Research Article

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Thermo-viscoelastic orthotropic constraint cylindrical cavity with variable thermal properties heated by laser pulse via the MGT thermoelasticity model

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Abstract: In the past few decades, many models have been proposed to address the shortcomings found in the classical theories of thermoelasticity and to allow limited speeds of heat waves. In this context, in the current paper a new generalized model of thermoelasticity based on the Moore–Gibson–Thompson (MGT) equation has been introduced. This new model can be derived by introducing the relaxation time factor into the third type of Green–Naghdi model (GN-III). In contrast to the previous works, it was taken into account that the physical properties of the material are dependent on temperature and on the viscous type. The viscoelastic medium has been assumed to obey the Kelvin–Voigt model. On the basis of the present model, thermo-viscoelastic interactions have been investigated in an unbounded orthotropic body with a cylindrical cavity. The surface of the cavity is restricted and exposed to a pulse-formed heat flow that dissolves exponentially. The characteristic thermal modulus of the material is assumed to be a linear function of temperature. The Laplace transform can be used to eliminate time dependency from control equations. Using a

suitable approximate method, the transformed equations have been finally inverted by numerical inversion of the Laplace transform. Certain comparisons have been introduced to estimate the effects of the viscosity, pulsed heat, and thermal temperature-independent properties on all studied fields. A comparison with previous models of thermoelasticity is also performed in tables to verify the accuracy of the proposed model. We found from the results that the physical fields strongly depend on the viscoelastic parameter, the change of the thermal conductivity, and pulsed heat, so it is not possible to neglect their effect on the manufacturing process of machines and devices.

Keywords: thermo-viscoelasticity, constraint, orthotropic cylinder, variable thermal properties, MGT model

1 Introduction

In materials science, research and electronics, building insulation, and related fields, thermal conductivity is important, particularly when achieving high temperatures. For metals and nonmetals, the effect of temperature on thermal conductivity varies. The conductivity of metals is mainly caused by free electrons. The thermal conductivity of metals is roughly proportional to times of absolute temperature (in Kelvin) with electrical conductivity. The electrical conductivity decreases with increasing temperatures in pure metals, so the product of both, the thermal conductivity, is nearly constant. Typically, alloys reduce the change in electrical conductivity, so the thermal conductivity of temperature also increases in proportion to the temperature.

The propagation of infinite speed heatwaves leads in classical thermal conduction theory to a paradox of physical phenomena which means that the thermal disruption is felt from the point of the application directly across the material body. In the last few decades, many

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ideas have been put forward to address this shortcoming, allowing for finite speeds for heatwaves. Biot [1] initially formulated the coupled dynamical theory of thermoelasticity, on the basis of the classical laws of Fourier, which consequently enables parabolic-hyperbolic governing equations in this theory to propagate heat waves at infinite speed. Therefore, the generalization of the theories of thermoelasticity has been addressed using modified Fourier's law. Several attempts have been made also to eradicate the "so-called paradox" involved in Biot's classical combined dynamic thermoelasticity theory, which was based on the Fourier heat conduction law.

Lord and Shulman [2] made a remarkable modification to eliminate a deficiency in coupled theory under the Cattaneo–Vernotte model of heat management [3–5]. By incorporating the parameter of relaxation time with respect to the heat flow vector, Cattaneo–Vernotte provided a broader form of Fourier law in the following form:

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right) \vec{q}(\vec{x}, t) = -K \vec{\nabla} \theta(\vec{x}, t). \quad (1)$$

Here $\vec{q}$ refers to the heat flux vector, K refers to the thermal conductivity, θ refers to the variant of the temperature, $\vec{x}$ is the position vector, and τ_0 indicates the non-negative time delay called the relaxation time parameter. There are other widespread theories that researchers have developed without using Fourier's nonclassical law. A few books and articles from the review show some groundbreaking progress made by many researchers in this direction [6–10].

