Research Article

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Fundamental solutions for the long–short-wave interaction system

<https://doi.org/10.1515/phys-2020-0220>

received August 03, 2020; accepted November 27, 2020

Abstract: In this article, the system for the long–short-wave interaction (LS) system is considered. In order to construct some new traveling wave solutions, He’s semi-inverse method is implemented. These solutions may be applicable for some physical environments, such as physics and fluid mechanics. These new solutions show that the proposed method is easy to apply and the proposed technique is a very powerful tool to solve many other nonlinear partial differential equations in applied science.

Keywords: He’s semi inverse method, LS system, traveling wave solutions, exact solutions, MATLAB 18

1 Introduction

Nonlinear partial differential equations (NLPDEs) are encountered in many fields of applied science, e.g., optics, plasma physics solid state physics, fluid mechanics and chemical engineering [1–14]. Moreover, by developing a specific transformation, the NLPDEs are converted into an ordinary differential equation (ODE). This conversion

causes the produced ODE to be solved by means of a group of powerful techniques, e.g., homogeneous balance method [15], tanh–sech method [16], extended tanh method [17], sine–cosine method [18], trigonometric function series method [19], Riccati–Bernoulli sub-ODE method [20,21], F-expansion method [22], $\left(\frac{G'}{G}\right)$ -expansion method [23,24], perturbation method [25,26], exp-function method [27,28] and Jacobi elliptic function method [29]. Recently, there is huge development in analytical methods as well as numerical methods in the field of mathematical modelling and applied sciences, see ref. [30–33].

This article is concerned with ref. [34]:

$$u_{xx} - uv + iu_t = 0, \quad v_x + (|u|^2)_x + v_t = 0, \quad (1.1)$$

where $u(x, t)$ and $v(x, t)$ are complex and real functions, respectively.

Expanding the methods to the search of NLPDEs for traveling wave solutions seems to be interesting and helpful to the mathematicians, physicists and engineers. In this study, we employ He’s semi-inverse technique to gain the solutions for the long–short-wave interaction (LS) system. It is worthwhile to note that the achieved solutions are given in three families. Most other papers concerning He’s semi-inverse technique give only one family.

This article is organized as follows: in Section 2, we recall He’s semi-inverse technique. In Section 3, we employ this technique to solve the LS equations. Finally, we give the conclusions about the results in Section 4.

2 He’s semi-inverse technique

For a given nonlinear evolution equation system with some physical fields $\chi_i(x, t)$

$$G(\chi, \chi_t, \chi_x, \chi_{tt}, \chi_{xx}, \dots) = 0, \quad (2.1)$$

we seek its solitary wave solutions by taking

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$$\chi = \chi(\xi), \quad \xi = kx - ct, \quad (2.2)$$

where k is the wave number and c is the wave speed. Using the aforementioned transformation, equation (2.1) will be transformed to the ODE:

$$E(\chi, \chi', \chi'', \chi''', \dots) = 0. \quad (2.3)$$

Integrating equation (2.3), if possible, term by term one or more times where the integration constant(s) can be put to zero for simplicity, we give [35,37]

$$J(\chi) = \int L(\chi, \chi') d\xi, \quad (2.4)$$

where L is the Lagrangian function, of the problem described by equation (2.3).

By the Ritz method, the solution takes one of the following forms: $\chi(\xi) = a \operatorname{sech}(b \xi)$, $\chi(\xi) = a \operatorname{csch}(b \xi)$, $\chi(\xi) = a \tanh(b \xi)$ and $\chi(\xi) = a \coth(b \xi)$. Here, we search for:

$$\begin{aligned} \chi(\xi) &= a \operatorname{sech}(b \xi), \chi(\xi) = a \operatorname{sech}^2(b \xi), \\ \chi(\xi) &= a \tanh(b \xi), \end{aligned} \quad (2.5)$$

where a and b are constants that must be determined. Substituting equation (2.5) in equation (2.4) and taking J stationary with respect to a and b we obtain

$$\frac{\partial J}{\partial a} = 0, \quad (2.6)$$

$$\frac{\partial J}{\partial b} = 0. \quad (2.7)$$

Solving simultaneously equations (2.6) and (2.7) gives a and b . Thus, the solitary wave solution equation (2.5) is achieved.

