Research Article

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Crystallization law of karst water in tunnel drainage system based on DBL theory

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Abstract: When a tunnel is constructed in a karst area, crystallization of the drainage pipe caused by karst water often threatens the normal operation of the tunnel. This work contributes to this field of research by proposing a functional model based on the diffusion boundary layer (DBL) theory proposed by Dreybrodt in the 1990s. The model is formed by determining the flow rate distribution of the drainage pipe in a laminar flow state and turbulent state, and then by applying Fick's diffusion law and Skelland's approximate formula. Then, to further verify the applicability of the functional model, a model test was carried out in the laboratory and the test results are compared to the theoretical results. The results show that the crystallization rate of karst water is mainly affected by the roughness of the pipe wall, followed by the slope of pipes. The slope can affect flow state by controlling the flow rate, which in turn affects the crystallization rate of karst water. When the slope of the drainage pipe is 3, 4, and 5%, the error between the experimental results and the theoretical calculation results is 24.7, 8.07, and 27.9%, respectively, and when the liquid level in the pipe is 7.2, 10.2, and 13.3 mm, the error is 27.9, 9.82, and 2.07%, respectively. Considering that the flow will take away the crystalline deposits on the pipe wall in the experiment, although some results have certain errors, they do not affect the overall regularity.

Keywords: tunnel engineering, DBL theory, crystallization blockage, crystallization rate model, model test

1 Introduction

China has the largest karst area in the world, accounting for 13.5% of the total area of the country; in particular, the proportion of the karst area in the southwest is as high as 43% of the total karst landform area in China. In view of the mountainous geological environment in Southwest China, tunnels and bridges are widely used in the traffic development process; the proportion of bridges and tunnels in the traffic infrastructure is more than 70%, and as high as 90% in some mountainous areas [1]. In addition to the widely distributed karst caves and underground rivers, accidents are prone to occur during the construction of tunnels, which increases the construction and operation costs [2,3].

Water is the natural antagonist of tunnel engineering. In the early stage of tunnel construction, water and mud inrush caused by confined water poses a serious threat to tunnel excavation activities [4–7]. In the later stage, the leakage of lining water and the blockage of drainage pipes will not only affect the safety and normal service life of the tunnel [8–10], but will also damage the ground buildings and water environment around the tunnel [11,12]. Due to the unique geological and hydrological conditions of the karst geomorphic area of Southwest China, there are large amounts of HCO_3^- and Ca^{2+} (the ion content is more than 82%). In the weakly alkaline environment (the pH value is 7.8–8.3), crystal precipitation easily occurs. In addition, the distribution of underground karst water is very uneven, and the “tunnel life pipelines,” which are the vertical, horizontal, and circular drainage pipes, are always affected by crystal precipitation. Therefore, it is necessary to study the plugging mechanism of tunnel drainage pipes in karst areas. The main karst landforms and related engineering locations in China are shown in Figure 1.

The transportation and accumulation of solid particles by flow [13] and the dissolution and redeposition of soluble rock will cause the blockage of drainage pipes [14–17]. The former is a physical action, while the latter is a chemical action and often lasts for a long time; in fact, the latter is calcite calcification. To investigate the

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Figure 1: The distribution of the main karst landforms in China and the research location.

crystal blockage of tunnel drainage pipes, it is essential to study the dynamic deposition of calcite in the pipes, and the two well-known models by which to study this are the PWP surface reaction control model [18] proposed in the 1970s and the diffusion boundary layer (DBL) flow system control model [19] proposed in the early 1990s. Compared with the PWP model, the DBL model considers the influences of hydrodynamic conditions (system fluidity) on the dissolution and deposition rates of calcite, the control of the CO_2 conversion rate in solution, and the molecular diffusion effect of reactants near the solid-liquid boundary layer; these considerations improve the accuracy of the calculation results. The field test conducted by Liu *et al.* [20,21] further validated the accuracy of this model. Since the beginning of the twenty-first century, the introduction of numerical methods, such as the finite element method and discrete element method, has provided convenience for the study of this problem [22,23]. However, too many assumptions in the diffusion model result in the numerical simulation results being quite different from practical engineering, and it can therefore only be used for qualitative analysis.

In recent years, the research on tunnel drainage system crystallization has mainly been conducted from two aspects. The first is the influencing factors, such as the water hardness, alkalinity, salinity, temperature,

pH, hydraulic gradient, flow rate, soil and water, micro-organism, etc. [24–28,13]; however, these factors are relatively single, and the comprehensive influence of multiple factors has not been considered. The other aspect is the analysis of methods for the elimination of crystal precipitates. Many scholars have studied the effect of flexible shock [29,30], geotextile filtration [31,32], and flocking drainpipe [33,34] on the elimination of crystal-line deposits in drainage pipes from the perspective of theory and experiment.