Likewise, Muller [11] was the first to establish a generalized theory of thermoelastic without any such change to the Law of Fourier and to generalize entropy iniquity. Because of the difficulty of this theory's implementations, Green and Lindsay [12] proposed this theory-based generalization. The theory of Green and Lindsay (GL) involves an inadequacy between Green and Laws in the development of entropy [13]. The GL model imposes in its constituent equations two constants known as thermal relaxation parameters. This theory of GL is often known as the theory that relies on temperature due to the inclusion of temperature conditions.

Green and Naghdi have developed a generalized theory of thermoelasticity based on the introduction to Fourier's law of a new constituent variable [14–16]. The three theories based on this reform were proposed by Green and Naghdi; one is GN-I, the second is GN-II, and the third is GN-III. Recently, some efforts have been made to change the classical Fourier law through Abouelregal [17–22] using time-derivative of a higher-order.

The improved Fourier's Law according to the GN-III Model can be expressed as follows:

$$\vec{q}(\vec{x}, t) = -[K \vec{\nabla} \theta(\vec{x}, t) + K^* \vec{\nabla} \vartheta(\vec{x}, t)], \quad (2)$$

where ϑ is the thermal displacement with $\dot{\vartheta} = \theta$ and K^* denotes a material parameter that determines the conductivity rate. The energy balance equation is given by

$$\rho C_E \frac{\partial \theta}{\partial t} + T_0 \frac{\partial}{\partial t} (\beta_{ij} e_{ij}) = -q_{i,i} + Q, \quad (3)$$

where C_E denotes the specific heat at constant strain, $\beta_{ij} = c_{ijk} \alpha_{kl}$ are the coefficients of thermal coupling, α_{kl} are the linear thermal expansion tensor, Q is the heat source, and ρ denotes the material density of the material.

Combining the modified Fourier law (2) with the energy Equation (3) has been shown to lead to a series of elements in the continuum of points, which makes the actual component infinitely dependent, thus making solutions continuously dependent. The model (2) has the same deficiency as the usual theory of Fourier, predicting the immediate spread of thermal conduction waves. The causality principle was not observed. Therefore, this suggestion is also generally modified and a relaxing factor is included to solve this problem [23]. In this presented work, we will present a new effort to remove the paradox inherent in the classical coupled theory that is based on Fourier's law of thermal conductivity as well as the third type of Green and Naghdi theories.

The Moore–Gibson–Thompson Equation (MGT) has become extremely interesting in recent years, with many papers aimed at researching and gaining insight. This principle was derived from a differential equation from a third-order which is included in the importance of many fluid dynamic considerations [24]. Quintanilla [23] is constructing a new thermoelastic heat conduction model given by the equation MGT.

After introducing the relaxation parameter τ_0 into Green–Naghdi model of type III (GN-III), Quintanilla [23] has proposed a modified Fourier's law. Then the modified Fourier's law of heat conduction can be expressed as follows: [23,25,26]

$$\left(1 + \tau_0 \frac{\partial}{\partial t}\right) \vec{q}(\vec{x}, t) = -[K \vec{\nabla} \theta(\vec{x}, t) + K^* \vec{\nabla} \vartheta(\vec{x}, t)]. \quad (4)$$

When the Equations (3) and (4) are combined together, we obtain a linear form of the heat conduction equation which is based on generalized Moore–Gibson–Thompson (MGTE) thermoelasticity [23,26] for isotropic material. The theory of MGTE is the generalization of the theory of Lord–Shulman (LS) [2] and of type III Green–Naghdi theory of thermoelasticity (GN-III) [23,26,27]

$$\begin{aligned} & \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \left[\frac{\partial}{\partial t} \left(\rho C_E \frac{\partial \theta}{\partial t} \right) + T_0 \frac{\partial^2}{\partial t^2} (\beta_{ij} e_{ij}) - \frac{\partial Q}{\partial t} \right] \\ &= \frac{\partial}{\partial t} [\nabla \cdot (K \nabla \theta)] + \frac{\partial}{\partial t} [\nabla \cdot (K^* \nabla \theta)]. \end{aligned} \quad (5)$$

The theoretical study of the thermoelastic and mechanical behavior of several structures has successfully applied MGT equation and nanofluids with thermal radiation in recent years [28–42].