3 The LS system

We have

$$\begin{aligned} u(x, t) &= e^{i\xi} u(\xi), \quad v(x, t) = v(\xi), \\ \xi &= \alpha x + \beta t + \zeta, \end{aligned} \quad (3.1)$$

where the constants α and β are real and $u(\xi)$ is a real function, the system of equation (1.1) will take the following ODEs:

$$u'' - uv - (\alpha^2 + \beta)u = 0, \quad (3.2)$$

$$(1 - 2\alpha)v' + 2uv' = 0. \quad (3.3)$$

Integrating equations (3.2) and (3.3) with respect to ξ one time and neglecting the constant of integration, we get:

$$(1 - 2\alpha)v + u^2 = 0. \quad (3.4)$$

Then, we get:

$$u'' - \frac{1}{2\alpha - 1}u^3 - (\alpha^2 + \beta)u = 0. \quad (3.5)$$

Equation (3.5) can be written in the form:

$$u'' = -\frac{\partial V}{\partial u}, \quad (3.6)$$

where

$$\begin{aligned} V &= -(\alpha_4 u^4 + \alpha_2 u^2 + \alpha_0), \\ \alpha_4 &= \frac{1}{4(2\alpha - 1)}, \\ \alpha_2 &= \frac{1}{2}(\alpha^2 + \beta) \end{aligned} \quad (3.7)$$

and α_0 is the arbitrary constant.

For solving equation (3.5) using He's semi-inverse method [35–37], from equation (3.5), the variational formulation is

$$J = \int_0^\infty \left[\frac{1}{2}u'^2 + \alpha_4 u^4 + \alpha_2 u^2 + \alpha_0 \right] d\xi. \quad (3.8)$$

We have:

$$u(\xi) = a \operatorname{sech} \theta; \quad \theta = b\xi. \quad (3.9)$$

Then, we obtain:

$$\begin{aligned} J &= \frac{1}{b} \int_0^\infty \left[\frac{1}{2}a^2 b^2 \operatorname{sech}^2 \theta \tanh^2 \theta + a^4 \alpha_4 \operatorname{sech}^4 \theta \right. \\ &\quad \left. + a^2 \alpha_2 \operatorname{sech}^2 \theta + \alpha_0 \right] d\theta. \end{aligned}$$

Since α_0 is a constant of integration we can let $\alpha_0 = 0$ and consequently:

$$V = -(\alpha_4 u^4 + \alpha_2 u^2), \quad (3.10)$$

$$J = \frac{a^2}{6b} [b^2 + 4\alpha_4 a^2 + 6\alpha_2]. \quad (3.11)$$

This yields

$$\frac{\partial J}{\partial a} = \frac{a}{3b} [8\alpha_4 a^2 + 6\alpha_2 + b^2] = 0, \quad (3.12)$$

$$\frac{\partial J}{\partial b} = -\frac{a^2}{6b^2} [4\alpha_4 a^2 + 6\alpha_2 - b^2] = 0. \quad (3.13)$$

From equations (3.12) and (3.13) we get:

$$a = \pm \sqrt{-\frac{\alpha_2}{\alpha_4}}, \quad b = \pm \sqrt{2\alpha_2}.$$

Thus, the solutions of equation (3.5) take the form:

$$u(\xi) = \pm \sqrt{\frac{\alpha_2}{\alpha_4}} \operatorname{sech}(\sqrt{2\alpha_2} \xi).$$

The traveling wave transformation will be given by:

$$\begin{aligned} u(x, t) &= \pm \sqrt{2(1 - 2\alpha)(\alpha^2 + \beta)} e^{i(\alpha x + \beta t + \zeta)} \\ &\quad \times \operatorname{sech}(\sqrt{\alpha^2 + \beta}(\alpha x + \beta t + \zeta)), \\ v(x, t) &= -2(\alpha^2 + \beta) e^{2i(\alpha x + \beta t + \zeta)} \operatorname{sech}^2(\sqrt{\alpha^2 + \beta}(\alpha x \\ &\quad + \beta t + \zeta)). \end{aligned} \quad (3.14)$$

This solution is depicted in Figures 1 and 2.

