter
Nt	thermophoresis parameter
Nu	Nusselt number
P	viscosity parameter
Pr	Prandtl number
RK45	Runge–Kutta method
Sc	Schmidt number
T	temperature K^{-1}
$u_1 = z\chi_1$	velocity in ms^{-1}
u_1, u_2	velocity components in ms^{-1}
x_1, x_2	Cartesian coordinates in m
z	stretching rate in s^{-1}
α	thermal diffusivity in $\text{m}^2 \text{ s}^{-1}$
ν	viscosity (kinematic) in $\text{m}^2 \text{ s}^{-1}$
ρ	density in kg m^{-3}

References

- [1] Stefan MJ. Versuch über die scheinbare Adhäsion, Sitzungsber, Abt. II, Osterr. Anzeiger der Akademie der Wissenschaften in Wien. Wien: Mathematische-naturwissenschaftliche Klasse; 1874;69:713–21.
- [2] Rashidi MM, Shahmohamadi H, Dinarvand S. Analytic approximate solutions for unsteady two-dimensional and axisymmetric squeezing flows between parallel plates. *Math Probl Eng.* 2008;2008:935095.
- [3] Hayat T, Qayyum A, Alsaedi A. Three-dimensional mixed convection squeezing flow. *Appl Math Mech.* 2015;36:47–60.
- [4] Hayat T, Yousaf A, Mustafa M, Obaidat S. MHD squeezing flow of second grade fluid between parallel disks. *Int J Numer Methods Fluid.* 2012;69:399–410.
- [5] Hayat T, Muhammad T, Qayyum A, Alsaedi A, Mustafa M. On squeezing flow of nanofluid in the presence of magnetic field effects. *J Mol Liq.* 2016;213:179–85.
- [6] Shahmohamadi H, Rashidi MM. VIM solution of squeezing MHD nanofluid flow in a rotating channel with lower stretching porous surface. *Adv Powder Technol.* 2016;27:171–8.
- [7] Rostami AK, Hosseinzadeh K, Ganji DD. Hydrothermal analysis of ethylene glycol nanofluid in a porous enclosure with complex snowflake shaped inner wall. *Waves Random Complex Media.* 2020. doi: 10.1080/17455030.2020.1758358.
- [8] Shah Z, Sheikhholeslami M, Ikramullah, Kumam P. Influence of nanoparticles inclusion into water on convective magneto hydrodynamic flow with heat transfer and entropy generation through permeable domain. *Case Stud Therm Eng.* 2020;21:100732.
- [9] Choi SUS, Eastman JA. Enhancing thermal conductivity of fluids with nanoparticles. ASME international mechanical engineering congress and exposition. 66, San Francisco: American Society of Mechanical Engineers; 1995. p. 99–105.
- [10] Parvin S, Chamkha AJ. An analysis on free convection flow, heat transfer and entropy generation in an odd-shaped cavity filled with nanofluid. *Int Commun Heat Mass Transf.* 2014;54:8–17.
- [11] Zaraki A, Ghalambaz M, Chamkha AJ, Rossi DD. Theoretical analysis of natural convection boundary layer heat and mass transfer of nanofluids: Effects of size, shape and type of nanoparticles, type of base fluid and working temperature. *Adv Powder Technol.* 2015;26:935–46.
- [12] Reddy S, Chamkha AJ. Soret and Dufour effects on MHD convective flow of Al_2O_3 -water and TiO_2 -water nanofluids past a stretching sheet in porous media with heat generation/absorption. *Adv Powder Technol.* 2016;27:1207–18.
- [13] Chamkha AJ, Ismael M, Kasaeipoor A, Armaghani T. Entropy generation and natural convection of CuO -water nanofluid in C-shaped cavity under magnetic field. *Entropy.* 2016;18:50. doi: 10.3390/e18020050.
- [14] Rasool G, Shafiq A, Khalique CM, Zhang T. Magnetohydrodynamic Darcy Forchheimer nanofluid flow over nonlinear stretching sheet. *Phys Scr.* 2019;94:105221.
- [15] Ismael M, Armaghani T, Chamkha AJ. Conjugate heat transfer and entropy generation in a cavity filled with a nanofluid-saturated porous media and heated by a triangular solid. *J Taiwan Inst Chem Eng.* 2016;59:138–51.