Viscoelasticity is the property of materials with viscous as well as elastic properties when deformed. There are major viscoelastic influences of synthetic polymers, wood and human tissue, and metals at high temperatures. Even a limited viscoelastic reaction may be critical in some applications. The distinction between elastic and viscoelastic materials is that viscoelastic materials do have viscosity and elastic materials do not have viscosity coefficient. Since viscoelastic substances have a coefficient in viscosity, they have a time-dependent strain rate. Purely elastic materials, when a load is applied, may not dissipate energy (heat), but the viscoelastic material is eliminated. For vibration isolation, acoustic damping, and shock absorption, an elastic viscous material is used. They release the energy as heat is absorbed.

Elastic materials at room temperature often have substantial viscoelastic properties when heated. This is the case for jet engine metal turbo blades that reach very high temperatures and have to cope with exceptionally high tensile stresses. Conventional metals will crash dramatically at higher temperatures, resulting in the creeping of crack-resistant alloys; turbine blades now also consist of so-called nickel-coated, cobalt, chromium, aluminum, titanium, tungsten, and molybdenum-coated superalloys. Some tall buildings, for instance at Columbia Center in Seattle, use viscoelastic dampers where the dampers consist of a steel plate with viscoelastic polymers, which are attached to some of the diagonal strap members. Viscoelastic dampers are used. Often vibration control is needed, but in this case, the use of a polymer is not acceptable and some other material with good vibration control properties should be used. Various studies have suggested generalized or coupled thermo-viscoelastic problems for several applications including the thermal formation effect [43–53].

This research aims to construct a new mathematical model based on MGT equation that describes the thermal behavior of flexible bodies which allows thermal waves to propagate at limited speeds and solves the defects of traditional models and some generalized models of thermal elasticity. The study of thermal conductivity, especially if it depends on a change in temperature, is very important in many areas of physical, chemical, engineering, and

other applications. For this reason also, thermal and mechanical reactions were studied inside an elastic, viscous, infinite body with a cylindrical cavity, taking into account that the coefficient of thermal conductivity is dependent on the change of temperature. This differs from many previous studies which considered the coefficient of thermal conductivity to be constant, even though the physical properties of elastic and thermoelastic materials depend on the temperature change.

The inner surface of the cylindrical cavity is fixed and heated by laser pulse. The basic equations are solved by applying the Laplace transform method as well as the inverse Laplace transform is obtained using a numerical analytical technique. Some special cases may be obtained when the effect of viscosity and thermal conductivity changes is neglected. The numerical results have been collected and discussed theoretically and graphically for all physical quantities in separate cases. The results showed that the new MGT thermoelasticity model fits the experimental data and physical phenomena better in contrast to the traditional thermoelastic models.

2 Basic equations and problem formulation

The additional governing equations for a homogenous isotropic thermo-viscoelastic medium without external body forces are given below [47,52]:

$$\sigma_{ij} = \tau_m c_{ijkl} e_{kl} - \beta_{ij} \theta, \quad (6)$$

$$2e_{ij} = u_{j,i} + u_{i,j}, \quad (7)$$

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (8)$$

where σ_{ij} denotes the components of the stress tensor, e_{ij} denotes the components of the strain tensor, u_i denotes the component of the displacement vector, β_{ij} are the thermal elastic coupling components, c_{ij} are isothermal elastic constants, $\tau_m = 1 + t_0 \frac{\partial}{\partial t}$, and t_0 is the mechanical relaxation time due to the viscosity.

Consider an orthotropic viscoelastic body, with a cylindrical cavity at uniform T_0 temperature, with a constraint surface and a thermal shock which relies on time. In defining the viscoelastic nature of the material, we can use the linear viscoelasticity model from Kelvin–Voigt. We use the cylindrical coordinates system (r, ξ, z) where z is the axial coordinate of the cylinder. The disturbances are assumed to be limited and are restricted to the interface $r = R$ and thus disappear as $r \rightarrow \infty$.