Now we have:

$$u(\xi) = a \operatorname{sech}^2 \theta; \quad \theta = b\xi. \quad (3.15)$$

Substituting (3.15) into equation (3.8) yields

$$\begin{aligned} J &= \frac{a}{b} \int_0^\infty [2ab^2 \operatorname{sech}^4 \theta \tanh^2 \theta + \alpha_4 a^3 \operatorname{sech}^8 \theta \\ &\quad + \alpha_2 a \operatorname{sech}^4 \theta + \alpha_0] d\theta. \end{aligned}$$

For $\alpha_0 = 0$, we find that

$$V = -(\alpha_4 \psi^4 + \alpha_2 \psi^2), \quad (3.16)$$

$$J = \frac{2a^2}{105b} [14b^2 + 24\alpha_4 a^2 + 35\alpha_2], \quad (3.17)$$

$$\frac{\partial J}{\partial a} = \frac{4a}{105b} [48\alpha_4 a^2 + 35\alpha_2 + 14b^2], \quad (3.18)$$

$$\frac{\partial J}{\partial b} = \frac{-2a^2}{105b^2} [24\alpha_4 a^2 + 35\alpha_2 - 14b^2]. \quad (3.19)$$

Then, we have

$$48\alpha_4 a^2 + 35\alpha_2 + 14b^2 = 0, \quad (3.20)$$

$$24\alpha_4 a^2 + 35\alpha_2 - 14b^2 = 0. \quad (3.21)$$

Solving equations (3.20) and (3.21) for a and b gives

$$a = \pm \sqrt{-\frac{35\alpha_2}{36\alpha_4}}, \quad b = \pm \sqrt{\frac{5}{6}\alpha_2}.$$

Thus, for equation (3.5) there exist the solutions:

$$u(\xi) = \pm \sqrt{\frac{-35\alpha_2}{36\alpha_4}} \operatorname{sech}^2\left(\sqrt{\frac{5}{6}\alpha_2} \xi\right). \quad (3.22)$$

Figure 1: Graph of real part of u in (3.14) with $\alpha = 1.3$, $\beta = -1.2$ and $\zeta = 1$.

Figure 2: Graph of imaginary part of u in (3.14) with $\alpha = 0.6$, $\beta = -1.2$ and $\zeta = 1$.

Hence, the traveling wave transformation for constants α and β is given as follows:

$$\begin{aligned} u(x, t) &= \pm \sqrt{\frac{35}{18}(1 - 2\alpha)(\alpha^2 + \beta)} e^{i(\alpha x + \beta t + \zeta)} \operatorname{sech}^2 \left(\sqrt{\frac{5}{12}(\alpha^2 + \beta)} (\alpha x + \beta t + \zeta) \right), \\ v(x, t) &= \frac{-35(\alpha^2 + \beta)}{18} e^{2i(\alpha x + \beta t + \zeta)} \operatorname{sech}^4 \left(\sqrt{\frac{5}{12}(\alpha^2 + \beta)} (\alpha x + \beta t + \zeta) \right). \end{aligned} \quad (3.23)$$

This solution is depicted in Figures 3 and 4.

We choose

$$u(\xi) = a \tanh \theta; \quad \theta = b\xi. \quad (3.24)$$

Then, we get

$$\begin{aligned} J &= \frac{a}{b} \int_0^\infty \left[\frac{1}{2} ab^2 \operatorname{sech}^4 \theta + \alpha_4 a^3 \tanh^4 \theta + \alpha_2 a^2 \tanh^2 \theta + \alpha_0 \right] d\theta \\ J &= \frac{a}{b} \int_0^\infty \left[a \left(\alpha_4 a^2 + \frac{1}{2} b^2 \right) \operatorname{sech}^4 \theta - a^2 (2\alpha_4 a + \alpha_2) \operatorname{sech}^2 \theta \right] d\theta + \frac{a}{b} (\alpha_4 a^3 + \alpha_2 a^2 + \alpha_0) \int_0^\infty d\theta. \end{aligned}$$

Figure 3: Graph of real part of u in (3.23) with $\alpha = 0.9$, $\beta = -1.8$ and $\zeta = 1$.

Figure 4: Graph of imaginary part of u in (3.23) with $\alpha = 0.9$, $\beta = -1.8$ and $\zeta = 1$.

