Research Article

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W-Chirped optical solitons and modulation instability analysis of Chen–Lee–Liu equation in optical monomode fibres

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Abstract: This work was devoted to unearth W-chirped to the famous Chen–Lee–Liu equation (CLLE) in optical monomode fibres. The results obtained will be useful to explain wave propagating with the chirp component. To attempt the main goal, we have used the new sub-ordinary differential equation (ODE) technique which was upgraded recently by Zayed EME, Mohamed EMA. Application of newly proposed sub-ODE method to locate chirped optical solutions to the Triki–Biswas equation equation. Optik. 2020;207:164360. On the other hand, we have used the modulation analysis to study the steady state of the obtained chirped soliton solutions in optical monomode fibres.

Keywords: W-chirped solitons, Chen–Lee–Liu equation, modulation instability

1 Introduction

Today, many authors focused their interest on treatment of the nonlinear physical system to unearth the wave called “soliton.” Solitons have been found in many

different physical systems such as bright soliton, dark soliton, breather-like solitons, and abundant vector solitons [1–14]. The prototypical model for the study of these solitons is the nonlinear Schrödinger (NLS) equation with attractive or repulsive interactions [15]. Many experiments suggested that the NLS model described well the evolution dynamics of many different nonlinear systems, such as water wave tank, optical fibre, and Bose–Einstein condensate [16]. The propagation of chirped soliton in the nonlinear physical system has become nowadays a wide field of research. During the spread of the latter, the pulse is loosed in nonlinear physical system because of many kinds of perturbations and properties of materials [17]. Today, with the event of chirped soliton which is able to remain in its form after perturbation or collision, it is possible to make pulse uniform in the nonlinear physical system [18]. By exciting chirped soliton, it is possible to amplify or reduce a pulse which propagates in the nonlinear physical system [19]. Otherwise, the W-chirped soliton as mentioned in the title is a chirped soliton with W-shape. In short, the solitons are full of many interests in various fields of applications such as communication, medicine, hydrodynamic just to name a few [20].

In addition, other models have been designed in recent years to illustrate the propagation of optical waves, among which the the Triki–Biswas equation [21], Fokas–Lenells equation [22] and so on. In this logic, the CLLE model was built to describe soliton in monomode optical fibres. It is given in the following form [23]:

$$iq_t + aq_{xx} + ib|q|^2q_x = 0, \quad (1)$$

where the quantity $q(x, t)$ is the wave profile in the spatial and temporal representation. Regarding the parameters a and b they are group velocity dispersion (GVD) and the contribution of the self-phase modulation (SPM), respectively [23]. More recently, the model has been the subject of a treatment taking into account the dual power low of nonlinearity, as a result the chirped solitons have been obtained [23]. Our objective in this article is to construct

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new forms of chirped solitons using the improved sub-ordinary differential equation (ODE) method by Zayed and Mohamed [21]. To achieve this, we will use the following diagram: Section 1 uses the usual transformation hypothesis to obtain the ordinary differential equation, while Section 2 is devoted to the application of the sub-ODE method. In Section 3, graphical representations in two-dimensional (2D) and three-dimensional (3D) are made to illustrate the physical explanation of the obtained results and it will be followed by the conclusion of the work in Section 4.

2 Mathematical investigation and chirped soliton to CLLE

To turn to the ODE, the following transformation is assumed:

$$q(x, t) = \phi(\xi)e^{i(f(\xi) - \Omega t)}, \quad (2)$$

where $\xi = \kappa x - vt$. The parameters $\phi(\xi)$ and $f(\xi)$ are reals, while the chirp component is written as follows:

$$\delta\omega(x, t) = -\frac{\partial}{\partial x}[f(\xi) - \Omega t] = -\kappa f'(\xi). \quad (3)$$

Using equations (2) and (1), it leads to two nonlinear ordinary equations, where the first one is the imaginary part and the second one is real.

$$-v\phi' + 2a\kappa^2 f' \phi' + a\kappa^2 f'' \phi + b\kappa \phi' \phi^2 = 0 \quad (4)$$

and

$$vf' \phi + \Omega \phi + a\kappa^2 \phi'' - a\kappa^2 f'^2 \phi - b\kappa f' \phi^3 = 0. \quad (5)$$

To obtain the analytical form of chirps, we multiply equation (4) by ϕ , and then integrate once by simultaneously considering the integration constant as zero as follows:

$$-\frac{v}{2}\phi^2 + a\kappa^2 \phi^2 f' + \frac{b\kappa}{4}\phi^4 = 0. \quad (6)$$