The previous research on the crystallization problem of tunnel drainage pipes is mostly based on the physical properties of water itself and studying the crystallization phenomenon occurring in the water. While in actual tunnel engineering, the crystallization mostly occurs on the wall of the drainage pipe, which shows that the crystallization is not only related to water quality, but also the contact state between the water and the pipe wall. This paper, based on the DBL theory, analyzes the flow rate distribution of karst water in the drainage pipe under different flow patterns and establishes the crystallization rate function of karst water on drainage pipe wall on the basis of considering the protrusion length and friction coefficient of drainage pipe wall. Then, a model test is carried out for three influencing factors, namely, the slope of the drainage pipe, the height of the free liquid

level in the pipe, and the roughness of the inner wall of the drainage pipe. The experimental results are collected and the validity of the function is verified, hence providing a theoretical basis for the study of the crystallization blockage problem in karst areas.

2 Functional model of the karst water crystallization rate in a tunnel drainage pipe

2.1 Functional model of the karst water crystallization rate in a laminar flow state

2.1.1 Karst water laminar flow rate distribution in a tunnel drainage pipe

The DBL model is primarily used to predict the calcification deposition rate of a flow system [19], and the flow characteristics (hydrodynamic conditions) are very important for model establishment. The DBL thickness in different hydrodynamic conditions is significantly different and will seriously affect the precipitation crystallization rate. Therefore, it is necessary to determine the karst water flow rate distribution before analyzing the crystallization law.

The karst water flow state in the tunnel drainage pipe is complex, and whether the flow fills the entire drainage pipe is unknown. In fact, when the flow fills the entire pipe, it is accompanied by a substantial amount of kinetic energy, and it is difficult for the flow to maintain a laminar state under the action of the internal momentum. Therefore, it is reasonable to consider that the tunnel drainage pipe is not full to study the karst water flow rate distribution in a laminar flow state. The general flow state of karst water in a drainage pipe in a laminar state is presented in Figure 2.

Most tunnel drainage pipes are round, and the viscous force is greater than the particle inertial force in the laminar state. In this case, the flow state is very stable, the fluid particles are not doped with each other, the momentum and energy are not obviously exchanged, and the liquid velocity in the pipe has an approximately parabolic distribution under the viscous force. The friction binding force is the largest at the pipe wall, and the flow rate is close to zero; in contrast, the friction binding

Figure 2: Height of the free liquid level in drainage pipe.

Figure 3: Flow rate distribution in the laminar flow state.

force near the axis of the drainage pipe is the least, and the flow rate is the largest, as shown in Figure 3. The quadratic function expression of water in a drainage pipe is given as follows:

$$u = u_{\max}(R^2 - x^2 - h^2) \quad (1)$$

where $u_{\max}$ is the largest flow rate in the section when the tunnel drainage pipe is full, h is the height of the free liquid level in the drainage pipe, and R is the radius of the drainage pipe.

The laminar shear stress can be calculated according to equation (2):

$$\tau = \rho g i R' \quad (2)$$

where τ is the laminar shear stress, ρ is the liquid density, g is the acceleration of gravity and is generally considered to be 9.8 m/s^2 , i is the slope of the drainage pipe, R' is the hydraulic radius, and when the pipe is filled with water, $R' = R/2$, where R is the drainage pipe radius.

When a Newtonian fluid moves in a circular pipe, the inner part of the liquid follows the Newtonian friction law. When karst water flows in a drainage pipe, the following is true:

$$\tau = -\mu \frac{du}{dr} \quad (3)$$

where μ is the dynamic viscosity coefficient, which is related to liquid properties, and we take the value as the dynamic viscosity of water; u is the flow rate at a liquid-free surface which away from the pipe axis is r . Substitution of equation (2) into equation (3) yields the following:

$$du = -\frac{\rho g i R}{2\mu} dr \quad (4)$$

Simultaneously integrating both sides of this equation yields the following:

$$u = -\frac{\rho g i R^2}{4\mu} + C \quad (5)$$

By substituting the critical condition $r = R$, $u = 0$, the constant $C = \frac{\rho g i R^2}{4\mu}$ can be obtained. By substituting C into equation (5), the following can be obtained:

$$u(r) = \frac{\rho g i}{4\mu} (R^2 - r^2) \quad (6)$$

Substitution of $r = 0$ into equation (6) yields the following:

$$u_{\max} = \frac{\rho g i R^2}{4\mu} \quad (7)$$

The flow rate quadratic rectangular coordinate function of the height of the free water level can then be determined as follows:

$$u = \frac{\rho g i}{4\mu} (R^2 - x^2 - z^2) \quad z \leq h \quad (8)$$

2.1.2 Functional model of the karst water crystallization rate in a laminar flow state

The dissolution or deposition flux of calcite in the DBL model depends on the DBL thickness. To calculate the DBL thickness, the Skelland equation can be introduced [35]:

$$\Delta x = 2.7 L^{\frac{1}{5}} \left(\frac{\nu}{u} \right)^{\frac{4}{5}} \quad (9)$$

where Δx is the DBL thickness, L is the length of the test piece along the flow direction, ν is the viscosity coefficient, and $\nu = \mu/\rho$.