For axially symmetric problem, the displacements are reduced to

$$u_r = u(r, t), \quad u_\xi(r, t) = u_z(r, t) = 0. \quad (9)$$

The nonvanishing strains may be written as

$$e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\xi\xi} = \frac{u}{r}, \quad e_{r\xi} = e_{rz} = e_{z\xi} = 0. \quad (10)$$

The constitutive relations for a Kelvin–Voigt type solid take the form [52]

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\xi\xi} \\ \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \tau_m c_{11} & \tau_m c_{12} & \beta_{11} \\ \tau_m c_{12} & \tau_m c_{22} & \beta_{22} \\ \tau_m c_{13} & \tau_m c_{23} & \beta_{33} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial r} \\ \frac{u}{r} \\ -\theta \end{bmatrix}, \quad (11)$$

where σ_{rr} , $\sigma_{\xi\xi}$, and σ_{zz} are the normal mechanical stresses.

The dynamic equation of motion of the cylindrical cavity with neglecting the body forces is expressed as

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\sigma_{rr} - \sigma_{\xi\xi}}{r} = \rho \frac{\partial^2 u}{\partial t^2}. \quad (12)$$

With the aid of Equation (11), the above equation of motion becomes

$$\begin{aligned} & \tau_m c_{11} \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) \left(\frac{\partial u}{\partial r} \right) - \tau_m c_{22} \frac{u}{r^2} \\ &= \beta_{11} \frac{\partial \theta}{\partial r} + (\beta_{11} - \beta_{22}) \frac{\theta}{r} + \rho \frac{\partial^2 u}{\partial t^2}. \end{aligned} \quad (13)$$

The generalized MGTE heat conduction equation (modified Fourier's law) without any heat sources ($Q = 0$) will be in the following form [23,27]

$$\begin{aligned} & \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \left[\frac{\partial}{\partial t} \left(\rho C_E \frac{\partial \theta}{\partial t} \right) + T_0 \frac{\partial^2}{\partial t^2} \left(\beta_{11} \frac{\partial u}{\partial r} + \beta_{22} \frac{u}{r} \right) \right] \\ &= \frac{\partial}{\partial t} [\nabla \cdot (K \nabla \theta)] + \frac{\partial}{\partial t} [\nabla \cdot (K^* \nabla \theta)]. \end{aligned} \quad (14)$$

The field equations in linear generalized thermoelasticity with one relaxation time can be written from Equations (11), (13), and (14) by configuring the parameter $K^* = 0$. After taking the thermal parameters $\tau_0 = K = K^* = 0$, the governing equations are obtained for coupled theory of thermoelasticity. Even on setting parameters $\tau_0 = K = K^* = 0$, and the thermomechanical coupling parameters $\beta_{11} = \beta_{22} = 0$, the governing equations for the theory of uncoupled thermoelasticity can be obtained.

3 Variable thermal conductivity

The thermal properties of the thermal sensitivity bodies must vary with temperature and result in a nonlinear

problem in heat conduction equation. The exact solution to this problem comes from the premise that the material only has merely nonlinear properties. This means that the thermal material K and K^* and the specific heat coefficient C_E are linearly dependent on the temperature, but the thermal diffusion coefficient k , ($k = K/(\rho C_E)$), is supposed to be fixed. Our objective is to examine the influence of the thermal conductivity dependence on temperature and to ensure that elastic and other thermal parameters keep them stable. The thermal conductivity K and thermal rate K^* vary depending on the temperature as [53]

$$\begin{aligned} K &= K(\theta) = K_0(1 + K_1\theta), \\ K^* &= K^*(\theta) = K_0^*(1 + K_2\theta), \end{aligned} \quad (15)$$

where K_0 and K_0^* are, respectively, the thermal conductivity and thermal rate at ambient temperature T_0 and K_1 , K_2 are the slope of the thermal conductivity/rate-temperature curves divided by the intercepts K_0 and K_0^* .