From equation (6), it is obtained that

$$f' = \frac{v}{2a\kappa^2} - \frac{b}{4a\kappa}\phi^2, \quad (7)$$

and this allows us to get the expression of the chirp as follows:

$$\delta\omega(x, t) = -\frac{v}{2a\kappa} + \frac{b}{4a}\phi^2. \quad (8)$$

Introducing equation (7) into equation (5) leads to the following nonlinear ordinary equation:

$$\ell_0 \phi + \ell_1 \phi^3 + \ell_2 \phi'' + \ell_3 \phi^5 = 0, \quad (9)$$

where

$$\ell_0 = \frac{1}{4} \frac{v^2 + 4\Omega a\kappa^2}{a\kappa^2}, \quad \ell_1 = -\frac{1}{2} \frac{bv}{a\kappa}, \quad (10)$$

$$\ell_2 = a\kappa^2, \quad \ell_3 = \frac{3b^2}{16a}.$$

To arrive at an integrable form of equation (9), we conjecture $\phi^2 = H$, which makes it possible to obtain the following form:

$$4\ell_0 H^2 + 4\ell_1 H^3 + \ell_2 [2H''H - H'^2] + 4\ell_3 H^4 = 0. \quad (11)$$

At the moment, we assume the solutions of equation (11) as in [21],

$$H = \mu F^n(\xi), \quad (12)$$

with m a parameter and $F(\xi)$ is expressed as follows see ref. [21],

$$F'^2(\xi) = AF^{2-2p}(\xi) + BF^{2-p}(\xi) + CF^2(\xi) + DF^{2+p}(\xi) + EF^{2+2p}(\xi), \quad p > 0. \quad (13)$$

We will now use the balance principle between the terms $H''H$ and H^4 , which leads to write $n + n + 2p = 4n \Rightarrow p = n$.

From there we get the solutions of equation (11), which is written as follows:

$$H = \mu F^p(\xi). \quad (14)$$

Making use of equation (12) with equation (13) in equation (11), the system of equations depending on $F^j(\xi)$, $j = (0, 2, 3, 4)$ is built as follows:

$$(4\ell_0 \mu^2 + \ell_2 \mu^2 p^2 C)F^{2p}(\xi) + (2\ell_2 \mu^2 p^2 D) + 4\ell_1 \mu^3 F^{3p}(\xi) + (3\ell_2 \mu^2 p^2 E + 4\ell_3 \mu^4)F^{4p}(\xi) - \ell_2 \mu^2 p^2 A = 0. \quad (15)$$

Obtaining the constants defined in equation (13) is dependent on the resolution of equation (15) by making use of the mathematical tool MAPLE.

Next, we employ equation (10) to point out the following result:

$$A = 0, \quad B = B, \quad C = -\frac{v^2 + 4\Omega a\kappa^2}{a^2 \kappa^4 p^2}, \quad D = \frac{vb\mu}{a^2 \kappa^3 p^2},$$

$$E = -\frac{1}{4} \frac{b^2 \mu^2}{a^2 \kappa^2 p^2}.$$

Case 1. $A = 0$, and assume that $B = 0$, the following case is obtained

According to the new type of algorithm set out in ref. [21], the following solutions should be revealed

- **i.1** $v^2 + 4a\Omega\kappa^2 < 0$, $D < 2C$, and $\mu = \pm \frac{1}{2} \frac{\sqrt{\Omega a(v^2 + 4\Omega a\kappa^2)} p}{\Omega b}$, what follows from $E = \frac{D^2}{4C} - C$.

$$q_1(x, t) = \left[\frac{\mu}{\cosh(\sqrt{C}\xi) - \frac{D}{2C}} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (16)$$

$$\delta\omega_1(x, t) = -\frac{\nu}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{1}{\cosh(\sqrt{C}\xi) - \frac{D}{2C}} \right]. \quad (17)$$

- **i.2** $\nu^2 + 4a\Omega\kappa^2 = 0$, $E < 0$.

$$q_2(x, t) = \frac{2a\kappa}{b} \sqrt{b\kappa D} \left[\frac{4D}{(D\xi)^2} - 4E \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (18)$$

thus the chirp solutions are obtained

$$\delta\omega_2(x, t) = -\frac{\nu}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{4D}{(D\xi)^2} - 4E \right]. \quad (19)$$