The DBL is very thin at the solid-liquid interface. To calculate the DBL thickness, the flow rate in the limit region must be investigated. While equation (8) can be used to determine the flow rate distribution in a laminar state, it is necessary to further obtain the flow rate near the pipe wall. With the assistance of the Taylor series, the second-order expansion of equation (6) at $r = R$ can be determined:

$$u = 0 + \frac{\rho g i}{2\mu} (R - r) + \frac{\rho g i}{2\mu} (R - r)^2 + o[(R - r)^2] \quad (10)$$

Because the DBL is very thin, the higher-order trace can be omitted, and the flow rate near the pipe wall can be determined:

$$u = \frac{\rho g i R}{2\mu} \Delta r \quad (11)$$

where $\Delta r = (r_0 - r)$ is a distance extremely close to the pipe wall, and it can be approximated that $\Delta r = \Delta x$. The flow rate is then approximately proportional to the distance in the DBL. Therefore, equation (11) can be expressed by equation (12):

$$u = \frac{\rho g i R}{2\mu} \Delta x \quad (12)$$

Substitution of equation (12) into equation (9) yields the following:

$$\Delta x = 2.36 \left(\frac{L \mu^8}{(g i R)^4} \right)^{\frac{1}{9}} \quad (13)$$

where L is considered to be the internal protrusion length of the drainage pipe in this work.

In equation (13), the factors that affect the molecular diffusion effect in the DBL are comprehensively considered; these include not only the external environmental factors (the slope of the drainage pipe i , the diameter of the drainage pipe R , the internal protrusion length of the drainage pipe L), but also the internal viscous effect (the fluid density ρ and dynamic viscosity coefficient μ). According to equation (13), with the increase of the slope and radius of the pipe, the flow rate near the pipe wall will increase, and the DBL thickness will decrease. Additionally, with the increase of the internal protrusion length of the drainage pipe, the DBL thickness will increase.

In the limited space of the drainage pipe, the diffusion speed of components in the karst water is very slow. In the DBL area, because of the viscous resistance of the

water, the solution is almost static and the diffusion speed of the solute is slower, so it can be approximately considered that the diffusion process in the DBL is stable diffusion. Based on this assumption and Fick's stable diffusion law, the deposition rate in the DBL can be expressed by equation (14):

$$F = \frac{D}{\Delta x} (C_{\text{Ca}^{2+}(\text{solution})} - C_{\text{Ca}^{2+}(\text{surface})}) \quad (14)$$

where D is the molecular diffusion coefficient, and its value is $5 \times 10^{-4} \text{ mm}^2/\text{s}$ [13]; $C_{\text{Ca}^{2+}(\text{solution})}$ is the concentration of Ca^{2+} in the aqueous solution in the pipe, $C_{\text{Ca}^{2+}(\text{surface})}$ is the concentration of Ca^{2+} on the surface of the crystal precipitates, and it is determined by $C_{\text{Ca}^{2+}(\text{surface})} = \sqrt{K_{\text{CaCO}_3}}$.

By combining equations (13) and (14), the karst water crystallization rate in a laminar state can be determined as follows:

$$F = \frac{D(giR)^{\frac{4}{9}}}{2.36(L\mu^8)^{\frac{1}{9}}} (C_{\text{Ca}^{2+}(\text{solution})} - C_{\text{Ca}^{2+}(\text{surface})}) \quad (15)$$

2.2 Functional model of the karst water crystallization rate in a turbulent state

2.2.1 Karst water turbulent flow rate distribution in a tunnel drainage pipe

According to different flow conditions, the karst water in a tunnel drainage pipe can be divided into three areas, namely, the turbulent core area, transition area, and viscous bottom layer, as illustrated in Figure 4.

Figure 4: Schematic diagram of the flow rate zone in a turbulent state.

- (1) Turbulent core area: In this area, the fluid pulsation rate is very large and the momentum of fluid particles is frequently exchanged; therefore, the flow rate distribution is quite different from that of the laminar state. Based on the mixed length theory, Prandtl proposed the flow rate empirical logarithmic function, and the dimensionless equation of this function is given by equation (16).

$$\frac{u}{u_*} = \frac{1}{k} \ln \frac{u_* y}{\nu} + c \quad (16)$$

where u_* is the friction flow rate, and $u_* = \sqrt{\tau_0/\rho}$, τ_0 is the wall friction; y is the horizontal coordinate along the direction of water; k and c are constants that must be determined by experiments, and under the full-pipe condition, k is 0.4 and c is 5.5 [36].