- [16] Rasool G, Zhang T, Shafiq A. Second grade nanofluidic flow past a convectively heated vertical Riga plate. *Phys Scr.* 2019;94:125212.
- [17] Lund LA, Omar Z, Khan I, Raza J, Bakouri M, Tlili I. Stability analysis of Darcy-Forchheimer flow of Casson type Nanofluid over an exponential sheet: Investigation of critical points. *Symmetry.* 2019;11:412.
- [18] Rasool G, Zhang T. Darcy-Forchheimer nanofluidic flow manifested with Cattaneo-Christov theory of heat and mass flux over non-linearly stretching surface. *PLoS One.* 2019;14:e0221302.
- [19] Lund LA, Omar Z, Khan I, Dero S. Multiple solutions of $\text{Cu-C}_6\text{H}_5\text{NaO}_7$ and $\text{Ag-C}_6\text{H}_5\text{NaO}_7$ nanofluids flow over non-linear shrinking surface. *J Cent South Univ.* 2019;26:1283–93.
- [20] Rasool G, Shafiq A, Tlili I. Marangoni convective nano-fluid flow over an electromagnetic actuator in the presence of first

- order chemical reaction. *Heat Transf Asian Res.* 2019;49:274–89.
- [21] Rasool G, Shafiq A, Durur H. Darcy-Forchheimer relation in Magnetohydrodynamic Jeffrey nanofluid flow over stretching surface. *Discrete Contin Dyn Syst Ser S.* 2019. doi: 10.3934/dcdss.2020399.
- [22] Rasool G, Wakif A. Numerical spectral examination of EMHD mixed convective flow of second grade nanofluid towards a vertical riga plate using an advanced version of the revised Buongiorno's nanofluid model. *J Therm Anal Calorim.* 2020. doi: 10.1007/s10973-020-09865-8.
- [23] Sohail M, Naz R, Abdelsalam SI. On the onset of entropy generation for a nanofluid with thermal radiation and gyrotactic microorganisms through 3D flows. *Physica Scripta.* 2019;95:045206.
- [24] Sohail M, Naz R. Modified heat and mass transmission models in the magnetohydrodynamic flow of Sutterby nanofluid in stretching cylinder. *Phys A Stat Mech Appl.* 2020;549:124088. doi: 10.1016/j.physa.2019.124088.
- [25] Rasool G, Zhang T, Chamkha AJ, Shafiq A, Tlili I, Shahzadi G. Entropy generation and consequences of binary chemical reaction on MHD Darcy-Forchheimer Williamson nanofluid flow over non-linearly stretching surface. *Entropy.* 2020;22:18.
- [26] Tlili I, Tawfeeq AA. Nanotechnology for water purification: Electrospun nanofibrous membrane in water and wastewater treatment. *J Water Reuse Desalination.* 2019;24:232–48. doi: 10.2166/wrd.2019.057.
- [27] Tlili I, Khan WA, Ramadan I. MHD flow of Nanofluid flow across horizontal circular cylinder: Steady forced convection. *J Nanofluids.* 2019;8:179–86.
- [28] Wakif A, Boulahia Z, Ali F, Eid MR, Sehaqui R. Numerical analysis of the unsteady natural convection MHD couette nanofluid flow in the presence of thermal radiation using single and two-phase nanofluid models for Cu-water nanofluids. *Int J Appl Comput Math.* 2018;4:81.
- [29] Wakif A, Boulahia Z, Sehaqui R. A semi-analytical analysis of electro-thermo-hydrodynamic stability in dielectric nanofluids using Buongiorno's mathematical model together with more realistic boundary conditions. *Results Phys.* 2018;9:1438–54.
- [30] Shafiq A, Rasool G, Masood CM. Significance of thermal slip and convective boundary conditions on three dimensional rotating Darcy-Forchheimer nanofluid flow. *Symmetry.* 2020;12(5):741. doi: 10.3390/sym12050741.
- [31] Hosseinzadeh K, Roghani S, Mogharrebi AR, Asadi A, Waqas M, Ganji DD. Investigation of cross-fluid flow containing motile gyrotactic microorganisms and nanoparticles over a three-dimensional cylinder. *Alex Eng J.* 2020;59(5):3297–307.
