Research Article

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On the optical solutions to nonlinear Schrödinger equation with second-order spatiotemporal dispersion

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Abstract: In this article, the sine-Gordon expansion method is employed to find some new traveling wave solutions to the nonlinear Schrödinger equation with the coefficients of both group velocity dispersion and second-order spatiotemporal dispersion. The nonlinear model is reduced to an ordinary differential equation by introducing an intelligible wave transformation. A set of new exact solutions are observed corresponding to various parameters. These novel soliton solutions are depicted in figures, revealing the new physical behavior of the acquired solutions. The method proves its ability to provide good new approximate solutions with some applications in science. Moreover, the associated solution of the presented method can be extended to solve more complex models.

Keywords: solitary wave, Schrödinger equation, sine-Gordon expansion method

1 Introduction

The nonlinear Schrödinger equation (NLSE) is one of the most powerful generic family of models, fascinating great attention of both mathematicians and physicists because of their potential applications in the recent era of the optical theory. A lot of natural complex phenomena can be described by this model of the nonlinear type. A good understanding of the solutions, configurations, interdependence, and supplementary features may contribute to a more study of more complex models in several areas of science and engineering. For example, electromagnetic theory, condensed matter physics, acoustics, cosmology, and plasma physics are some of the areas that benefit from studying this type of equation. With these above-mentioned applications, the need to further study the NLSE is of interest and this was the motivation to investigate more about the behavior of this model. The study with an effective method which may provide accurate results with physical meaning is an ongoing research for such model and similar ones. In the sense of fractional calculus, an extended model of the NLSE can be proposed and studied to take into account the effect of the fractional term. Fractional calculus has a great amount of work for solving models with applications and continues to prove the ability to provide more realistic models. For example, optimal control of diabetes [1], blood ethanol concentration system modeling [2], and dengue fever modeling [3] are some of the real-life applications of models with fractional derivatives. We are interested in the future to simulate the fractional NLSE. For more details regarding other areas of application, one may see refs. [4–30].

Many researchers were interested in nonlinear models due to their complexity. Analytical solutions can elucidate the physical behavior of a natural system more accurately corresponding to a particular process. New, innovative, and accurate techniques are being developed to find a

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new solution to nonlinear equations, which may contribute in recent areas of science and technology. Recently, many numerical and analytical approaches are being developed such as the auxiliary equation method [31], Cole-Hopf transformation, exp-function method [32], sine-cosine method [33], Darboux transformation [34], Hirota method [35], Lie group analysis [36], modified simple equation method [37], similarity reduced method, tanh method, inverse scattering scheme [38], Bäcklund transform method [39], homogeneous balance scheme [40], sine-cosine method, tanh-coth method, extended FAN sub-equation method [41], auxiliary equation method [42], and many more.

One of these important and effective methods that may provide good solutions with important physical behaviors is the sine-Gordon expansion method. The method has been used numerous times for solving different science and engineering models of physical importance. For example, Baskonus in ref. [43] applied this method for investigating the behavior of a Davey–Stewartson equation with power-law nonlinearity, which has some applications in fluid dynamics. Also, Yel et al. [44] adopted the same method for solving the new coupled Konno–Oono equation acquiring new solitons like solutions. In ref. [45], the method is used to find new dark-bright solitons for the shallow water wave model. Other related models that have been solved using this method including Fokas–Lenells equation [46], nonautonomous NLSEs equations [47], conformable time-fractional equations in RLW-class [48], 2D complex Ginzburg–Landau equation [49], time-fractional Fitzhugh–Nagumo equation [50], and references therein. It is worth mentioning that this study is the first to be dealing with finding the solution to the Schrödinger equation with the coefficients of both group velocity dispersion and second-order spatiotemporal dispersion using this method.

In the present article, we use the sine-Gordon expansion method to derive exact traveling wave solutions for the NLSE with its coefficients of both group velocity and spatiotemporal dispersion. The model can take the following form:

$$i\left(\frac{\partial q}{\partial x} + \alpha \frac{\partial q}{\partial t}\right) + \beta \frac{\partial^2 q}{\partial t^2} + \gamma \frac{\partial^2 q}{\partial x^2} + |q|^2 q = 0, \quad (1)$$

where $q(x, t)$, α , β , and γ are defined in refs. [51–53].

This article is organized as follows. In Section 2, we describe the sine-Gordon expansion method. The application of the method is presented in Section 3. The conclusions are drawn in Section 4.