- (2) Transition area: The particle movement state in this area is between the turbulent core area and viscous bottom layer. This area is very thin and its flow rate distribution and thickness are similar to the viscous bottom layer. The thickness of the transition area cannot be considered when calculating viscous bottom layer.
- (3) Viscous bottom layer: As the boundary layer is close to the solid particles, the flow rate in this area is slower due to the obstruction of the drainage pipe wall, but the flow rate gradient changes substantially. The fluid particles are mainly subjected to viscous shear stress, the flow pattern is basically laminar, and the flow rate is distributed in a straight line. According to Newton's internal friction law, the flow rate in this area can be calculated by equation (17):

$$\frac{u}{u_*} = \frac{u_* y}{\nu} \quad (17)$$

2.2.2 Functional model of the karst water crystallization rate in a turbulent state

The viscous bottom layer is very thin, and it is very similar to a DBL in properties and genesis, so the viscous bottom layer can be assumed as DBL [37]. If y' is set as the DBL thickness of the full pipe and y is set as the DBL thickness of the unfilled pipe, because the flow rates in the bottom of the turbulent core area are continuous in the full-pipe state, the equation (18) is yielded by combining equations (16) and (17):

$$\frac{u_* y'}{v} = \frac{1}{k} \ln \frac{u_* y'}{v} + c \quad (18)$$

Equation (18) is a nonlinear equation, and it is difficult to accurately solve it. To more conveniently fix the DBL thickness in a full pipe, u/u_* is taken as a vertical coordinate and $\ln[(u_* y')/v]$ is taken as a horizontal coordinate. The function image of equations (16) and (17) in the same coordinate system is presented in Figure 5.

According to Figure 5, the critical interval of the cohesive bottom layer (DBL) in the full pipe can be determined by equations (19) and (20).

$$\text{Cohesive bottom layer (DBL)} : \frac{u_* y'}{v} \leq 11.6 \quad (19)$$

$$\text{Turbulent core area} : \frac{u_* y'}{v} > 11.6 \quad (20)$$

Therefore, the DBL thickness in the full pipe can be calculated by equation (21):

$$y' = \frac{11.6v}{u_*} \quad (21)$$

According to the Darcy–Weisbach equation, equation (21) can be transformed into equation (22):

$$y' = \frac{32.8R}{\text{Re} \sqrt{\lambda}} \quad (22)$$

where R is the diameter of the drainage pipe, λ is the head loss coefficient, and the Reynolds number (Re) can be calculated by equation (23):

$$\begin{aligned} \text{Re} = \psi(i, h) &= \frac{i\varphi(h)}{\mu} \\ &= -\frac{\rho g i}{l \mu^2} \int_{-\sqrt{R^2-h^2}}^{\sqrt{R^2-h^2}} \int_{-h}^{-h} (R^2 - x^2 - z^2) dz dx \end{aligned} \quad (23)$$

Equation (23) is the calculation equation of Re in an underfilled pipe, where l is the wetted perimeter of the liquid surface. According to equation (23), the critical

Figure 5: Distribution function of turbulent flow rate under the full-pipe condition.

Reynolds number Re^* is related to the height of the free liquid level h and the slope of the drainage pipe i , and the viscous shear stress consumes the work of gravity during the flow process.

Compared with the full-pipe state, equation (17) is still valid, and only the values of coefficient k and c in equation (16) change in an underfilled pipe; therefore, the DBL thickness in an underfilled pipe can be calculated by equation (24):

$$y = \frac{kR}{\text{Re} \sqrt{\lambda}} \quad (24)$$

After the tunnel drainage pipe is determined, with the increase of the flow rate, the flow state in the pipe will gradually change from laminar to turbulent, and the process is continuous. Therefore, the critical Reynolds number Re^* is continuous in its neighborhood when the flow state in the DBL changes from laminar to turbulent. Then, by combining equations (13) and (24), the following is obtained:

$$2.36 \left(\frac{L \mu^8}{(g i R)^4} \right)^{\frac{1}{9}} = \frac{KR}{\text{Re}^* \sqrt{\lambda}} \quad (25)$$

The coefficient K can then be determined by equation (26):

$$k = \frac{2.36 \text{Re}^* \sqrt{\lambda}}{R} \left(\frac{L \mu^8}{(g i R)^4} \right)^{\frac{1}{9}} \quad (26)$$

Then, by substituting equation (26) into equation (24), the DBL thickness in a turbulent state is calculated as follows:

$$y = \frac{2.36 \text{Re}^*}{\text{Re}} \left(\frac{L \mu^8}{(g i R)^4} \right)^{\frac{1}{9}} \quad (27)$$

Finally, by combining equations (27) and (14), the karst water crystallization rate in a turbulent state can be calculated as follows:

$$F = \frac{Re}{2.36 Re^*} \frac{D(giR)^{\frac{4}{9}}}{(L\mu^8)^{\frac{1}{9}}} (C_{Ca^{2+}(solution)} - C_{Ca^{2+}(surface)}) \quad (28)$$

