Research Article

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Intensification of thermal stratification on dissipative chemically heating fluid with cross-diffusion and magnetic field over a wedge

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Abstract: This paper aims to examine the heat and mass characteristics for thermally stratified chemically heated, dissipative flow under the cross-diffusion and imposed Lorentz forces. A self-similar model is obtained employing suitable similarity transformations. Then, the RK technique is used for mathematical analysis. The stimulations of pertinent physical quantities in the flow regimes, shear stresses, and the Nusselt number were examined graphically. It is noted that more radiative flow favors the thermal behavior of the fluid and increases in the Prandtl number causes the decrease in thermal characteristics. Moreover, decreases in mass characteristics were examined by the fluctuating chemical reaction and Schmidt parameters. Lastly, key outcomes of the work are pinpointed.

Keywords: Lorentz force, radiative heat flux, wedge flow, cross-diffusion, shear stresses

1 Introduction

The investigation of boundary layer flow is one of the significant motives and an important part in the field of thermal transportation and fluid dynamics. Currently, regular flow characteristics over a wedge (either flat or porous wedge) are of potential interest.

The earlier work in this direction was examined by Falkner and Skan [1]. They discussed incompressible flow over a stationary wedge and tackled the resulting nonlinear flow model approximately. Later on, Rajagopal *et al.* [2] extended the study of Falkner and Skan over a stationary wedge situated inside the fluid. Lin and Lin [3] investigated thermal transportation in the convectively heated stationary wedge. They discussed the model for higher values of Prandtl number and found self-similar solutions for the model. Investigation of the characteristics of the Falkner–Skan flow in the presence and absence of convection over a wedge was reported in refs [4,5], respectively. Furthermore, the impacts of radiative heat flux and Lorentz force in the flow regimes are discussed there. Theoretical investigation of fluid under porosity effects and in the presence of mixed convection was carried out by Kumari *et al.* [6].

Ahmad and Khan [7] discussed the impacts of internally heated viscous dissipative flow over a convectively heated wedge; the wedge is capable of moving. They noted that the thermal field increases in the presence of an internal heat source. In 2014, Khan *et al.* [8] examined flow characteristics over a wedge. Further, flow along a vertical wedge was reported in Ganapathirao *et al.* [9]. The colloidal flow over a nonmoving wedge was reported in ref. [10]. The presence of the convection flow condition on the dissipative flow cannot be overlooked because of its direct effect on thermal transportation. The applications of the aforementioned flow with conditions were composed of thermal insulation, rocket engine, cooling of a nuclear reactor, electronic chips, and semiconductor wafers.

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Keeping the aforementioned applications in mind, Gebhart [11] explored the stimulus of viscous dissipation on flow characteristics. Afterward, the influence of viscous dissipation over a wedge was discussed by Yih [12]. He encountered the effects of the imposed Lorentz force and solved the flow model by employing the Keller-box method.

Recently, Ullah *et al.* [13] inspected the non-Newtonian model over a wedge. The stimulus of the applied Lorentz force on flow characteristics was part of their discussion. The unsteady Casson model over stretching surfaces was explored in Ullah *et al.* [14]. Their study contained the novel analysis of radiation, chemical, and mixed convection parameters in the flow regimes. Ahmed *et al.* [15] reported the study of nanofluids in the rotating system by considering ferromagnetics as nanoparticles. Ishak *et al.* [16] pointed out the Falkner–Skan flow with the addition of suction or blowing properties. The influence of the magnetohydrodynamic flow with forced convection over a wedge by encountering time-dependent viscosity was discussed by Pal and Mondal [17]. The MHD slip flow over a convectively heated wedge by considering heat generation/absorption was examined by Rahman *et al.* [18].

Recently, Hussanan *et al.* [19] examined the unsteady Falkner–Skan flow by encountering the influences of ohmic heating. The impacts of chemical parameters on the porous radiative wedge were reported in ref. [20]. Chambre and Acrivos [21] revealed the boundary layer model by considering the chemical parameters. The Falkner–Skan flow model for the wedge saturated with a colloidal mixture was reflected by Yacob *et al.* [22]. Recently, Khan *et al.* [23] investigated a bio-convection dissipative flow model. They highlighted the impacts of Joule heating and applied magnetic field over the flow of gyrotactic microorganisms and studied the model numerically. Furthermore, they analyzed the influence of pertinent flow parameters in flow regimes.