2 sine-Gordon expansion method

The main steps of the sine-Gordon expansion method are described below to determine an exact solution for the partial differential equation. The sine-Gordon equation can take the following form [48,54]:

$$u_{xx} - u_{tt} = m^2 \sin(u), \quad (2)$$

where $u = u(x, t)$ and m is a constant. Next, equation (2) can be reduced into a nonlinear ordinary differential equation with the aid of a traveling wave transform $u(x, t) = U(\xi)$, $\xi = x - vt$ into the following:

$$U'' = \frac{m^2}{1 - v^2} \sin(U), \quad (3)$$

where v is the wave velocity in the aforementioned wave transform. Then, by multiplying both sides of equation (3) with the term U' and integrating one, we reach the following:

$$\left(\left(\frac{U}{2}\right)'\right)^2 = \frac{m^2}{1 - v^2} \sin^2\left(\frac{U}{2}\right) + C, \quad (4)$$

where C is an integration constant. Assuming that $C = 0$, $\frac{U}{2} = H(\xi)$, and $\frac{m^2}{1 - v^2} = a^2$ in equation (4), we obtain

$$H' = a \sin(H), \quad (5)$$

and by replacing the coefficient $a = 1$ into equation (5), we acquire the following equation:

$$H' = \sin(H). \quad (6)$$

As can be seen, equation (6) can be considered as the known sine-Gordon equation with a simplified form. Now, to solve equation (6), we adapt the separation of variables method and with some simplifications, one can find the following relations:

$$\sin(H(\xi)) = \operatorname{sech}(\xi), \quad \cos(H(\xi)) = \tanh(\xi), \quad (7)$$

$$\sin(H(\xi)) = i \operatorname{csch}(\xi), \quad \cos(H(\xi)) = \coth(\xi). \quad (8)$$

Now, consider a nonlinear partial differential equation as follows:

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \dots) = 0, \quad (9)$$

by using the transformation $u(x, t) = U(\xi)$ with $\xi = x - vt$, equation (8) can be converted into the following form:

$$G(U, U', U'', \dots) = 0. \quad (10)$$

The trial solution to equation (9) is assumed to be of the following form:

$$U(H) = \sum_{j=1}^N \cos^{j-1}(\xi) [B_j \sin(H) + A_j \cos(H)] + A_0. \quad (11)$$

Based on equations (7) and (8), the solution of equations (11) can be written as follows:

$$U(\xi) = \sum_{j=1}^N \tanh^{j-1}(\xi) [B_j \operatorname{sech}(\xi) + A_j \tanh(\xi)] + A_0 \quad (12)$$

and

$$U(\xi) = \sum_{j=1}^N \cos^{j-1}(\xi) [iB_j \operatorname{csch}(\xi) + A_j \coth(\xi)] + A_0, \quad (13)$$

where N is an integer value that can be calculated by balancing the terms of the highest derivative with the non-linear terms. Inserting equation (11) into (10) and some algebra, yields a polynomial equation in $\sin^j(H)\cos^j(H)$. Then, by setting the coefficients of $\sin^j(H)\cos^j(H)$ to zero will result in a set of over-determined algebraic equations in A_j , B_j , and v . Next, the algebraic system is tried to be solved for the coefficients A_j , B_j , and v . For the last step, A_j , B_j values are substituted into equations (12) and (13), which will result in the new solution to equation (9) in the form of a traveling wave.

3 Application of the method

To begin, we take the travelling wave transformation as:

$$q(x, t) = U(\xi)e^{i\phi}, \quad \xi = x - vt, \quad \phi = -\kappa x + \omega t + \theta_0, \quad (14)$$

where

$$q_x = (U' - i\kappa U)e^{i\phi}, \quad q_t = (-vU' + i\omega U)e^{i\phi}, \quad (15)$$

$$\begin{aligned} q_{xx} &= (U'' - 2i\kappa U' - \kappa^2 U)e^{i\phi}, \\ q_{tt} &= (v^2 U'' - 2i\omega v U' - \omega^2 U)e^{i\phi}. \end{aligned} \quad (16)$$