Case 2. Considering $A = B = 0$, $\nu^2 + 4a\Omega\kappa^2 < 0$, we have gained combined bright soliton and hyperbolic functions as solutions

- **ii.1** $D^2 - 4CE > 0$,

$$q_3(x, t) = \left[\frac{4\mu C \operatorname{sech}^2\left(\frac{1}{2}\sqrt{C}\xi\right)}{2\sqrt{D^2 - 4CE} - (\sqrt{D^2 - 4CE} + D)\operatorname{sech}^2\left(\frac{1}{2}\sqrt{C}\xi\right)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (20)$$

$$q_4(x, t) = \left[\frac{4\mu C \operatorname{csch}^2\left(\frac{1}{2}\sqrt{C}\xi\right)}{2\sqrt{D^2 - 4CE} + (\sqrt{D^2 - 4CE} - D)\operatorname{csch}^2\left(\frac{1}{2}\sqrt{C}\xi\right)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (21)$$

$$q_5(x, t) = \left[\frac{2\mu C}{\pm\sqrt{D^2 - 4CE} \cosh(\sqrt{C}\xi) - D} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (22)$$

chirp solutions correspond to

$$\delta\omega_3(x, t) = -\frac{\nu}{2a\kappa} + \frac{b}{4a} \left[\frac{4C \operatorname{sech}^2\left(\frac{1}{2}\sqrt{C}\xi\right)}{2\sqrt{D^2 - 4CE} - (\sqrt{D^2 - 4CE} + D)\operatorname{sech}^2\left(\frac{1}{2}\sqrt{C}\xi\right)} \right], \quad (23)$$

$$\delta\omega_4(x, t) = -\frac{\nu}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{4C \operatorname{csch}^2\left(\frac{1}{2}\sqrt{C}\xi\right)}{2\sqrt{D^2 - 4CE} + (\sqrt{D^2 - 4CE} - D)\operatorname{csch}^2\left(\frac{1}{2}\sqrt{C}\xi\right)} \right], \quad (24)$$

$$\delta\omega_5(x, t) = -\frac{\nu}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{2C}{\pm\sqrt{D^2 - 4CE} \cosh(\sqrt{C}\xi) - D} \right]. \quad (25)$$

Figure 1 plots (h_1) the corresponding W-chirp bright soliton of equation (17) and the bright soliton equation (16) when the speed of the soliton is $\nu^2 + 4a\Omega\kappa^2 < 0$, and the parameter of the sub-ODE method is given by $\mu = \pm \frac{1}{2} \frac{\sqrt{\Omega a (\nu^2 + 4a\Omega\kappa^2)} p}{\Omega b}$. Furthermore, Figure 2 is the spatiotemporal plot 2D of equations (16) and (17), respectively, at the same constraint relation on the velocity and free parameter of the sub-ODE algorithm. Moreover, Figure 3 is the spatiotemporal plot of the dark soliton solutions of analytical solution equation (25).

Figure 1: Plot of the corresponding (h_1) W-chirp bright soliton and (h_2) the bright soliton of equation (16) at $a = 1$, $\mu = 1$, $\kappa = 3.15$, $b = 0.2003$, $\Omega = -0.304215$, $\nu = 0.12$, $C = 0.00307$.

Figure 2: Plot of the corresponding (h_3) W-chirp bright soliton 2-D and (h_4) the bright soliton 2-D of equation (16) at $a = 1$, $\mu = 1$, $\kappa = 3.15$, $b = 0.2003$, $\Omega = -0.304215$, $v = 0.12$, $C = 0.00307$.

Figure 3: Spatiotemporal plot of equation (25) at $a = 1$, $\mu = 0.0001$, $\kappa = 1.47$, $b = 0.0023$, $\Omega = -8.89$, $v = -2.75$, $D = -1.72$, $E = -4.63$, $C = 3.70$.