3 Model test

3.1 Water sample investigation and test mechanism

The water sample used in this study was taken from the Annaga tunnel of the Guangna Expressway in Yunnan Province, China. The tunnel construction area is located in a subtropical zone, which has a mild climate and abundant rainfall. Moreover, the underground water is rich and the surface water is relatively developed. The groundwater is mainly Quaternary pore water and carbonate karst water, and the karst is widely developed. A test of a groundwater sample revealed that there are many different kinds of ions in the groundwater; the main cations are Na^+ , K^+ , Ca^{2+} , and Mg^{2+} , and the main anions are Cl^- , SO_4^{2-} , and HCO_3^- . Of all the ions, the content of Ca^{2+} is the highest of the cations, accounting for 26.52% of the total ion content, while the content of HCO_3^- is the highest of the anions, accounting for 61.74% of the total ion content. In addition, Al^{3+} , Zn^{2+} , Fe^{2+} , and Fe^{3+} were also found in the water sample, but the proportions were very low; thus, they can be ignored. The specific composition of the water sample is presented in Table 1.

Karst water crystallization experiments take a long time and require higher water quality, and it is difficult

Figure 6: Mechanism of crystallization and precipitation of carbonate solution.

to maintain the cleanliness of an experimental environment and a continuous flow in a construction site. Because these factors would affect the accuracy of experimental results, a model test was carried out. The water sample analysis results show that HCO_3^- and Ca^{2+} are the main ions. Based on the principle of the main contradiction, $Ca(HCO_3)_2$ was chosen as the main solute to prepare the solution and NaOH was used to adjust the pH value of the solution. The experimental mechanism is presented in Figure 6.

3.2 Experimental equipment and solution preparation

A self-developed test system for crystal precipitation in a tunnel drainage pipe that can realize self-circulation was adopted in the model test. The water solution can circulate in the system without human intervention.

The whole system mainly comprised four parts, namely, a water storage system, power water supply system, drainage pipe system, and support system. The water storage system was composed of three customized water tanks, including one main water tank with a capacity of 2,000 L, the size of which was $0.8 \times 1.0 \times 2.5$ m, and two auxiliary water tanks with a capacity of 528 L and a size of $0.4 \times 0.5 \times 2.2$ m. The three water tanks had closed covers and were equipped with the solution. The dynamic system included a KQL80 pump with a rated voltage of 380 V and a power of 220 kW, and the power of the pump could be controlled by a Sanji S1100 frequency converter. The drainage pipe system mainly included an experimental drainage pipe and connecting pipe, and the experimental drainage pipe was the most important component in the experiment; the crystallization quality of the solution on the inner wall was the focus of the experiment. The experimental drainage

Table 1: Analysis of the composition of a water sample from the Annaga tunnel (average)

Polarity	Type	Concentration (mmol/L)	Content percentage (%)
Cation	Na^+	0.003	0.04
Cation	K^+	0.021	0.34
Cation	Ca^{2+}	1.635	26.52
Cation	Mg^{2+}	0.286	4.64
Anion	Cl^-	0.405	6.57
Anion	SO_4^{2-}	0.005	0.08
Anion	HCO_3^-	3.801	61.74

Note: The pH value of the water sample was 7.86.

pipe was a PVC corrugated pipe with a diameter of 100 mm, and the quantity was 10; the connecting pipe was DN110 circular pipe with a diameter of 110 mm, and the quantity was based on the experimental situation. The support system mainly included the main water tank support and the test drainage pipe adjustment support, which were used to adjust the slope of the drainage pipe. The experimental device is presented in Figure 7. In the experiment, the frequency converter was used to adjust water pump 2 to control the solution from the auxiliary tank 4 to flow into the main tank 1 through the connecting conduit 3 at a certain flow rate. The pipe ball valve was adjusted to control the solution flow from the main tank 1 to the auxiliary tanks 4 and 5 through the corrugated pipe 8 at a certain rate, and the solution moved through the whole system according to this mode during the experimental process. The solid crystallization quality was weighed after every cycle by a precision electronic scale.

The selection of the solute is a crucial step in the process of preparing a solution and has a significant impact on the final experimental results. In this experiment, the solution was prepared mainly based on Table 1. However, the stability of $\text{Ca}(\text{HCO}_3)_2$ is not good, and it easily produces CaCO_3 in water. In contrast, the mixture solution of CaCl_2 and NaHCO_3 can maintain the stability

of HCO_3^- and Ca^{2+} at a normal temperature, so they were ideal solutes to make the solution; the details of the solution preparation process are presented in Figure 8. During the process of solution preparation, distilled water produced by a laboratory-level electric distilled water generator was used as a solvent. In process 5, all valves in the test drainage pipe were closed when pouring the solution that was dissolved in a small beaker into the main water tank 1. At this time, the water pump was opened to keep the distilled water in water tank 4 continuously flowing into the main water tank 1 to ensure that the solute in the main water tank 1 could be distributed evenly as soon as possible. In process 6, a pH meter was used to monitor the solution pH in real-time, and the addition of NaOH was stopped when the pH value reached approximately 7.86. During the entire experiment, the solution of each water tank was sampled regularly to ensure that the ion concentration and the pH value were stable.