Case I:

$$A_0 = 0, \quad A_1 = \pm \sqrt{\frac{-2(\alpha^2 \gamma + \beta)}{4\beta^2 \omega^2 + 4\alpha\beta\omega + \alpha^2 + 8\beta\gamma}}, \quad B_1 = 0,$$

$$\kappa = \frac{1}{2\gamma} \left[1 \pm 8 \left(\left(-\left(\gamma\omega^2\beta + \gamma\omega\alpha + 2\gamma^2 - \frac{1}{4} \right) \left(\beta\omega + \frac{1}{2}\alpha \right)^2 \left(\beta^2\omega^2 + \beta(\alpha\omega + 2\gamma) + \frac{1}{4}\alpha^2 \right) \right) (4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 8\beta\gamma)^{-1} \right)^{\frac{1}{2}} \right].$$

Substituting equation (14) into equation (1), we have

$$i(1 - \alpha v)U' - (\alpha\omega - \kappa)U + (\beta v^2 + \gamma)U'' - 2i(\omega v\beta + \gamma\kappa)U' - (\beta\omega^2 + \gamma\kappa^2)U + U^3 = 0. \quad (17)$$

Imaginary part:

$$1 - \alpha v - 2(\omega v\beta + \gamma\kappa) = 0 \quad \Rightarrow \quad v = \frac{1 - 2\gamma\kappa}{\alpha + 2\omega\beta}. \quad (18)$$

Real part:

$$(\beta v^2 + \gamma)U'' + (\kappa - \alpha\omega - \beta\omega^2 + \gamma\kappa^2)U + U^3 = 0. \quad (19)$$

By applying equation (18) in equation (19), we get

$$\left(\beta \left(\frac{1 - 2\gamma\kappa}{\alpha + 2\omega\beta} \right)^2 + \gamma \right) U'' + (\kappa - \alpha\omega - \beta\omega^2 + \gamma\kappa^2)U + U^3 = 0. \quad (20)$$

Thus, we obtain

$$\begin{aligned} &(\beta(1 - 2\gamma\kappa)^2 + \gamma(\alpha + 2\omega\beta)^2)U'' \\ &+ (\alpha + 2\omega\beta)^2((\kappa - \alpha\omega - \beta\omega^2 + \gamma\kappa^2)U + U^3) = 0. \end{aligned} \quad (21)$$

With the aid of the homogenous principle, and by balancing the two terms U'' and U^3 will yield $N = 1$.

With $N = 1$, equations (11), (12), and (13) take the form

$$U(H) = B_1 \sin(H) + A_1 \cos(H) + A_0, \quad (22)$$

$$U(\xi) = B_1 \operatorname{sech}(\xi) + A_1 \tanh(\xi) + A_0, \quad (23)$$

and

$$U(\xi) = iB_1 \operatorname{csch}(\xi) + A_1 \coth(\xi) + A_0. \quad (24)$$

Then, by substituting the form of equation (22) along with its second derivative into (21), a polynomial in powers of a hyperbolic function form will result. By setting the summation of the coefficients of the trigonometric identities with the same power to zero, we find a group of algebraic equations. This set of equations is simplified and the parameter values can be found. For each case, the solution of equation (1) can be found by substituting the values of the parameters into equations (23) and (24) and then, into equation (14).

From (14), we deduce the following exact solutions:

$$q_1(x, t) = \pm \sqrt{\frac{-2(\alpha^2\gamma + \beta)}{4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 8\beta\gamma}} \tanh(x - vt) \\ \times \exp \left[i \left(-\frac{1}{2\gamma} \left[1 \pm 8 \left(-\left(\gamma\omega^2\beta + \gamma\omega\alpha + 2\gamma^2 - \frac{1}{4} \right) \left(\beta\omega + \frac{1}{2}\alpha \right)^2 \right. \right. \right. \right. \\ \left. \left. \left. \times \left(\beta^2\omega^2 + \beta(\alpha\omega + 2\gamma) + \frac{1}{4}\alpha^2 \right) \right) (4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 8\beta\gamma)^{-1} \right]^{\frac{1}{2}} \right] x + \omega t + \theta_0 \right),$$

and

$$q_2(x, t) = \pm \sqrt{\frac{-2(\alpha^2\gamma + \beta)}{4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 8\beta\gamma}} \coth(x - vt) \\ \times \exp \left[i \left(-\frac{1}{2\gamma} \left[1 \pm 8 \left(-\left(\gamma\omega^2\beta + \gamma\omega\alpha + 2\gamma^2 - \frac{1}{4} \right) \left(\beta\omega + \frac{1}{2}\alpha \right)^2 \right. \right. \right. \right. \\ \left. \left. \left. \times \left(\beta^2\omega^2 + \beta(\alpha\omega + 2\gamma) + \frac{1}{4}\alpha^2 \right) \right) (4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 8\beta\gamma)^{-1} \right]^{\frac{1}{2}} \right] x + \omega t + \theta_0 \right).$$