- **ii.2** $D^2 - 4CE < 0$,

$$q_6(x, t) = \left[\frac{2\mu C}{\pm \sqrt{-(D^2 - 4CE)} \sinh(\sqrt{C}\xi) - D} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (26)$$

and the chirp

$$\delta\omega_6(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{2C}{\pm \sqrt{-(D^2 - 4CE)} \sinh(\sqrt{C}\xi) - D} \right]. \quad (27)$$

- **ii.3** $D^2 - 4CE = 0$,

$$q_7(x, t) = \left[-\frac{\mu C}{D} \left(1 \pm \tanh\left(\frac{p}{2}\sqrt{C}\xi\right) \right) \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (28)$$

$$q_8(x, t) = \left[-\frac{\mu C}{D} \left(1 \pm \coth\left(\frac{\sqrt{C}}{2}\xi\right) \right) \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (29)$$

chirps are expressed as

$$\delta\omega_7(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[-\frac{C}{D} \left(1 \pm \tanh\left(\frac{\sqrt{C}}{2}\xi\right) \right) \right], \quad (30)$$

$$\delta\omega_8(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[-\frac{C}{D} \left(1 \pm \coth\left(\frac{\sqrt{C}}{2}\xi\right) \right) \right], \quad (31)$$

and two other solutions when the constraint condition is limited to $A = B = 0$, $v^2 + 4a\Omega\kappa^2 < 0$,

$$q_9(x, t) = \left[\frac{\mu CD \operatorname{sech}^2\left(\frac{\sqrt{C}}{2}\xi\right)}{D^2 - CE \left(1 \pm \tanh\left(\frac{\sqrt{C}}{2}\xi\right) \right)^2} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (32)$$

$$q_{10}(x, t) = \left[\frac{\mu CD \operatorname{csch}^2\left(\frac{\sqrt{C}}{2}\xi\right)}{D^2 - CE \left(1 \pm \coth\left(\frac{\sqrt{C}}{2}\xi\right) \right)^2} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (33)$$

and the corresponding chirp

$$\delta\omega_9(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{CD \operatorname{sech}^2\left(\frac{\sqrt{C}}{2}\xi\right)}{D^2 - CE \left(1 \pm \tanh\left(\frac{\sqrt{C}}{2}\xi\right)\right)^2} \right], \quad (34)$$

$$\delta\omega_{10}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{CD \operatorname{csch}^2\left(\frac{\sqrt{C}}{2}\xi\right)}{D^2 - CE \left(1 \pm \coth\left(\frac{\sqrt{C}}{2}\xi\right)\right)^2} \right]. \quad (35)$$

Case 3. Considering $A = B = 0$, and $v^2 + 4a\Omega\kappa^2 > 0$, the following bright soliton and hyperbolic functions as solutions is obtained

- **iii.1** $D^2 - 4CE > 0$,

$$q_{11}(x, t) = \left[\frac{-2C\mu \sec^2\left(\frac{\sqrt{-C}}{2}\xi\right)}{2\sqrt{D^2 - 4CE} - (\sqrt{D^2 - 4CE} - D) \sec^2\left(\frac{\sqrt{-C}}{2}\xi\right)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (36)$$

$$q_{12}(x, t) = \left[\frac{2C\mu \csc^2\left(\frac{\sqrt{-C}}{2}\xi\right)}{2\sqrt{D^2 - 4CE} + (\sqrt{D^2 - 4CE} + D) \csc^2\left(\frac{\sqrt{-C}}{2}\xi\right)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (37)$$

$$q_{13}(x, t) = \left[\frac{2C\mu \sec(\sqrt{-C}\xi)}{\pm\sqrt{D^2 - 4CE} - D \sec(\sqrt{-C}\xi)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (38)$$

$$q_{14}(x, t) = \left[\frac{2\mu C \csc(\sqrt{-C}\xi)}{\pm\sqrt{D^2 - 4CE} - D \csc(p\sqrt{-C}\xi)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (39)$$

the corresponding chirp-like solution

$$\delta\omega_{11}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{-2C \sec^2\left(\frac{\sqrt{-C}}{2}\xi\right)}{2\sqrt{D^2 - 4CE} - (\sqrt{D^2 - 4CE} - D) \sec^2\left(\frac{\sqrt{-C}}{2}\xi\right)} \right], \quad (40)$$

$$\delta\omega_{12}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{2C \csc^2\left(\frac{\sqrt{-C}}{2}\xi\right)}{2\sqrt{D^2 - 4CE} + (\sqrt{D^2 - 4CE} + D) \csc^2\left(\frac{\sqrt{-C}}{2}\xi\right)} \right], \quad (41)$$

$$\delta\omega_{13}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{2C \sec(\sqrt{-C}\xi)}{\pm\sqrt{D^2 - 4CE} - D \sec(\sqrt{-C}\xi)} \right], \quad (42)$$

$$\delta\omega_{14}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{2C \csc(\sqrt{-C}\xi)}{\pm\sqrt{D^2 - 4CE} - D \csc(\sqrt{-C}\xi)} \right]. \quad (43)$$

