Research Article

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On solitary wave solutions of a peptide group system with higher order saturable nonlinearity

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Abstract: In this article, a new continuous model of a peptide group system with higher order saturable nonlinearity is derived. The model is constructed up on discrete Schrodinger equation with higher order saturable nonlinearity. New exact solutions of the proposed model are investigated. Via the generalized Riccati equation mapping method travelling wave and soliton solutions are derived. For certain values of parameters kink, antikink, breathers, and dark and bright solitons are presented.

Keywords: Riccati equation mapping method, exact solutions, solitons, peptide system, saturable nonlinearity

1 Introduction

Recently, nonlinear systems have a great attention [1–7]. Solitons are robust localized modes generated by this type of nonlinear evolution equations and have a great deal of interest. They have been observed in many such

systems. Two soliton solutions for a fifth-order nonlinear evolution equation have been derived [8]. Exact soliton solutions for a discrete nonlinear electrical transmission line in $(2 + 1)$ dimension have been derived [9]. Variational approximation is used to predict the form of the discrete solitons of a generalized version of the Ablowitz–Ladik model with a power-law nonlinearity [10]. Soliton solutions of the Camassa–Holm equation are studied [11]. Optical solitons in birefringent fibers have been discussed [12]. Riccati equation mapping method is one of the most important methods of constructing travelling wave solutions [13–17]. Using Davydov’s idea about energy transfer along protein molecules [18], discrete nonlinear Schrodinger equation (DNLSE) with saturable nonlinearity has been used to model the state of amide-I excitations in proteins [19,20]. In this work, the generalized Riccati equation mapping algorithm is used to investigate the solitary wave solutions of the system introduced by ref. [21].

The article is organized as follows. Generalized Riccati algorithm is given in Section 2. The mathematical model is derived in Section 3, and Section 4 is devoted to present solitary solutions and numerical example. Conclusion is given in Section 5.

2 Generalized Riccati algorithm

Considering the partial differential equation:

$$L(\xi, \xi_x, \xi_t, \xi_{xx}, \xi_{xt}, \xi_{tt}) = 0, \quad (2.1)$$

where $\xi = \xi(x, t)$, the algorithm can be summarized in the following steps [17].

- 1) Using traveling wave transformations $\omega = \gamma x + \kappa t$, $\xi(x, t) = F(\omega)$, equation (2.1) can be transformed into an ordinary differential equation:

$$G(F, F', F'', \dots) = 0, \quad (2.2)$$

where primes denote the derivatives with respect to δ .

- 2) The solution $F(\delta)$ of equation (2.2) is considered in the following form:

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$$F(\omega) = \sum_{j=0}^m \epsilon_j \chi^j(\omega), \quad (2.3)$$

where m and $\epsilon_j, j = 1, 2, \dots, m$, are parameters to be evaluated and the function $\chi(\delta)$ be such that

$$\frac{d\chi(\omega)}{d\omega} = a\chi^2(\omega) + b\chi(\omega) + c. \quad (2.4)$$

- 3) Substituting equation (2.3) along with equation (2.4) into equation (2.2), we get a system of algebraic equations corresponding to the powers of $\chi(\delta)$ to be solved in order to obtain constants. Exact solutions are depending on the discriminant $b^2 - 4ac$.

3 Mathematical system

The one-dimensional DNLS lattice model with higher order saturable nonlinearity [21] is investigated:

$$i \frac{\partial \xi_n}{\partial t} = -(\xi_{n+1} + \xi_{n-1} - 2\xi_n) + \alpha_1 \frac{\xi_n}{1 + |\xi_n|^2} - \alpha_2 \frac{|\xi_n|^2 \xi_n}{(1 + |\xi_n|^2)^2}, \quad (3.1)$$

where ξ_n is the dimensionless state in the n th lattice ($n = 0, 1, 2, \dots, N-1$), α_1 and α_2 are real nonlinearity parameters, and t corresponds to temporal coordinates.