In 1937, Hartree [24] discussed the boundary layer equation approximately. Later on, researchers turned toward the study of wedge flow under different flow conditions. In 1961, Koh and Hartnett [25] explored the shear stresses and Nusselt number for streamline flow by considering the porosity parameter. Lin and Lin [26] discussed the approximate solutions for the wedge flow of any Prandtl number. Analysis of cylindrically shaped nanoparticles in a vertical channel was reported in ref. [27]. Various flow parameters and various channels were investigated in refs [28–31]. Furthermore, the flow model with a Brinkman-type nanofluid over a vertical plate was discussed by Ali *et al.* [32]. A natural convection model over a nonlinear sheet surface was studied by Ullah *et al.* [33].

For the nanofluid flow between Riga plates and second-grade fluid flow between an oblique channel, see refs [34,35], respectively. The analysis of the effective Prandtl model on squeezed flow nanofluids and other fruitful studies in various geometries are given in refs [36,37]. For further studies, one can refer the work presented in refs [38–44].

The behavior of third-grade fluid by incorporating the Cattaneo–Christov constitutive model, investigation of entropy for a viscoelastic nanofluid, effects of thermal radiation and ohmic heating, cross-flow of mixed convective fluid, and analysis of multiple nanofluids are studied in refs [45–49], respectively. Furthermore, significant studies are found in refs [50–55] and references therein.

From the intense science literature survey, it is perceived that the flow of a Newtonian fluid over a flat wedge by incorporating the cross-diffusion gradients and chemical reaction effects is not yet pointed out. For the novelty of the study, the phenomena of Lorentz forces and thermal radiation are taken in the momentum and energy conservation laws. The modeling of the particular flow over a wedge leads to a nonlinear self-similar problem, which is treated numerically. For numerical treatment, the Runge–Kutta technique coupled with the shooting technique is operated. The impacts of various physical dimensionless quantities emerging in the model are perceived *via* graphical results.

The flow problem and local heat and mass transport rate are modeled in Section 2. Afterward, the particular flow model is treated numerically in Section 3 and the problem is successfully solved over a semi-infinite region. Next, the impacts of the various ingrained flow quantities on the fluid velocity, temperature, concentration trends, shear stresses, and local rate of mass and heat transport are considered in Section 4. Finally, the major results of the presented study are highlighted in the last section.

2 Model formulation

The magnetized streamline flow is considered past a wedge in the presence of radiative heat flux, cross-diffusion, and chemical reaction. The fluid is flowing with the mainstream velocity $U(x)$. The variable temperature at the wedge is denoted by $T_w(x)$ and $C_w(x)$ is the concentration. Further, the ambient temperature and concentration are denoted by T and C , respectively. The imposed magnetic field $B(x)$ is in the form $B(x) = B_0 x^{\frac{m-1}{2}}$, where B_0 is uniform (for instance, see refs [56,57]), and it is supposed that the magnetic Reynolds number and

Figure 1: Flow configuration.

induced magnetic field are almost inconsequential comparative to Lorentz forces. The physical theme of the problem is elucidated in Figure 1.

The dimensional wedge flow model is as follows [58]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U(x) \frac{dU(x)}{dx} \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial y^2} \right) + \frac{\sigma}{\rho} \mathbf{B}^2(\mathbf{x})(U(x) - u), \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \left(\frac{\partial^2 T}{\partial y^2} \right) + \frac{DK_T}{\rho c_p C_s} \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{1}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{16\sigma^* T_\infty^3}{3k^* \rho c_p} \left(\frac{\partial^2 T}{\partial y^2} \right), \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial y^2} \right) + \frac{DK_T}{T_m} \left(\frac{\partial^2 T}{\partial y^2} \right) - k_1(C - C_\infty). \quad (4)$$

Here, the continuity equation satisfies identically. Equation (2) represents the momentum equation with mainstream velocity $U(x)$, imposed magnetic field $\mathbf{B}(\mathbf{x})$, dynamic viscosity μ , the density of the fluid ρ , and electrical conductivity σ ; u and v denote the velocities. Equation (3) is the thermal equation with Rd. Here, k is the thermal conductivity, c_p is the specific heat capacity, C_s is the concentration susceptibility, K_T is the thermal diffusion, D is the mass diffusivity, and T is the temperature. In equation (3), C is the fluid concentration, k_1 and T_m are the chemical reaction and mean temperature, respectively.