Case 4. For $A = 0$, $B = \frac{8C^2}{27D}$, $E = \frac{D^2}{4C}$, we obtain the following cases

- **iv.1** $v^2 + 4a\Omega\kappa^2 > 0$, we gained hyperbolic function solutions

$$q_{15}(x, t) = \left[\frac{8\mu C \tanh^2\left(\frac{1}{2}\sqrt{\frac{-C}{3}}\xi\right)}{3D \left(3 + \tanh^2\left(\frac{1}{2}\sqrt{\frac{-C}{3}}\xi\right)\right)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (44)$$

$$q_{16}(x, t) = \left[\frac{8\mu C \coth^2\left(\frac{1}{2}\sqrt{\frac{-C}{3}}\xi\right)}{3D \left(3 + \coth^2\left(\frac{1}{2}\sqrt{\frac{-C}{3}}\xi\right)\right)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (45)$$

the chirp hyperbolic function solutions

$$\delta\omega_{15}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{8C \tanh^2\left(\frac{1}{2}\sqrt{\frac{-C}{3}}\xi\right)}{3D \left(3 + \tanh^2\left(\frac{1}{2}\sqrt{\frac{-C}{3}}\xi\right)\right)} \right], \quad (46)$$

$$\delta\omega_{16}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[-\frac{8C \coth^2 \left(\frac{1}{2} \sqrt{\frac{-C}{3}} \xi \right)}{3D \left(3 + \coth^2 \left(\frac{1}{2} \sqrt{\frac{-C}{3}} \xi \right) \right)} \right], \quad (47)$$

- **iv.2** $v^2 + 4a\Omega\kappa^2 < 0$, we gained trigonometric function solutions

$$q_{17}(x, t) = \left[\frac{8\mu C \tan^2 \left(\frac{1}{2} \sqrt{\frac{C}{3}} \xi \right)}{3D \left(3 - \tan^2 \left(\frac{1}{2} \sqrt{\frac{C}{3}} \xi \right) \right)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (48)$$

$$q_{18}(x, t) = \left[\frac{8\mu C \cot^2 \left(\frac{1}{2} \sqrt{\frac{C}{3}} \xi \right)}{3D \left(3 - \cot^2 \left(\frac{1}{2} \sqrt{\frac{C}{3}} \xi \right) \right)} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (49)$$

the equivalent chirp is obtained

$$\delta\omega_{17}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{8\mu C \tan^2 \left(\frac{1}{2} \sqrt{\frac{C}{3}} \xi \right)}{3D \left(3 - \tan^2 \left(\frac{1}{2} \sqrt{\frac{C}{3}} \xi \right) \right)} \right], \quad (50)$$

$$\delta\omega_{18}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{8\mu C \cot^2 \left(\frac{1}{2} \sqrt{\frac{C}{3}} \xi \right)}{3D \left(3 - \cot^2 \left(\frac{1}{2} \sqrt{\frac{C}{3}} \xi \right) \right)} \right]. \quad (51)$$

Case 5. For $A = B = 0$,

- **v.1** $v^2 + 4a\Omega\kappa^2 < 0$,

$$q_{19}(x, t) = \left[\frac{4\mu C e^{(\sqrt{C}\xi)}}{(e^{\pm\sqrt{C}\xi} - D) - 4CE} \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (52)$$

$$\delta\omega_{19}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[\frac{4C e^{(p\sqrt{C}\xi)}}{(e^{\pm\sqrt{C}\xi} - D) - 4CE} \right]. \quad (53)$$

Case 6. For $A = 0$, in what follows, Jacobian elliptic function solutions will be presented. It should be noted that these solutions can turn to soliton solutions such as bright, dark solitons and periodic or singular functions when $m \rightarrow 1$ or $m \rightarrow 0$. Here the parameter m is the Jacobian modulus.