Considering the stationary solution of equation (2.1) in the form $\xi_n = \gamma_n e^{-i\beta t}$, we get the system:

$$\beta \gamma_n + (\gamma_{n+1} + \gamma_{n-1} - 2\gamma_n) - \alpha_1 \frac{\gamma_n}{1 + \gamma_n^2} + \alpha_2 \frac{\gamma_n^3}{(1 + \gamma_n^2)^2} = 0. \quad (3.2)$$

Following [17] for a continuous variable x the system (2.2) can be approximated as:

$$\frac{d^2 \gamma}{dx^2} - (\alpha_1 - \beta) \gamma + (\alpha_1 + \alpha_2) \gamma^3 - (\alpha_1 + 2\alpha_2) \gamma^5 = 0. \quad (3.3)$$

4 Exact solutions

Here we investigate solitary wave solutions of the system (3.3) using the generalized Riccati algorithm. Considering the traveling wave transformations

$$\delta = \sqrt{l}(x - at), \gamma(x, t) = \Gamma(\delta), \Gamma(\delta) = \pm \sqrt{H(\delta)}. \quad (4.1)$$

Equation (3.3) can be written as:

$$\begin{aligned} & \frac{-l}{4H} \left(\frac{dH}{d\delta} \right)^2 + \frac{l}{2} \frac{d^2 H}{d\delta^2} - (\alpha_1 - \beta)H + (\alpha_1 + \alpha_2)H^2 \\ & - (\alpha_1 + 2\alpha_2)H^3 = 0. \end{aligned} \quad (4.2)$$

From (2.3) and (2.4), we obtain the following system of algebraic equations:

$$\begin{aligned} \chi^0 : & (\alpha_1 + \alpha_2)\epsilon_0^3 - (\alpha_1 + 2\alpha_2)\epsilon_0^4 + (\beta - \alpha_1)\epsilon_0^2 - \frac{l}{4}c^2\epsilon_1^2 \\ & + \frac{l}{2}bce_0\epsilon_1 = 0, \\ \chi^1 : & 2(\beta - \alpha_1)\epsilon_0\epsilon_1 + 3(\alpha_1 + \alpha_2)\epsilon_0^2\epsilon_1 - 4(\alpha_1 + 2\alpha_2)\epsilon_0^3\epsilon_1 \\ & + \frac{l}{2}(b^2 + 2ac)\epsilon_0\epsilon_1 = 0, \\ \chi^2 : & (\beta - \alpha_1)\epsilon_1^2 + 3(\alpha_1 + \alpha_2)\epsilon_0\epsilon_1^2 - 6(\alpha_1 + 2\alpha_2)\epsilon_0^2\epsilon_1^2 \\ & + \frac{l}{4}(b^2 + 2ac)\epsilon_1^2 + \frac{3l}{2}ace_0\epsilon_1 = 0, \\ \chi^3 : & (\alpha_1 + \alpha_2)\epsilon_1^3 - 4(\alpha_1 + 2\alpha_2)\epsilon_0\epsilon_1^3 + l(a^2\epsilon_0 + ab\epsilon_1)\epsilon_1 = 0, \\ \chi^4 : & \frac{3l}{4}a^2\epsilon_1^2 - (\alpha_1 + 2\alpha_2)\epsilon_1^4 = 0. \end{aligned}$$

By solving the preceding system under the condition

$\beta = \alpha_1 - \frac{3(\alpha_1 + \alpha_2)^2}{16(\alpha_1 + \alpha_2)}$, we get the needed constants:

$$\begin{aligned} l &= \frac{3(\alpha_1 + \alpha_2)^2}{4(\alpha_1 + 2\alpha_2)} \frac{1}{b^2 - 4ac}, \epsilon_0 = \frac{3(\alpha_1 + \alpha_2)}{8(\alpha_1 + 2\alpha_2)} \\ &+ \frac{3}{8} \sqrt{\frac{b^2}{b^2 - 4ac}}, \\ \epsilon_1 &= \frac{3(\alpha_1 + \alpha_2)}{8(\alpha_1 + 2\alpha_2)} \sqrt{\frac{a^2}{b^2 - 4ac}}, \alpha_1 \neq -2\alpha_2. \end{aligned} \quad (4.3)$$

New types of solutions of equation (3.3) are given in the following [12].

Family 1. For $v = b^2 - 4ac > 0$, $\delta = \sqrt{l}(x - at)$ and $ab \neq 0$ (or $ac \neq 0$).

$$\gamma_1(x, t) = \pm \left[\epsilon_0 - \frac{\epsilon_1}{2a} \left(b + \sqrt{v} \tanh \left(\frac{\sqrt{v}}{2} \delta \right) \right) \right]^{1/2}, \quad (4.4)$$

$$\gamma_2(x, t) = \pm \left[\epsilon_0 - \frac{\epsilon_1}{2a} \left(b + \sqrt{v} \coth \left(\frac{\sqrt{v}}{2} \delta \right) \right) \right]^{1/2}, \quad (4.5)$$