3.3 Experimental setup

The experiment took place in a hydraulic laboratory. The environmental temperature was relatively stable, the solution temperature was 19.6°C, the CO_2 concentration

Figure 7: Experimental equipment design.

Figure 8: Solution preparation.

was between 520 and 570 ppm, and the effective diameter of the inner wall of the pipes was 97 mm. To avoid errors, the experimental drainage pipes were numbered from No. 1 to No. 10, as shown in Figure 7. The concentration of Ca^{2+} in the solution was 1.635 mmol/L and the concentration of HCO_3^- was 3.801 mmol/L. The measurement cycle of the experiment was 5 days, and the experiment lasted for 30 days. The crystallization quality in each cycle was determined to infer the crystal volume, and then the crystallization thickness in each cycle was calculated by the cross-sectional area of liquid overflow. On this basis, the crystallization rate of the experimental pipes was determined. The initial properties of the experimental pipes are reported in Table 2.

Table 2: Initial properties of experimental pipes

Pipe No.	Initial mass m_0 (g)	Effective length l_1 (cm)	Protuberance length l_2 (cm)	Protuberances quantity (n)
1	476.66	72.05	0.59	55
2	626.43	73.52	0.62	55
3	466.03	73.61	0.62	55
4	438.07	73.62	0.64	55
5	633.52	73.73	0.62	55
6	610.34	73.37	0.68	55
7	442.33	71.26	0.59	55
8	606.38	73.84	0.62	55
9	379.34	73.12	0.61	55
10	467.26	73.13	0.59	55

Note: The protuberance length l_2 is the test piece length L in equation (9).

Previous theoretical investigation has shown that, among the factors that affect the karst water crystallization law in tunnel drainage pipes, the environmental temperature, the physical index of the liquid, and the flow pattern in the drainage pipes are uncontrollable factors. These factors are affected by the geological conditions and are not controlled by manpower, whereas factors such as the drainage pipe diameter, the drainage pipe slope, and the length of the protuberance in the inner wall can be controlled artificially. From this point of view, a control variable experiment was set up for the height of the free liquid level in the pipe, the slope of the drainage pipe, and the friction coefficient, and the setting of these variables is reported in Table 3. The adjustment of the slope of the drainage pipe was realized by adjusting the height of the adjusting bracket under the experimental drainage pipe. The height of the free liquid level in the pipe was adjusted by controlling the valve rotation and the pump power. The inner wall friction coefficient was set by placing cloth with different friction coefficients in the drainage pipe. The inner wall of pipe No. 6 was smoothed.

4 Results and discussion

4.1 Analysis of experimental results

The crystallization blockage of a tunnel drainage pipe is the result of the joint action of internal factors (water quality, water temperature, solution pH, etc.) and external

Table 3: Experimental variable setting

Experiment group	Pipe No.	Height of the free liquid level (mm)	Slope (%)	Inner wall roughness	Initial flow rate (cm/s)
Slope	1	7.2	3	0.26	41.36
	2	7.2	4	0.21	43.73
	3	7.2	5	0.23	45.28
Height of the free liquid level	3	—	—	—	—
	4	10.2	5	0.27	41.83
	5	13.3	5	0.24	39.86
Inner wall roughness	3	—	—	—	—
	6	7.2	5	0.12	43.36
	7	7.1	5	0.53	42.11
	8	6.8	5	0.65	40.41
	9	7.0	5	0.69	43.78
	10	7.3	5	0.83	39.19

factors (drainage pipe material, drainage pipe construction process, etc.). Internal factors are difficult to change due to the influence of geological conditions, whereas external factors can be adjusted. In the experiment, due to the sticking effect of the bulge on the inner wall of the pipe, the flow rate at the bulge was less than the depression. According to equations (15) and (29), the crystallization rate at the bulge is faster and the crystallization effect is more obvious, and these phenomena were verified by the experimental results, as presented in Figure 9. In Figure 9, the yellow areas are crystal precipitates; the darker the color, the more precipitate there is. The precipitate in the drainage pipe was distributed in strips.

Figure 9: Crystallization distribution.

The thickness of the precipitate at the convex position of the inner wall was the thickest and its color was more obvious, whereas there was less precipitate at the concave position of the inner wall and the color was not obvious.

During the experiment, the crystallization quality in the drainage pipe was measured every 5 days, and the change trend of the crystallization quality with time in each cycle is presented in Figure 10. From Figure 10, we can found that the crystallization quality generally increased with time, but the development varied under different experimental conditions.