Having already obtained bright and dark soliton solutions, periodic and singular solutions above, we will limit ourselves strictly to the Jacobian elliptic solutions type. They look like

- **vi.1** For $E < 0$, $B = \frac{D^3}{32m^2E^2}$, $C = \frac{D^2(4m^2 + 1)}{16m^2E}$, it is revealed

$$q_{20}(x, t) = \left[-\frac{\mu D}{4E} \left(1 \pm m \operatorname{cn} \left(\frac{D}{4m} \sqrt{-\frac{1}{E}} \xi \right) \right) \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (54)$$

$$q_{21}(x, t) = \left[-\frac{\mu D}{4E} \left(1 \pm \frac{\sqrt{1 - m^2} \operatorname{sn} \left(\frac{D}{4m} \sqrt{-\frac{1}{E}} \xi \right)}{\operatorname{dn} \left(\frac{D}{4m} \sqrt{-\frac{1}{E}} \xi \right)} \right) \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (55)$$

$$\delta\omega_{20}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \left[-\frac{D}{4E} \left(1 \pm m \operatorname{cn} \left(\frac{D}{4m} \sqrt{-\frac{1}{E}} \xi \right) \right) \right], \quad (56)$$

$$\delta\omega_{21}(x, t) = -\frac{v}{2a\kappa} + \frac{b\mu}{4a} \times \left[-\frac{D}{4E} \left(1 \pm \frac{\sqrt{1 - m^2} \operatorname{sn} \left(\frac{D}{4m} \sqrt{-\frac{1}{E}} \xi \right)}{\operatorname{dn} \left(\frac{D}{4m} \sqrt{-\frac{1}{E}} \xi \right)} \right) \right]^{\frac{1}{2}}. \quad (57)$$

- **vi.2** For $E < 0$, $B = \frac{m^2 D^3}{32(m^2 - 1)E^2}$, $C = \frac{D^2(5m^2 - 41)}{16(m^2 - 1)E}$, it is revealed

$$q_{22}(x, t) = \left[-\frac{\mu D}{4E} \left(1 \pm \frac{1}{\sqrt{1 - m^2}} \operatorname{dn} \left(\frac{D}{4} \sqrt{-\frac{1}{(1 - m^2)E}} \xi \right) \right) \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (58)$$

$$q_{23}(x, t) = \left[-\frac{\mu D}{4E} \left(1 \pm \frac{1}{\operatorname{dn} \left(\left(\frac{D}{4} \sqrt{-\frac{1}{(1 - m^2)E}} \xi \right) \right)} \right) \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (59)$$

then the chirps

$$\delta\omega_{22}(x, t) = -\frac{\nu}{2a\kappa} + \frac{b\mu}{4a} \left[-\frac{D}{4E} \left(1 \pm \frac{1}{\sqrt{1-m^2}} \operatorname{dn} \left(\frac{D}{4} \sqrt{-\frac{1}{(1-m^2)E}} \xi \right) \right) \right] \quad (60)$$

$$\delta\omega_{23}(x, t) = -\frac{\nu}{2a\kappa} + \frac{b\mu}{4a} \left[-\frac{D}{4E} \left(1 \pm \frac{1}{\operatorname{dn} \left(\frac{D}{4} \sqrt{-\frac{1}{(1-m^2)E}} \xi \right)} \right) \right] \quad (61)$$

- **vi.3** For $E < 0$, $B = \frac{m^2 D^3}{32E^2}$, $C = \frac{D^2(m^2 + 41)}{16E}$, hence

$$q_{24}(x, t) = \frac{2a\kappa}{b} \sqrt{bkD} \left[-\frac{D}{4E} \left(1 \pm \operatorname{dn} \left(\frac{D}{4} \sqrt{-\frac{1}{E}} \xi \right) \right) \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (62)$$

$$q_{25}(x, t) = \frac{2a\kappa}{b} \sqrt{bkD} \left[-\frac{D}{4E} \left(1 \pm \frac{\sqrt{1-m^2}}{\operatorname{dn} \left(\frac{D}{4} \sqrt{-\frac{1}{E}} \xi \right)} \right) \right]^{\frac{1}{2}} \times e^{i(f(\xi) - \Omega t)}, \quad (63)$$

which leads to chirp solutions as follows:

$$\delta\omega_{24}(x, t) = -\frac{\nu}{2a\kappa} + \frac{b\mu}{4a} \left[-\frac{D}{4E} \left(1 \pm \operatorname{dn} \left(\frac{D}{4} \sqrt{-\frac{1}{E}} \xi \right) \right) \right], \quad (64)$$

$$\delta\omega_{25}(x, t) = -\frac{\nu}{2a\kappa} + \frac{b\mu}{4a} \left[-\frac{D}{4E} \left(1 \pm \frac{\sqrt{1-m^2}}{\operatorname{dn} \left(\frac{D}{4} \sqrt{-\frac{1}{E}} \xi \right)} \right) \right] \quad (65)$$