The feasible conditions are the following [58]:

At $y = 0$:

$$u = 0, \quad v = 0, \quad T = T_w(x), \quad C = C_w, \quad (5)$$

At $y \rightarrow \infty$:

$$u \rightarrow U(x) = u_0 x^m, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty. \quad (6)$$

The stream functions $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$, and similarity transformations are defined as follows ref. [58]:

$$\begin{aligned} \psi &= \sqrt{\nu u_0 x^{m+1}} F \left(\sqrt{\frac{u_0 x^{m+1}}{\nu}} y x^{-1} \right), \\ \eta &= \sqrt{\frac{u_0 x^{m+1}}{\nu}} y x^{-1}, \quad \beta \left(\sqrt{\frac{u_0 x^{m+1}}{\nu}} y x^{-1} \right) = \frac{T - T_\infty}{T_w - T_\infty}, \\ \phi \left(\sqrt{\frac{u_0 x^{m+1}}{\nu}} y x^{-1} \right) &= \frac{C - C_\infty}{C_w - C_\infty}. \end{aligned} \quad (7)$$

Note that $B(x) = B_0 x^{\frac{m-1}{2}}$ is the form of an applied magnetic field according to refs [56,57], $U(x) = \frac{u_0}{x^{-m}}$, where $0 \leq m \leq 1$. The expression $\beta^* = \beta^*(2 - \beta^*)^{-1}$ (where β^* represents the Hartree pressure) corresponds to $\beta^* = \frac{\Omega}{\pi}$.

After some necessary calculations, the attained dimensionless model is

$$F''' - \left[mF'^2 - \frac{m+1}{2} FF'' - m \right] + M(1 - F') = 0, \quad (8)$$

$$(1 + \text{Rd})\beta'' + \frac{\text{Pr}(m+1)}{2} F\beta' + \text{PrEc}F'^2 + \text{PrDf}\phi'' = 0, \quad (9)$$

$$\phi'' + \frac{\text{Sc}(m+1)}{2} F\phi' + \text{SrSc}\beta'' - \gamma\text{Sc}\phi = 0. \quad (10)$$

The corresponding boundary conditions for our flow model are the following:

$$\begin{aligned} F(\eta \uparrow \eta=0) &= 0, \quad F'(\eta \uparrow \eta=0) = 0, \\ \beta(\eta \uparrow \eta=0) &= 1, \quad \phi(\eta \uparrow \eta=0) = 1, \end{aligned} \quad (11)$$

$$F'(\eta \uparrow \eta=\infty) \rightarrow 1, \quad \beta(\eta \uparrow \eta=\infty) \rightarrow 0, \quad \phi(\eta \uparrow \eta=\infty) \rightarrow 0. \quad (12)$$

The dimensionless formulas against the involved quantities are

$$\begin{aligned} M &= \frac{\sigma B_0^2}{\rho u_0}, \quad \text{Pr} = \frac{\nu \rho c_p}{k}, \quad \text{Rd} = -\frac{16\sigma^* T_\infty^3}{3kk^*}, \\ D_f &= \frac{DK_T(C_w - C_\infty)}{C_s C_p \nu (T_w - T_\infty)}, \quad \text{Sr} = \frac{D_m k_T (T_w - T_\infty)}{\nu T_m (C_w - C_\infty)}, \\ \text{Sc} &= \frac{\nu}{D} \quad \text{and} \quad \gamma = \frac{\nu k_1}{U^2}. \end{aligned}$$

The quantities of physical interest in the dimensional form are described as follows [58]:

$$C_F = \frac{\mu}{\rho U^2(x)} \left(\frac{\partial u}{\partial y} \right) \Big|_{y=0}, \quad (13)$$

$$\text{Nu}_x = \left[\frac{kx}{k(T_w - T_\infty)} \right] \left(\frac{\partial T}{\partial y} \right) \Big|_{y=0}, \quad (14)$$

$$\text{Sh}_x = \left[\frac{x}{C_w - C_\infty} \right] \left(\frac{\partial C}{\partial y} \right) \downarrow_{y=0}. \quad (15)$$

In self-similar form, these expressions are reduced as follows:

$$C_F \sqrt{\text{Re}_x} = F''(0),$$

$$\text{Nu}_x(\text{Re}_x)^{-\frac{1}{2}} = -\beta'(0),$$

$\text{Sh}(\text{Re}_x)^{-\frac{1}{2}} = -\phi'(0)$, where $\text{Re}_x = \frac{xU(x)}{\nu}$ represents the local Reynold number.