In the inner wall roughness group, the crystallization quality showed an obvious increasing trend with the increase of the friction coefficient. The crystallization qualities of pipes No. 7–No. 10 were concentrated in 25.0–27.0 g; it was obviously larger than No. 3 and No. 6 with lower friction coefficient. Compared to the other two experiment groups, the crystallization phenomenon of the inner wall roughness group is also obvious, which means the friction coefficient has the greatest impact on crystallization among all the influencing factors, and this is consistent with the expectations. From the microscopic point of view, the greater the friction coefficient, the rougher the water surface, the larger the surface area of karst water contacting the inner wall, the stronger the resistance of the solute in the water flow. This will slow the movement speed of anions and cations in the solution, thereby making the two easier to combine and ultimately resulting in the inevitable acceleration of the crystallization rate, which are complementary.

In the slope group, the crystallization quality of pipeline No. 3 was greater than those of No. 2 and No. 1, and the crystallization quality increased with the increase of the slope, which is consistent with inference of ref. [13]

Figure 10: Crystallization quality of the drainage pipe in the test period.

and the findings of the theoretical calculation model derived previously in this paper. In fact, the gradient controls the flow rate of the solution; the greater the gradient, the greater the flow rate. Thus, the flow in the pipe will convert to a turbulent state, and the critical flow rate will also be faster, thereby thinning the DBL layer [19]. Therefore, the ions in the solute can more easily diffuse through the DBL to the precipitation layer to crystallize out, ultimately accelerating the precipitation rate. The crystallization rate of pipe No. 3 slowed down obviously in the later period of the experiment because the inner wall friction coefficient increased with the continuous precipitation of crystals and their adhesion on the inner wall of the drainage pipe, which slowed down the flow rate near the pipe wall and thickened the DBL, ultimately slowing down the crystallization rate. When the friction coefficient reaches a certain degree, the effect of the friction effect on solution crystallization is greater than that of the diffusion layer effect, and the crystallization rate will be accelerated again.

To further understand the change of the crystallization rate with time, the increasing rate of the crystallization quality in each cycle during the experiment was calculated, as shown in Figure 11. It can be seen that, with the increase of time, the increasing rate of the quality of each drainage pipe gradually decreased. The rapid-growth period of crystalline quality was mainly concentrated in the first cycle (the first 5 days), and starting from the third cycle, the growth rate began to decrease rapidly, and finally stabilized. This phenomenon was particularly obvious in the inner wall roughness group. The third-cycle crystallization quality growth rate in this group exhibited a

Figure 11: Crystallization quantity growth rate in each cycle.

cliff-like decline that was much greater than those of the other two experimental groups, and the growth rate in the first cycle was also much larger than those of the other two groups. These findings confirm the previous inference that the effect of the inner wall roughness on the crystallization rate is obvious and is the most important factor considered in this work. Compared with the inner wall roughness group, the slope group and the free liquid level height group exhibited slower growth rates in the first cycle, but the rapid-growth time was longer. The rapid accumulation of crystallization quality within two cycles (10 days) then stabilized. In the second stage, the crystallization quality growth rate of pipe No. 2 decreased rapidly, which is most likely due to a measurement error of the crystallization quality caused by the weak adhesion of the tiny crystal nuclei and washing away by the water flow at the beginning of crystallization.

4.2 Comparison of the experimental and theoretical values

Because the inner wall of the bellows is uneven, the distribution of crystal precipitation in the bellows is uneven. In order to facilitate the comparison, the crystal thickness needs to be corrected; the corrected crystal thickness can be calculated as follows:

$$\overline{\Delta h} = c \overline{\Delta h'} = c \frac{m}{\rho_l S} \quad (29)$$

where $\overline{\Delta h'}$ is the average thickness of the pipe crystallization, and $\overline{\Delta h'} = m / \rho_l S$, m is the crystallization quality

measured in the experiment (see Figure 10), S is the area where the solution flows in the pipe, and ρ is the sediment density ($\rho = 2.71 \text{ g/cm}^3$, which is calculated by a density test experiment). Additionally, c is the error correction coefficient of the sediment thickness caused by the inner protrusion of the pipe wall, and $c = l/nl_2$, l is the wet water perimeter and l_2 is the length of the inner wall protuberance of the pipe.

According to equations (15) and (28), it can be discerned that the crystallization rate is a determined value when all factors are determined. When the crystallization precipitation area does not change significantly, the crystallization thickness is proportional to the elapsed time. Taking pipes No. 1 through No. 5 as an example, the theoretical crystallization thickness is presented in Table 4.

By linearly fitting the experimental results, the $H_n' - t$ function (which describes the relationship between the experimental crystallization thickness and time) was obtained. The experimental fitting function was then compared with the theoretical model function in one graph, as shown in Figure 12. From the figure, it is evident that the dispersion of the experimental results is not high; excluding the lower regression coefficient of fitting function 3, the regression coefficients of the other functions are basically maintained at above 90%. This indicates that most of the experimental results are basically linearly distributed, which is consistent with the assumption that the crystallization rate is constant when all factors are determined.

