https://doi.org/10.1515/phys-2021-0030
received February 24, 2021; accepted April 19, 2021

Abstract: The fast marching method (FMM) is an efficient, stable and adaptable travel time calculation method. In the realization of this method, it is necessary to select the minimum travel time node from the narrow band many times. The selection method has an important influence on the calculation efficiency of FMM. Traditional FMM adopts the binary tree heap sorting method to achieve this step. Fibonacci heap sort method to FMM will be applied in this study. Compared with the binary tree heap sorting method, the Fibonacci heap sort method can realize the minimum travel time node selection in the narrow band in a more efficient way when the number of the narrow-band nodes is huge. The new method will be verified through error analysis and two numerical model calculations.

Keywords: fast marching method, Fibonacci heap sort, travel time calculation, seismic wave propagation, eikonal equation

1 Introduction

Seismic travel time is an important physical parameter describing the kinematics of seismic waves. More specially, travel time illustrates the time required for the seismic wave to reach the coordinate points of the various positions in the underground space and is closely related to the spatial distribution of the velocity parameters. Travel time calculation thus plays an important role in seismic processing, such as travel time inversion, seismic tomography, earthquake location and pre-stack migration [1–3]. Therefore, the calculation accuracy of the travel time is a decisive factor in the efficiency and precision of the various seismic processing. Two typical seismic calculation methods of the travel time are the ray tracing and the wavefront tracing. The wavefront tracing methods mainly include the wavefront construction method based on the kinematic ray tracing system and the finite difference method based on the Huygens principle and the solution equation, including the fast sweeping method [4], the group marching method [5], the fast iterative method [6] and the fast marching method (FMM) [7,8]. In this article, our research goal is the FMM. The FMM was first preferred by Sethian, which yields consistent, accurate and highly efficient algorithms. They are optimal in the sense that the computational complexity of the algorithms is \( O(N \log N) \), where \( N \) is the total number of points in the domain. The schemes are utilized in a variety of aspects, including problems in shape offsetting, computing distances from complex curves and surfaces, shape-from shading, photolithographic development, computing first arrivals in seismic travel times, construction of shortest geodesics on surfaces, optimal path planning around obstacles and visibility and reflection calculations. In recent years, this method has been rapidly developed and widely used in the field of seismic wave travel time calculation [9–13], and different computational schemes have been presented [14,15].

The FMM includes two key techniques in the calculation of the travel time, i.e. the narrow-band expansion and heap sort. The narrow-band expansion is to sweep the front ahead in a downwind fashion by considering a set of points in a narrow band around the existing front and to march this narrow band forward, freezing the values of the existing points and bringing new ones into the narrow-band structure. Since the heap sort
2 Methods

The research foundation of FMM is to solve the eikonal equation:

$$|\nabla t(x, z)| = s(x, z)$$  \hspace{1cm} (1)

where $x$ and $z$ stand for space coordinates, $t$ is the travel time, $s$ is the slowness and $\nabla$ is the symbol of gradient. FMM adopted the upwind differential method to solve the equation. The gradient item in (1) can be further transformed to:

$$|\nabla t|_{ij} = \begin{cases} \max (D_{ij}^x t + D_{ij}^z t, 0)^2 + \min (D_{ij}^x t + D_{ij}^z t, 0)^2, \\ \max (D_{ij}^x t - D_{ij}^z t, 0)^2 + \min (D_{ij}^x t - D_{ij}^z t, 0)^2 \end{cases}$$  \hspace{1cm} (2)

where

$$D_{ij}^x t = \frac{t_{i+1} - t_i}{\Delta x}, \quad D_{ij}^z t = \frac{t_{j+1} - t_j}{\Delta z},$$  \hspace{1cm} (3)

$$D_{ij}^x t = \frac{t_{i+1} - t_i}{\Delta x}, \quad D_{ij}^z t = \frac{t_{j+1} - t_j}{\Delta z}$$