Consequently, the following system is attained:

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \\ y_5' \\ y_6' \\ y_7' \end{bmatrix} = \begin{bmatrix} y_2 \\ y_3 \\ \left[my_2^2 - \frac{m+1}{2} y_1 y_3 - m \right] - M(1 - y_2) \\ y_5 \\ \frac{1}{(1 + \text{Rd})} \left[-\frac{\text{Pr}(m+1)}{2} y_1 y_5 - \text{Pr} D_f y_7' - \text{Pr} \text{Ec} y_3^2 \right] \\ y_7 \\ -\frac{\text{Sc}(m+1)}{2} y_1 y_7 - \text{Sr} \text{Sc} y_5' + \gamma \text{Sc} y_6 \end{bmatrix}. \quad (20)$$

and the initial conditions are as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ n_1 \\ 1 \\ n_2 \\ 1 \\ n_3 \end{bmatrix}. \quad (21)$$

The particular model described by equations (8)–(10) with nonhomogenous auxiliary conditions in Equations (11) and (12) does not possess exact solutions due to non-linearity. Thus, the present flow model can be solved by the numeric technique; we used the RK technique [59,60]. To operate the technique, first, we need to make the following substitutions:

$$\begin{aligned} y_1 &= F, & y_2 &= F', & y_3 &= F', & y_4 &= \beta, \\ y_5 &= \beta', & y_6 &= \phi, & y_7 &= \phi'. \end{aligned} \quad (16)$$

Further, the model was reduced to the following version:

$$F''' = \left[mF'^2 - \frac{m+1}{2} FF'' - m \right] - M(1 - F'), \quad (17)$$

$$\beta'' = \frac{1}{(1 + \text{Rd})} \left[-\frac{\text{Pr}(m+1)}{2} F\beta' - \text{Pr} \text{Ec} F''^2 - \text{Pr} D_f \phi'' \right], \quad (18)$$

$$\phi'' = -\frac{\text{Sc}(m+1)}{2} F\phi' - \text{Sr} \text{Sc} \beta'' + \gamma \text{Sc} \phi. \quad (19)$$

Finally, the solutions of the model are as described in Table 1. This computation is carried out for step size 0.5.

4 Graphical results

This section highlights the impact of Eckert number, Prandtl number, thermal radiation, chemical reaction, magnetic number, Schmidt, Dufour, and Soret numbers on the flow characteristics. Here, it is worth mentioning that the wedge flow corresponds to $m = 0.5$, while the horizontal plate flow and stagnation point flow correspond to $m = 0$ and $m = 1.0$, respectively. Further,

Table 1: Model solutions over the domain of interest

η	$F'(\eta)$	$\beta(\eta)$	$\phi(\eta)$
0.0	0.0000000000	1.0000000000	1.0000000000
0.5	0.4575633648	0.8870454597	0.7286054919
1.0	0.7314206224	0.7440129051	0.5164061589
1.5	0.8803175946	0.5933443311	0.3533632878
2.0	0.9526029667	0.4490377056	0.2326710105
2.5	0.9834982230	0.3204734153	0.1471573757
3.0	0.9949960216	0.2133585775	0.0889701955
3.5	0.9986907397	0.1298089980	0.0505958003
4.0	0.9997098593	0.0687575165	0.0258071403
4.5	0.9999508991	0.0269210431	0.0099969650
5.0	1.0000007510	$-6.6748755860 \times 10^{-8}$	$2.6103811060 \times 10^{-8}$

the cross-diffusion in the flow regimes is also under consideration.

Figure 2 shows the alterations in $F'(\eta)$ for varied magnetic fields. It is evident that for the stagnation point flow ($m = 1$) flow, the velocity field increases rapidly. Maximum variations in the velocity of the fluid $F'(\eta)$ are noticed in the area $0.5 \leq \eta \leq 2.5$. The alterations in $F'(\eta)$ are almost similar away from the wedge surface.

The behavior of the temperature $\beta(\eta)$ of the fluid due to varying radiation parameters, Prandtl number, Eckert, and Dufour numbers are shown in Figures 3–5. From Figure 3a, it can be observed that for more thermally stratified fluid, the temperature starts increasing. For a horizontal plate flow, the thermal field increased rapidly. Near $\eta = 5.0$, these variations are maximum. In the region $\eta > 5.0$, $\beta(\eta)$ increases slowly for the wedge, horizontal plate, and stagnation point flow cases. Beyond the region $\eta \geq 15.0$, the fluid temperature vanishes asymptotically. For the stagnation point flow, alterations in temperature gradually decrease compared to those in the wedge flow case. Consequently, Figure 3b shows the influences of Pr on $\beta(\eta)$. The variable Prandtl number leads to a decrease in $\beta(\eta)$. Near the wedge, the decreasing behavior of $\beta(\eta)$ is almost inconsequential. But, in the region $2.0 \leq \eta \leq 6.0$, these effects are very clear, and maximum decreases in temperature are observed in this area. In the case of a horizontal plate flow, the fluid temperature decreases slowly as compared to the wedge flow and stagnation point flow. The three-dimensional view of the temperature for wedge, stagnation point flow, and horizontal plate flow due to varying Prandtl numbers Pr are shown in Figure 4a and b, respectively.