$$\begin{aligned} \gamma_3(x, t) = & \pm \left[\epsilon_0 - \frac{\epsilon_1}{4a} \left(2b + \sqrt{v} \left(\tanh \left(\frac{\sqrt{v}}{4} \delta \right) \pm \coth \left(\frac{\sqrt{v}}{4} \delta \right) \right) \right) \right]^{1/2}, \\ & (4.6) \end{aligned}$$

$$\begin{aligned} \gamma_4(x, t) = & \pm \left[\epsilon_0 + \frac{\epsilon_1}{2a} \left(-b + \frac{\sqrt{(a_1^2 - a_0^2)v} + a_0 \sqrt{v} \cosh(\sqrt{v}\delta)}{a_0 \sinh(\sqrt{v}\delta) + a_1} \right) \right]^{1/2}, \\ & (4.7) \end{aligned}$$

where a_0 and a_1 are real constants and $a_1^2 - a_0^2 > 0$.

$$\gamma_5(x, t) = \pm \left[\epsilon_0 + \frac{2c\epsilon_1 \cosh\left(\frac{\sqrt{v}}{2}\delta\right)}{\sqrt{v} \sinh\left(\frac{\sqrt{v}}{2}\delta\right) - b \cosh\left(\frac{\sqrt{v}}{2}\delta\right)} \right]^{1/2}, \quad (4.8)$$

$$\gamma_6(x, t) = \pm \left[\epsilon_0 - \frac{2c\epsilon_1 \sinh\left(\frac{\sqrt{v}}{2}\delta\right)}{b \sinh\left(\frac{\sqrt{v}}{2}\delta\right) - \sqrt{v} \cosh\left(\frac{\sqrt{v}}{2}\delta\right)} \right]^{1/2}, \quad (4.9)$$

$$\gamma_7(x, t) = \pm \left[\epsilon_0 + \frac{2c\epsilon_1 \cosh\left(\frac{\sqrt{v}}{2}\delta\right)}{\sqrt{v} \sinh(\sqrt{v}\delta) - b \cosh(\sqrt{v}\delta) \pm i\sqrt{v}} \right]^{1/2}, \quad (4.10)$$

$$\gamma_8(x, t) = \pm \left[\epsilon_0 + \frac{2c\epsilon_1 \sinh\left(\frac{\sqrt{v}}{2}\delta\right)}{-b \sinh(\sqrt{v}\delta) + \sqrt{v} \cosh(\sqrt{v}\delta) \pm \sqrt{v}} \right]^{1/2}, \quad (4.11)$$

$$\gamma_9(x, t) = \pm \left[\epsilon_0 + \frac{4c\epsilon_1 \sinh\left(\frac{\sqrt{v}}{4}\delta\right) \cosh\left(\frac{\sqrt{v}}{4}\delta\right)}{-2b \sinh\left(\frac{\sqrt{v}}{4}\delta\right) \cosh\left(\frac{\sqrt{v}}{4}\delta\right) + 2\sqrt{v} \cosh^2\left(\frac{\sqrt{v}}{4}\delta\right) - \sqrt{v}} \right]^{1/2}. \quad (4.12)$$

Family 2. For $b^2 - 4ac < 0$ and $ab \neq 0$ (or $ac \neq 0$).

$$\gamma_{10}(x, t) = \pm \left[\epsilon_0 + \frac{\epsilon_1}{2a} \left(-b + \sqrt{-v} \tan\left(\frac{\sqrt{-v}}{2}\delta\right) \right) \right]^{1/2}, \quad (4.13)$$

$$\gamma_{11}(x, t) = \pm \left[\epsilon_0 - \frac{\epsilon_1}{2a} \left(b + \sqrt{-v} \cot\left(\frac{\sqrt{-v}}{2}\delta\right) \right) \right]^{1/2}, \quad (4.14)$$

$$\gamma_{12}(x, t) = \pm \left[\epsilon_0 + \frac{\epsilon_1}{4a} \left(-2b + \sqrt{-v} \left(\tan\left(\frac{\sqrt{-v}}{4}\delta\right) \pm \cot\left(\frac{\sqrt{-v}}{4}\delta\right) \right) \right) \right]^{1/2}, \quad (4.15)$$

$$\gamma_{13}(x, t) = \pm \left[\epsilon_0 + \frac{\epsilon_1}{2a} \left(-b + \frac{\sqrt{(b_1^2 - b_0^2)(-v)} - a_0 \sqrt{-v} \cos(\sqrt{-v}\delta)}{b_1 \sin(\sqrt{-v}\delta) + a_1} \right) \right]^{1/2}, \quad (4.16)$$

where b_0 and b_1 are real constants and $b_1^2 - b_0^2 < 0$.