Under the condition

$$\alpha_0 = -a^2(\alpha_4 a + \alpha_2),$$

we get

$$V = -(\alpha_4 \psi^4 + \alpha_2 \psi^2 - a^2(\alpha_4 a + \alpha_2)), \quad (3.25)$$

$$J = \frac{-a^2}{3b} [4\alpha_4 a^2 + 3a\alpha_2 - b^2]. \quad (3.26)$$

$$\begin{aligned} \frac{\partial J}{\partial a} &= \frac{-2a}{3a} [8\alpha_4 a^2 + 3a\alpha_2 - b^2], \\ \frac{\partial J}{\partial b} &= \frac{a^2}{3b^2} [4\alpha_4 a^2 + 3a\alpha_2 + b^2]. \end{aligned} \quad (3.27)$$

Solving the two equations $\frac{\partial J}{\partial a} = 0$, $\frac{\partial J}{\partial b} = 0$, using (3.27), for a and b we get:

$$a = \pm \sqrt{\frac{-\alpha_2}{2\alpha_4}}, \quad b = \pm \sqrt{-\alpha_2}.$$

Thus, the solutions of equation (3.5) take the form

$$u(\xi) = \pm \sqrt{\frac{-\alpha_2}{2\alpha_4}} \tanh(\pm \sqrt{-\alpha_2} \xi). \quad (3.28)$$

The traveling wave transformation takes the form

$$\begin{aligned} u(x, t) &= \pm \sqrt{(1 - 2\alpha)(\alpha^2 + \beta)} e^{i(\alpha x + \beta t + \zeta)} \tanh\left(\pm \sqrt{\frac{-1}{2}(\alpha^2 + \beta)} (\alpha x + \beta t + \zeta)\right), \\ v(x, t) &= -(\alpha^2 + \beta) e^{2i(\alpha x + \beta t + \zeta)} \tanh^2\left(\pm \sqrt{\frac{-1}{2}(\alpha^2 + \beta)} (\alpha x + \beta t + \zeta)\right). \end{aligned} \quad (3.29)$$

This solution is depicted in Figures 5 and 6.

Wang et al. [34] introduced periodic wave solutions of system (1.1) by using the F-expansion method. Bekir et al. [38] obtained optical soliton solutions, utilizing the exp-function and ansatz methods. Bekir et al. [39]

Figure 5: Graph of real part of u in (3.29) with $\alpha = 0.9$, $\beta = -1.5$ and $\zeta = 1$.

Figure 6: Graph of imaginary part of u in (3.29) with $\alpha = 0.9$, $\beta = -1.5$ and $\zeta = 1$.

employed the (G'/G) -expansion technique for system (1.1) to construct traveling wave solutions expressed in terms of hyperbolic, trigonometric and rational functions. Fan *et al.* [40] presented some envelop traveling wave solutions in trigonometric and elliptic functions, utilizing the complete discrimination system for polynomial method. Baskonus *et al.* [41] applied the new modified expansion method to system (1.1) and found solutions of hyperbolic, complex and dark soliton solutions.

In this study, the exact solutions of the LS system were achieved in the explicit form, namely, hyperbolic function solutions. This study shows that the proposed method is reliable in handling NPDEs to establish a variety of exact solutions. These solutions have interesting applications in nonlinear sciences, for example, in the circular [42], the profile of a laminar jet [43]. These solutions represent the wave pictures in water waves, bio-physics, gravity, plasma and nonlinear optics. Indeed, these hyperbolic function solutions represent the ranges and altitudes of seismic sea waves. Some 2D and 3D graphics corresponding to the selected solutions have been plotted using MATLAB software by considering the suitable values for the parameters, namely Figures 1–6.

Remark 3.1. He's variational principle method can be easily implemented to solve various models of NLPDEs such as models given in ref. [44–47].

4 Conclusions

The basic goal of this work was to execute He's variational principle technique for solving the LS system. As a result, we have obtained three different families of solutions, which are hyperbolic functions solutions. The solutions contain free parameters. The calculations show that the proposed method is powerful, efficient and sturdy to get vital solutions. The obtained solutions will be extremely helpful in future investigations. We can say that He's semi-inverse technique can be extended to solve many other models of NLPDEs, which arise in applied science.

Funding source: This work was supported by the Natural Science Foundation of China (Grant No. 61673169, 11301127, 11701176, 11626101 and 11601485).

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships

that could have appeared to influence the work reported in this paper.

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