Case II:

$$A_0 = 0, \quad A_1 = 0, \quad B_1 = \pm \sqrt{\frac{-2(\alpha^2\gamma + \beta)}{-4\beta^2\omega^2 - 4\alpha\beta\omega - \alpha^2 + 4\beta\gamma}}, \quad B_1 = 0, \\ \kappa = \frac{1}{2\gamma} \left[1 \pm 8 \sqrt{\frac{-\left(\gamma\omega^2\beta + \gamma\omega\alpha - \gamma^2 - \frac{1}{4} \right) \left(\beta\omega + \frac{1}{2}\alpha \right)^2 \left(\beta^2\omega^2 + \beta(\alpha\omega - \gamma) + \frac{1}{4}\alpha^2 \right)}{-4\beta^2\omega^2 - 4\alpha\beta\omega - \alpha^2 + 4\beta\gamma}} \right].$$

From (14), we deduce the following exact solutions:

$$q_3(x, t) = \pm \sqrt{\frac{-2(\alpha^2\gamma + \beta)}{-4\beta^2\omega^2 - 4\alpha\beta\omega - \alpha^2 + 4\beta\gamma}} \operatorname{sech}(x - vt) \\ \times \exp \left[i \left(-\frac{1}{2\gamma} \left[1 \pm 8 \left(-\left(\gamma\omega^2\beta + \gamma\omega\alpha - \gamma^2 - \frac{1}{4} \right) \left(\beta\omega + \frac{1}{2}\alpha \right)^2 \right. \right. \right. \right. \\ \left. \left. \left. \times \left(\beta^2\omega^2 + \beta(\alpha\omega - \gamma) + \frac{1}{4}\alpha^2 \right) \right) (-4\beta^2\omega^2 - 4\alpha\beta\omega - \alpha^2 + 4\beta\gamma)^{-1} \right]^{\frac{1}{2}} \right] x + \omega t + \theta_0 \right),$$

and

$$q_4(x, t) = \pm \sqrt{\frac{2(\alpha^2\gamma + \beta)}{-4\beta^2\omega^2 - 4\alpha\beta\omega - \alpha^2 + 4\beta\gamma}} \operatorname{csch}(x - vt) \\ \times \exp \left[i \left(-\frac{1}{2\gamma} \left[1 \pm 8 \left(-\left(\gamma\omega^2\beta + \gamma\omega\alpha - \gamma^2 - \frac{1}{4} \right) \left(\beta\omega + \frac{1}{2}\alpha \right)^2 \right. \right. \right. \right. \\ \left. \left. \left. \times \left(\beta^2\omega^2 + \beta(\alpha\omega - \gamma) + \frac{1}{4}\alpha^2 \right) \right) (-4\beta^2\omega^2 - 4\alpha\beta\omega - \alpha^2 + 4\beta\gamma)^{-1} \right]^{\frac{1}{2}} \right] x + \omega t + \theta_0 \right).$$

Case III:

$$A_0 = 0, \quad A_1 = \pm \frac{1}{2} \sqrt{\frac{-2(\alpha^2\gamma + \beta)}{4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 2\beta\gamma}}, \quad B_1 = \pm \frac{1}{2} \sqrt{\frac{2(\alpha^2\gamma + \beta)}{4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 2\beta\gamma}},$$

$$\kappa = \frac{1}{2\gamma} \left[1 \pm \sqrt{\frac{-(4\gamma\omega^2\beta + 4\gamma\omega\alpha + 2\gamma^2 - 1)(2\beta\omega + \alpha)^2(4\beta^2\omega^2 + 2\beta(2\alpha\omega + \gamma) + \alpha^2)}{4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 2\beta\gamma}} \right].$$

From (14), we deduce the following exact solutions:

$$q_5(x, t) = \pm \sqrt{\frac{2(\alpha^2\gamma + \beta)}{4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 2\beta\gamma}} (\operatorname{sech}(x - vt) + i \tanh(x - vt))$$

$$\times \exp \left[i \left(-\frac{1}{2\gamma} (1 \pm ((-(4\gamma\omega^2\beta + 4\gamma\omega\alpha + 2\gamma^2 - 1)(2\beta\omega + \alpha)^2 \right. \right.