$$\gamma_{14}(x, t) = \pm \left[\epsilon_0 - \frac{2c\epsilon_1 \cos\left(\frac{\sqrt{-v}}{2}\delta\right)}{\sqrt{-v} \sin\left(\frac{\sqrt{-v}}{2}\delta\right) + b \cos\left(\frac{\sqrt{-v}}{2}\delta\right)} \right]^{1/2}, \quad (4.17)$$

$$\gamma_{15}(x, t) = \pm \left[\epsilon_0 - \frac{2c\epsilon_1 \sin\left(\frac{\sqrt{-v}}{2}\delta\right)}{b \sin\left(\frac{\sqrt{-v}}{2}\delta\right) - \sqrt{-v} \cos\left(\frac{\sqrt{-v}}{2}\delta\right)} \right]^{1/2}, \quad (4.18)$$

$$\gamma_{16}(x, t) = \pm \left[\epsilon_0 - \frac{2c\epsilon_1 \cos\left(\frac{\sqrt{-v}}{2}\delta\right)}{\sqrt{-v} \sin(\sqrt{-v}\delta) + b \cos(\sqrt{-v}\delta) \pm \sqrt{-v}} \right]^{1/2}, \quad (4.19)$$

$$\gamma_{17}(x, t) = \pm \left[\epsilon_0 + \frac{2c\epsilon_1 \sin\left(\frac{\sqrt{-v}}{2}\delta\right)}{-b \sin(\sqrt{-v}\delta) + \sqrt{-v} \cos(\sqrt{-v}\delta) \pm \sqrt{-v}} \right]^{1/2}, \quad (4.20)$$

Figure 1: For equation (4.4) with $\alpha_1 = 10$, $\alpha_2 = 21$, $a = 1.5$, $b = 4$, and $c = 2$, (a) kink solution and (b) antikink solution.

Figure 2: For equation (4.7) with $\alpha_1 = 10$, $\alpha_2 = 21$, $a = 1.5$, $b = 4$, $c = 2$, $a_0 = 1$, and $a_1 = 2$, (a) breather solution and (b) antibreather solution.

Figure 3: For equation (4.11) with $\alpha_1 = 10$, $\alpha_2 = 21$, $a = 1.5$, $b = 4$, and $c = 2$, (a) dark solution, (b) dark solution, (c) dark solution, and (d) dark solution.

$$y_{18}(x, t) = \pm \left[\epsilon_0 + \frac{4c\epsilon_1 \sinh\left(\frac{\sqrt{-v}}{4}\delta\right) \cosh\left(\frac{\sqrt{-v}}{4}\delta\right)}{-2b \sin\left(\frac{\sqrt{-v}}{4}\delta\right) \cos\left(\frac{\sqrt{-v}}{4}\delta\right) + 2\sqrt{-v} \cos^2\left(\frac{\sqrt{-v}}{4}\delta\right) - \sqrt{-v}} \right]^{1/2}, \quad (4.21)$$

Family 3. For $c = 0$, and $ab \neq 0$.

$$y_{19}(x, t) = \pm \left[\epsilon_0 - \frac{db\epsilon_1}{a(d + \cosh(b\delta) - \sinh(b\delta))} \right]^{1/2}, \quad (4.22)$$

$$y_{20}(x, t) = \pm \left[\epsilon_0 + \frac{b\epsilon_1(\cosh(b\delta) + \sinh(b\delta))}{a(d + \cosh(b\delta) - \sinh(b\delta))} \right]^{1/2}, \quad (4.23)$$

where d is a constant.

4.1 Numerical example

For $\alpha_1 = 10$, $\alpha_2 = 21$, $a = 1.5$, $b = 4$, and $c = 2$: (1) kink and antikink solutions of equation (4.4) are plotted in Figure 1(a) and (b), respectively; (2) with $a_1 = 2$ and $a_0 = 1$, breather of equation (4.7) is given in Figure 2(a) and antibreather is given in Figure 2(b); and (3) dark solutions for equation (4.11) are plotted in Figure 3(a) and (d) and bright solutions are plotted in Figure 3(b) and (e).

5 Conclusion and discussion

Using Davydov's idea about energy transfer along protein molecules, DNLSE with higher order saturable nonlinearity has been used to model the state of amide-I excitations in proteins. Here equation (3.3) is a new continuous derivation of DNLSE and Riccati equation mapping method is successfully employed for constructing the exact solutions for this system. The exact solutions of equation (3.3) have not been considered by any methods till now. Simulation is used to show kink, antikink, breather, antibreather, and dark solutions for equation (3.3).