- [32] Hosseinzadeh K, Asadi A, Mogharrebi AR, Azari ME, Ganji DD. Investigation of mixture fluid suspended by hybrid nanoparticles over vertical cylinder by considering shape factor effect. *J Therm Anal Calorim.* 2020. doi: 10.1007/s10973-020-09347-x.
- [33] Hosseinzadeh K, Mogharrebi AR, Asadi A, Sheikhshahrokhdeh M, Mousavisani S, Ganji DD. Entropy generation analysis of mixture nanofluid ($\text{H}_2\text{O}/\text{C}_2\text{H}_6\text{O}_2$)– Fe_3O_4 flow between two stretching rotating disks under the effect of MHD and nonlinear thermal radiation. *Int J Ambient Energy.* 2019. doi: 10.1080/01430750.2019.1681294.
- [34] Hosseinzadeh K, Roghani S, Asadi A, Mogharrebi A, Ganji DD. Investigation of micropolar hybrid ferrofluid flow over a vertical plate by considering various base fluid and nanoparticle shape factor. *Int J Numer Method Heat Fluid Flow.* 2020. doi: 10.1108/HFF-02-2020-0095.
- [35] Shah Z, Hajizadeh MR, Ikramullah, Alreshidi NA, Deebani W, Shutaywi M. Entropy optimization and heat transfer modeling for Lorentz forces effect on solidification of NEPCM. *Int Commun Heat Mass Transf.* 2020;117:104715.
- [36] Sohail M, Shah Z, Tassaddiq A, Kumam P, Roy P. Entropy generation in MHD Casson fluid flow with variable heat conductance and thermal conductivity over non-linear bi-directional stretching surface. *Sci Rep.* 2020;10(1):12530. doi: 10.1038/s41598-020-69411-2.
- [37] Deebani W, Tassaddiq A, Shah Z, Dawar A, Ali F. Hall effect on radiative casson fluid flow with chemical reaction on a rotating cone through entropy optimization. *Entropy.* 2020;22:480.
- [38] Shah Z, McCash LB, Dawar A, Bonyah E. Entropy optimization in Darcy-Forchheimer MHD flow of water based copper and silver nanofluids with Joule heating and viscous dissipation effects. *AIP Adv.* 2020;10:065137.
- [39] Shah Z, Kumam P, Dawar A, Alzahrani EO, Thounthong P. Study of the couple stress convective micropolar fluid flow in a hall MHD generator system. *Front Phys.* 2019;7:171.
- [40] Forchheimer P. Wasserbewegung durch Boden. *Z des Ver deutscher Ingenieure.* 1901;45:1782–8.
- [41] Muskat M. The flow of homogeneous fluids through porous media. *Edwards, MI: Ann Arbor;* 1946.
- [42] Seddeek MA. Influence of viscous dissipation and thermophoresis on Darcy-Forchheimer mixed convection in a fluid saturated porous media. *J Colloid Interface Sci.* 2006;293:137–42.
- [43] Hayat T, Khan MI, Khan TA, Alsaedi A. Entropy generation in Darcy-Forchheimer bidirectional flow of water-based carbon nanotubes with convective boundary conditions. *J Mol Liq.* 2018;265(1):629–38.
- [44] Sadiq MA, Hayat T. Darcy-Forchheimer flow of magneto Maxwell liquid bounded by convectively heated sheet. *Results Phys.* 2016;6:884–90.
- [45] Umavathi JC, Ojjela O, Vajravelu K. Numerical analysis of natural convective flow and heat transfer of nanofluids in a vertical rectangular duct using Darcy-Forchheimer-brinkman model. *Int J Therm Sci.* 2017;111:511–24.
- [46] Hayat T, Shah F, Khan MI, Alsaedi A. Framing the performance of heat absorption/generation and thermal radiation in chemically reactive Darcy-Forchheimer flow. *Results Phys.* 2017;7:3390–5.
- [47] Sheikhholeslami M, Rashidi MM, Al Saad DM, Firouzi F, Rokni HB, Domairry G. Steady nanofluid flow between parallel plates considering thermophoresis and Brownian effects. *J King Saud Univ Sci.* 2016;28:380–9.