3 Modulation instability (MI) analysis

After obtaining abundant analytical solutions, it is judicious to seek the stability of these solutions by using the

Figure 4: MI gain spectrum $G(\Omega)$ [m^{-1}] as a function of frequency Ω (Hz) in normal GVD ($a = 1$) with (d_1) the positive effect of SPM and (d_2) the negative effect of SPM at the power incident $P_0 = 1,500$.

famous linear stability technique in equation (1). To get there, we project the perturbed solution of equation (1) in the following form:

$$q(x, t) = [\sqrt{P_0} + A(x, t)] e^{i\phi_{NL}}, \quad \phi_{NL} = cP_0 x. \quad (66)$$

With the incident power P_0 , $A(x, t)$ is the so-called implicit perturbation function, while the nonlinear phase is represented by ϕ_{NL} . Introduce equation (70) into equation (1), then linearizing with respect to $A(x, t)$, it stems

$$iA_t + aA_{xx} + ibP_0 A_x = 0. \quad (67)$$

Let us estimate the form of the solution of equation (71) as follows:

$$A(x, t) = a_1 e^{i(Kx - \Omega t)} + a_2 e^{-i(Kx - \Omega t)}, \quad (68)$$

the parameters K and Ω represent the wave number and the perturbation frequency, respectively. By

inserting equation (72) into equation (71), a system of equations in terms of a_1 and a_2 is obtained after separating the coefficients of $e^{i(Kx-\Omega t)}$ and $e^{-i(Kx-\Omega t)}$.

$$(\Omega - aK^2 - bP_0K)a_1 = 0, \quad (69)$$

$$(-\Omega - aK^2 + bP_0K)a_2 = 0, \quad (70)$$

then the dispersion relation is as follows:

$$-\Omega^2 + 2\Omega bP_0K + a^2K^4 - b^2P_0^2K^2 = 0. \quad (71)$$

The solution of equation (75) can be obtained as follows:

$$K = \pm \frac{1}{2a} \sqrt{2P_0^2b^2 - 2\sqrt{P_0^4b^4 - 4a^2\Omega^2 - 8a^2\Omega bP_0}}, \quad (72)$$

The obtained solution depends on two essential terms of the monomode fibre, namely the GVD and the SPM. Furthermore, the obtained solution equation (76) makes it possible to locate the stability of the stationary state. So, for

$$2P_0^2b^2 - 2\sqrt{P_0^4b^4 - 4a^2\Omega^2 - 8a^2\Omega bP_0} \geq 0, \quad (73)$$

or

$$2P_0^2b^2 \geq 2\sqrt{P_0^4b^4 - 4a^2\Omega^2 - 8a^2\Omega bP_0}, \quad (74)$$

this solution is purely real, and there is a saturation of the stationary state in the face of small perturbations. On the other hand, if

$$2P_0^2b^2 < 2\sqrt{P_0^4b^4 - 4a^2\Omega^2 - 8a^2\Omega bP_0}, \quad (75)$$

the wave number K has an imaginary part, and this solution is unstable in the steady state because of the exponential growth of the perturbation. There is no doubt that the MI takes place. From there, we can get the MI gain spectrum as follows:

$$\begin{aligned} G(\Omega) &= a \operatorname{Im}(\Omega) \\ &= \sqrt{2P_0^2b^2 - 2\sqrt{P_0^4b^4 - 4a^2\Omega^2 - 8a^2\Omega bP_0}}. \end{aligned} \quad (76)$$

Figure 4 depicts the MI gain spectrum with the effect of SPM, which is related to the Kerr nonlinearity of the monomode optical fibres. Inspecting the curve, two regimes are setting out. In the first case, we assume the positive SPM and the second one with negative valued of SPM.

4 Conclusion

This article concerns the study of the W-chirped soliton and other solutions in optical monomode fibres by using

the dimensionless CLLE. More recently, chirped soliton to the model with dual power law of nonlinearity has been studied. The steady state of the obtained results has not been made. In view of the importance of the chirped signal in the communication system, we undertook the modulation analysis of the steady state of the obtained results in order to be able to define their domain of existence through graphical representations of the gain spectrum. The perturbation effect to will be added next the model to better circumscribe the constraints linked to the propagation of the chirped signal with perturbation effects.

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