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References

- [1] Marquié P, Bilbault JM, Remoissenet M. Observation of non-linear localized modes in an electrical lattice. *Phys Rev E*. 1995;5:6127.
- [2] Kenmogne F, Ndombou GB, Yemélé D, Fomethe A. Nonlinear supratransmission in a discrete nonlinear electrical transmission line: Modulated gap peak solitons. *Chaos Solitons Fractals*. 2015;75:263–71.
- [3] Elhassanein A. On the complex dynamics of functional-coefficients nonlinear autoregressive time series models. *Commun Statistics: Simul Computation*. 2017;46:2406–17.
- [4] Elhassanein A. On bifurcation and stochastic sensitivity analysis of forced functional-coefficients nonlinear autoregressive models and S&P 500 Data. *J Comput Theor Nanosci*. 2017;14:1862–75.
- [5] Elhassanein A. Complex dynamics of forced lstar model with delay. *Discret Contin Dyn Syst Ser B*. 2015;20:93.
- [6] Elhassanein A. Complex dynamics of logistic self-exciting threshold autoregressive model. *J Comput Theor Nanosci*. 2015;12(4):542–8.
- [7] Elhassanein A. On the control of forced process feedback nonlinear autoregressive model. *J Comput Theor Nanosci*. 2015;12(8):1519–26.
- [8] Wazwaz A. Kink solutions for three new fifth order nonlinear equations. *Appl Math Model*. 2014;38:110–8.
- [9] Tala-Tebue E, Zayed EME. New Jacobi elliptic function solutions, solitons and other solutions for the (2+1)-dimensional nonlinear electrical transmission line equation. *Eur Phys J Plus*. 2018;133:314.
- [10] Cuevas-Maraver J, Kevrekidis PG, Malomed BA, Guo L. Solitary waves in the Ablowitz–Ladik equation with power-law nonlinearity. *J Phys A: Math Theor*. 2019;52:065202. doi: 10.1088/17518121/aaf755 in press.
- [11] Berx J, Indekeu JO. Analytic iteration procedure for solitons and traveling wavefronts with sources. *J Phys A: Math Theor*. 2019;52:38LT01. doi: 10.1088/1751-8121/ab3914 in press.
- [12] Yıldırım Y, Biswas A, Jawad AJM, Ekici M, Zhou Q, Alzahrani AK, et al. Optical solitons with differential group delay for complex Ginzburg–Landau equation. *Results Phys*. 2020;16:102888.
- [13] Shun-don Z. The generalizing Riccati equation mapping method in non-linear evolution equation: application to (2+1)-dimensional Boiti–Leon–Pempinelle equation. *Chaos Solitons Fract*. 2008;37:1335–42.
- [14] Zheng CL. Comments on “The generalizing Riccati equation mapping method in nonlinear evolution equation: Application to (2+1)-dimensional Boiti–Leon–Pempinelle equation. *Chaos Solitons Fract*. 2009;39:1493–5.
- [15] Malwe BH, Betchewe G, Doka SY, Kofane TC. Travelling wave solutions and soliton solutions for the nonlinear transmission line using the generalized Riccati equation mapping method. *Nonlinear Dyn*. 2016;84:171–7.
- [16] Qin Z, Lan L, Huijuan Z, Mirzazadeh M, Alih B, Essaid Z, et al. Dark and singular optical solitons with competing nonlocal nonlinearities. *Opt Appl*. 2016;46:79–86.
- [17] Salathiel Y, Amadou Y, Betchewe G, Doka SY, Crepin KT. Soliton solutions and traveling wave solutions for a discrete electrical lattice with nonlinear dispersion through the

- generalized Riccati equation mapping method. *Nonlinear Dyn.* 2017;87:2435–43.
- [18] Davydov AS. Solitons and energy transfer along protein molecules. *J Theor Biol.* 1977;66:379–87.
- [19] Tchameu JDT, Motcheyo ABT, Tchawoua C. Mobility of discrete multibreathers in the exciton dynamics of the Davydov model with saturable nonlinearities. *Phys Rev E.* 2014;90:43203.
- [20] Tchameu JDT, Tchawoua C, Motcheyo ABT. Effects of next-nearest-neighbor interactions on discrete multibreathers corresponding to Davydov model with saturable nonlinearities. *Phys Lett A.* 2015;379:2984–90.
- [21] Tchameu JDT, Tchawoua C, Motcheyo ABT. Nonlinear supratransmission of multibreathers in discrete nonlinear Schrödinger equation with saturable nonlinearities. *Wave Motion.* 2016;65:112–8.