In Figure 12, the slope of the $H_n' - t$ function represents the experimental crystallization rate in each pipe, δ_n represents the relative error between the experimental crystallization rate and the theoretical crystallization rate in each pipe, and δ_n can be calculated by equation (30):

$$\delta_n = \frac{|F'_{(n)} - F_{(n)}|}{F_{(n)}} \times 100\% \quad (30)$$

Table 4: Theoretical crystallization thickness of each pipe

Experiment group	Pipe No.	Theoretical crystallization rate F_n	Theoretical crystallization thickness H_n ($H_n = F_n t$)
Slope	1	$F_1 = 0.0230$	$H_1 = 0.0230t$
	2	$F_2 = 0.0207$	$H_2 = 0.0207t$
	3	$F_3 = 0.0183$	$H_3 = 0.0183t$
Free liquid level height	3	—	—
	4	$F_4 = 0.0177$	$H_4 = 0.0177t$
	5	$F_5 = 0.0150$	$H_5 = 0.0150t$

From Figure 12, it is clear that the experimental crystallization rates of pipes No. 2, No. 4, and No. 5 are close to the theoretical results, and the relative errors are all within 10%. However, considering that pipe No. 2 had crystallization loss, its crystallization rate will be slightly greater than the fitting crystallization rate actually, and it is closer to or greater than the theoretical crystallization rate. Additionally, the experimental crystallization rates of pipes No. 1 and No. 3 differ greatly from the theoretical results with respective errors of 24.7 and 27.9%, but it cannot be proven that the theory is not applicable. In fact, from the perspective of the crystal thickness, even when considering the crystallization loss of pipe No. 2, the phenomenon that the experimental crystal thickness was greater than the theoretical crystal thickness is normal. On the one hand, the crystallization developed into a thin solid precipitation layer on the wall along with the continuation of the experiment, which increased the friction coefficient of the water cross-section because of the rough surface of the precipitation layer. The experimental conditions were then changed and the crystallization rate increased, which made the experimental crystallization thickness greater than the theoretical thickness. However, in the latter part of the experiment, the continuous development of the precipitation layer slowed down the flow rate in the DBL, which reduced the crystallization rate and slowed the growth rate of the crystallization thickness. On the other hand, to calculate the theoretical crystallization thickness, it was assumed that the coverage area of the crystallization precipitation layer did not change with time; however, this assumption is not perfect. With the continuous development of crystallization in the experiment, the attachment area of the precipitation layer gradually increased, and the crystallization quality also increased correspondingly. This caused the error to further expand, which is reflected in the figure by the fitting function curve that is higher than the function curve of the theoretical model.

The theoretical functional model indicates that the smoother the inner wall of the drainage pipe, the thinner the DBL, and the faster the crystallization rate. However, consideration should be given to the problem that tiny crystal nuclei stuck to the inner wall of the pipe at the beginning of the experiment. When the inner wall friction coefficient of the pipe is less than a certain value, the tiny crystal nuclei can be easily washed away before adhering to the inner wall, which will slow the crystallization rate to a certain degree. Research by Liu et al. [34] demonstrated that flocking in the inner wall of the pipe can significantly increase the inner wall friction coefficient, which will significantly slow the flow rate at the pipe

Figure 12: Comparison of experimental fitting function and theoretical model function. (a) No. 1 ($i = 3\%$, $h = 7.2$ mm) (b) No. 2 ($i = 4\%$, $h = 7.2$ mm) (c) No. 3 ($i = 5\%$, $h = 7.2$ mm) (d) No. 4 ($i = 5\%$, $h = 10.2$ mm) (e) No. 5 ($i = 3\%$, $h = 13.3$ mm).

wall, increase the DBL thickness, and eventually cause a decrease of the crystallization rate. Therefore, it can

be determined that the reference of the DBL theory is conditional. If the inner wall is too smooth or too rough,

the use of only the DBL theory cannot correctly handle the problem of the crystallization rate in the inner wall of the pipe.

5 Conclusion

Via the research presented in this paper, a functional model of the crystallization rate in a tunnel drainage pipe based on the DBL theory was established, and its rationality was verified by a model test. The following main conclusions can be drawn:

- (1) Under the action of molecular diffusion, the solute in karst water will pass through DBL and form crystallization on the pipe wall. When the liquid in drainage pipe transits from laminar flow to turbulent flow, the increase of flow rate will reduce the thickness of DBL, and then reduce the difficulty of solute passing through DBL, and ultimately accelerate the crystallization rate of karst water.
- (2) The crystallization rate of karst water in turbulent state depends on Reynolds number Re and critical Reynolds number Re^* . The higher the Re/Re^* , the faster the crystallization rate of karst water.
- (3) Among the factors considered in this paper, the most influential factor on the crystallization rate of karst water is the inner wall roughness, followed by the slope of the pipe, and finally the liquid level height. The slope can control flow rate, affect the liquid flow pattern, and then affect the crystallization rate of karst water; the liquid level height has a greater influence on the crystallization range, but has a less influence on the crystallization rate.
- (4) The application of DBL theory will be limited by the inner wall roughness. When the pipe wall is too rough or too smooth, the accuracy of the theoretical formula will be affected.

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