The stimuli of Dufour and Eckert numbers on the thermal characteristics $\beta(\eta)$ are shown in Figure 5a and b, respectively. It is noted that increasing Dufour and

Eckert numbers lead to an increase in $\beta(\eta)$. But, these effects are almost inconsequential for the wedge ($m = 1.0$), stagnation point flow ($m = 0.5$), and horizontal plate flow ($m = 0$). Although, increase in temperature is almost negligible, but, for the horizontal plate flow case, temperature increases quite rapidly compared to the wedge and stagnation point flow case.

Figures 6–8 show the impact of γ , Schmidt number Sc, and Sr on $\phi(\eta)$. The decrease in $\phi(\eta)$ is shown in Figure 6a and b, respectively, for γ and Sc. It is observed that for more chemically heated fluid, decrease in the $\phi(\eta)$ field is quite slow for the horizontal plate flow. On the contrary, Figure 6b shows a prompt decrease in $\phi(\eta)$. This shows that increasing Sc leads to a prompt decrease in the concentration profile. Further, far away from the wedge surface, influences of γ and Sc vanish asymptotically. Fascinating fluctuations in $\phi(\eta)$ are inspected for varying Sr values in Figure 8a. The concentration field increases due to increasing Soret number. These effects are almost inconsequential for the wedge, stagnation point flow, and horizontal plate flow. Further, 3D scenario of the dimensionless concentration field is shown in Figure 8b.

Influences of M and m on the shear stresses are shown in Figure 9a and b, respectively. For both varying M and m , the skin friction starts increasing. In the case of magnetic parameters, shear stresses increase quite slowly compared to the case of varying m values.

The impact of Dufour D_f , Rd, and Pr numbers on the local heat transport is shown in Figures 10 and 11, respectively. It can be seen that the local heat transport decreases for D_f and Rd. For the horizontal plate flow, the Nusselt number decreases promptly compared to the wedge and stagnation point flow cases. The reverse behavior of Nu is shown in Figure 11b at varying Prandtl numbers Pr.

Figure 2: Influence of M on $F'(\eta)$. (a) 2D and (b) 3D view.

Figure 3: Stimulus of (a) Rd and (b) Pr on $\beta(\eta)$.

Figure 4: 3D view of $\beta(\eta)$ for (a) Rd and (b) Pr .

Figure 5: Stimulus of (a) Df and (b) Ec on $\beta(\eta)$.

Figure 6: Stimulus of (a) γ and (b) Sc on $\phi(\eta)$.

Figure 7: 3D view of $\phi(\eta)$ for (a) γ and (b) Sc .

Figure 8: Influence of Sr on $\phi(\eta)$ for (a) 2D and (b) 3D view.

Figure 9: Stimulus of (a) m and (b) M on $F''(0)$. Figure 10: Stimulus of (a) D_f and (b) Rd on $-\beta'(0)$. Figure 11: Stimulus of (a) D_f and (b) Pr on $-\beta'(0)$.

Figure 12: Stimulus of (a) Sc and (b) γ on $-\phi'(0)$.

Figure 13: Stimulus of (a) Sr and (b) γ on $-\phi'(0)$.

Figure 14: Stimulus of (a) Sr and (b) Sc on $-\phi'(0)$.

Figure 12a and b highlight the variations in the Sherwood number as a function of Sc and γ , respectively. It is noted that at smaller Sc and chemical parameters γ , variations in local mass transfer are quite small. However, for Sc , these effects are prominent as compared to the chemically reacting fluid. A decreasing behavior of the local mass transfer coefficient is observed in Figures 13a and 14a, respectively, for increasing the chemical reaction parameter and the Soret number horizontally. Furthermore, it can be seen from Figures 13b and 14b that the mass transfer coefficient varies linearly.

5 Conclusion

The fluid flow characteristics over a wedge are reported by incorporating the influences of the imposed magnetic field, thermal radiation, and chemical reactions. From the analysis, it is supposed that the radiative heat flux leads to an increase in the fluid temperature, and decreases in the temperature are noted for the Prandtl number. The temperature $\beta(\eta)$ increases due to cross-diffusion of gradients and dissipative phenomena. The decrease in the concentration trends is reported for the higher Schmidt number while the Soret number increases the concentration field. Moreover, more surface shear stresses are noted for stronger magnetic effects, and local mass transfer increases for stronger Schmidt effects.

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