The narrow-band technology was used to calculate the wavefront in the implementation process of the FMM [16]. All the grid points were divided into three categories: accepted node, narrow-band node and far away node. When the initial narrow band is established, the source point is the accepted node, the nodes around it are the narrow-band nodes and the other nodes are the far away nodes. In the implementation process, it is necessary to continuously transform the narrow-band nodes into the accepted nodes and transform the far away nodes into narrow-band nodes. When converting a narrow-band node into an accepted node, we tend to select the narrow-band grid point with minimum trial value (as shown in Figure 1) and the selected narrow-band node becomes the accepted node. Since narrow band expansion needs to select the minimum travel time every time calculation process, the efficiency of the method to achieve the selection plays a significant role in the calculation efficiency of the FMM. The seismic wave travel time calculation process needs to repeat a large number of this process, thus the method of selecting the minimum travel time exerts a significant impact on the calculation efficiency of the entire FMM. Generally, the binary heap sort method has been used to select the minimum travel time grad point. In this study, we will use the Fibonacci heap to obtain the narrow-band grid point with minimum travel time [17]. In contrast to the binary heap sort, there will be a better computational efficiency of the Fibonacci heap, especially when it needs to rearrange the narrow-band grid points with large amounts of narrow grand points. Since both methods can successfully select the minimum travel time point in the narrow band, the new method and the classic method have exactly the same calculation accuracy. A flow diagram of the Fibonacci heap in the FMM is shown in Figure 2.

3 Numerical experiments

In this part, the calculation accuracy and efficiency of the Fibonacci heap in the FMM are tested by different models. To illustrate, there are three models, including the homogeneous model, the layered model and the complex

![Figure 1: The schematic diagram of selecting minimum travel time node in the narrow band.](image-url)
model, which are adopted to perform the test. The calculation of the homogeneous medium model by using the Fibonacci heap in the FMM is shown in Figure 3. The distribution of the isochron diagram corresponding to the Fibonacci heap is given in Figure 4. The horizontal and vertical grid points of the homogeneous medium model are both 501. The horizontal and vertical grid spacing are both 10 m. The seismic source is located at (2,500, 2,500) (meters) [13]. The isochrones from the homogeneous medium model are uniformly distributed in a circular shape, conforming to the law of seismic wave propagation [14]. The relative error is large near the seismic source, but they are not more than 1%, which proves that the application of the Fibonacci heap to FMM has good calculation accuracy.

Figure 5 shows the calculation results of the FMM based on Fibonacci in the layered model. The horizontal grid points of the layered model are 601. The vertical grid points are 361. The horizontal and vertical grid spacing are both 10 m. The seismic source is located at (0, 3,000) (meters). The calculation time of the original method and the new method for Figure 5 is 3.41 and 3.22 s, respectively. Figure 6 indicates the calculation results of the FMM based on Fibonacci heap in the complex model. The number of horizontal grid points of the complex

---

**Figure 2**: The implementation flow chart of FMM.

**Figure 3**: Isochronal graph of seismic travel time for homogeneous model.

**Figure 4**: Relative error distribution graph.

**Figure 5**: Isochronal graph of seismic travel time for the layered model.
model is 361. The number of vertical grid points is 501. The horizontal and vertical grid spacing are both 10 m. The seismic source is located at (0, 1,800) (meters). The calculation time of the original method and the new method for Figure 6 is 3.11 and 2.99 s, respectively. The isochron distribution of the layered model and the complex model conforms to the law of seismic wave propagation, proving that the FMM based on Fibonacci heap has good adaptability.

4 Conclusion

In this article, the method of selecting the minimum travel time node involved in the narrow-band technology in FMM is studied, and the Fibonacci heap method is applied to FMM. Compared with the binary heap method adopted by traditional FMM, Fibonacci heap has higher computational efficiency when there are a large number of narrow-band nodes. Error analysis and numerical model calculations prove that the FMM method using Fibonacci heap has good calculation accuracy and adaptability. This method has good application prospects in various seismic exploration methods.

Conflict of interest: Authors state no conflict of interest.

References