$$\left. \left. \left. \times (4\beta^2\omega^2 + 2\beta(2\alpha\omega + \gamma) + \alpha^2)(4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 2\beta\gamma)^{-1} \right)^{\frac{1}{2}} \right) x + \omega t + \theta_0 \right],$$$$

and

$$q_6(x, t) = \pm \sqrt{\frac{-2(\alpha^2\gamma + \beta)}{4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 2\beta\gamma}} (\operatorname{csch}(x - vt) + \coth(x - vt))$$

$$\times \exp \left[i \left(-\frac{1}{2\gamma} (1 \pm ((-(4\gamma\omega^2\beta + 4\gamma\omega\alpha + 2\gamma^2 - 1)(2\beta\omega + \alpha)^2 \right. \right.

$$\left. \left. \left. \times (4\beta^2\omega^2 + 2\beta(2\alpha\omega + \gamma) + \alpha^2)(4\beta^2\omega^2 + 4\alpha\beta\omega + \alpha^2 + 2\beta\gamma)^{-1} \right)^{\frac{1}{2}} \right) x + \omega t + \theta_0 \right].$$$$

Figure 1: Graphical representation of solution $q_1(x, t)$ with the parameter values as: $\varsigma_1 = 2, \varsigma_2 = 3, \varsigma_3 = 1, \vartheta_1 = 3, \vartheta_2 = 1, \vartheta_3 = 1, \alpha = 3, \beta = 2, \gamma = 4, \mu = 3, v = 2, \omega = 3$.

Figure 2: Graphical representation of solution $q_3(x, t)$ with the parameter values as: $\varsigma_1 = 2, \varsigma_2 = 3, \varsigma_3 = 1, \vartheta_1 = 5, \vartheta_2 = 3, \vartheta_3 = 2, \alpha = -1, \beta = 2, \gamma = -2, \mu = 2, v = -2, \omega = 3$.

4 Graphical representation of solutions

In this section, the solitons solution for the main equation for different cases and different values of the parameters is being investigated and represented throughout the following figures with the help of Mathematica 11.0.

5 Conclusions

In this study, the sine-Gordon expansion method was employed to integrate the NLSE with the coefficients of group velocity dispersion and second-order spatiotemporal dispersion. Some new traveling wave solutions are found while changing the values of the parameters.

Figure 3: Graphical representation of solution $q_3(x, t)$ with the parameter values as: $\varsigma_1 = 2$, $\varsigma_2 = 4$, $\varsigma_3 = 1$, $\vartheta_1 = 1$, $\vartheta_2 = 2$, $\vartheta_3 = 3$, $\alpha = 3$, $\beta = 2$, $\gamma = -2$, $\mu = 2$, $\nu = 3$, $\omega = 2$.

Figure 5: Graphical representation of solution $q_5(x, t)$ with the parameter values as: $\varsigma_1 = 2$, $\varsigma_2 = -3$, $\varsigma_3 = 4$, $\vartheta_1 = 0$, $\vartheta_2 = -1$, $\vartheta_3 = 1$, $\alpha = -4$, $\beta = 2$, $\gamma = -2$, $\mu = 2$, $\nu = 1$, $\omega = -3$, $\lambda_1 = -1$, $\lambda_2 = 1$.

Figure 4: Graphical representation of solution $q_4(x, t)$ with the parameter values as: $\varsigma_1 = 2$, $\varsigma_2 = 3$, $\varsigma_3 = 5$, $\vartheta_1 = 4$, $\vartheta_2 = 5$, $\vartheta_3 = 3$, $\alpha = 4$, $\beta = 3$, $\gamma = -4$, $\mu = 2$, $\nu = 2$, $\omega = 3$.

Figure 6: Graphical representation of solution $q_6(x, t)$ with the parameter values as: $\varsigma_1 = 2$, $\varsigma_2 = 1$, $\varsigma_3 = 3$, $\vartheta_1 = 0$, $\vartheta_2 = 3$, $\vartheta_3 = -2$, $\alpha = 4$, $\beta = 2$, $\gamma = 2$, $\mu = 2$, $\nu = 4$, $\omega = 2$.

The new form of solutions possesses some novel traveling wave behaviors. A graphical representation of these solutions is provided in Figures 1–6. The proposed method is shown to provide a solution with important physical representation which may help in dealing with similar complex nonlinear models with applications in contemporary science and other related areas. The method proves to be a reliable method for solving such models with high accuracy. This work, thus, provides a lot of encouragement for subsequent research in this area, and the results of that research will be reported in near future.

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