By substituting from Equation (15) in Equation (14), we obtain a partial nonlinear differential equation in the form

$$\begin{aligned} & \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \left[\frac{\partial}{\partial t} \left(\rho C_E \frac{\partial \theta}{\partial t} \right) + T_0 \frac{\partial^2}{\partial t^2} \left(\beta_{11} \frac{\partial u}{\partial r} + \beta_{22} \frac{u}{r} \right) \right] \\ &= K_0 \frac{\partial}{\partial t} [\nabla \cdot ((1 + K_1\theta) \nabla \theta)] + K_0^* \frac{\partial}{\partial t} [\nabla \cdot ((1 + K_2\theta) \nabla \theta)]. \end{aligned} \quad (16)$$

The previous equation can be converted to a linear equation by defining the mapping [46]

$$\begin{aligned} \psi_1 &= \int_0^\theta (1 + K_1\varphi) d\varphi, \\ \psi_2 &= \int_0^\theta (1 + K_2\varphi) d\varphi. \end{aligned} \quad (17)$$

After inserting Equation (13) in Equation (14) and integration, then we have [54]

$$\begin{aligned} \psi_1 &= \theta \left(1 + \frac{1}{2} K_1\theta \right), \\ \psi_2 &= \theta \left(1 + \frac{1}{2} K_2\theta \right). \end{aligned} \quad (18)$$

Through differentiating relation (17) once with regard to distances and also in terms of time, the following relationships can be deduced

$$\begin{aligned} \frac{K(\theta)}{K_0} \nabla \theta &= \nabla \psi_1, \\ \frac{K^*(\theta)}{K_0^*} \nabla \theta &= \nabla \psi_2, \end{aligned} \quad (19)$$

$$\begin{aligned}\frac{K}{K_0} \frac{\partial \theta}{\partial t} &= \frac{\partial \psi_1}{\partial t}, \\ \frac{\partial \psi_2}{\partial t} &= \frac{K^*}{K_0^*} \frac{\partial \theta}{\partial t} = \frac{K^*}{K_0^*} \theta.\end{aligned}\quad (20)$$

Differentiating again Equation (19) with respect to distances, we obtain

$$\begin{aligned}\nabla \cdot \left(\frac{K(\theta)}{K_0} \nabla \theta \right) &= \nabla^2 \psi_1, \\ \nabla \cdot \left(\frac{K^*(\theta)}{K_0^*} \nabla \theta \right) &= \nabla^2 \psi_2.\end{aligned}\quad (21)$$

Combining Equations (20) and (21) and using $|\theta/T_0| \ll 1$ where $\theta = T - T_0$, we get

$$\frac{\partial}{\partial t} \nabla^2 \psi_2 \approx K^* \nabla^2 \psi_1. \quad (22)$$

By using Equations (21) and (22) then, the heat conduction Equation (18) may be reduced to

$$\begin{aligned}\left(1 + \tau_0 \frac{\partial}{\partial t} \right) \left[\frac{1}{k} \frac{\partial \psi_1}{\partial t} + \frac{T_0}{K_0} \frac{\partial^2}{\partial t^2} \left(\beta_{11} \frac{\partial u}{\partial r} + \beta_{22} \frac{u}{r} \right) \right] \\ = \frac{\partial}{\partial t} \nabla^2 \psi_1 + \frac{K_0^*}{K_0} \nabla^2 \psi_1.\end{aligned}\quad (23)$$

From Equation (18), the equation of motion will be in the form

$$\begin{aligned}\tau_m c_{11} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \tau_m c_{22} \frac{u}{r^2} &= \frac{\beta_{11}}{1 + K_1 \theta} \frac{\partial \psi_1}{\partial r} \\ + \frac{(\beta_{11} - \beta_{22})}{K_1 r} (-1 + \sqrt{1 + 2K_1 \psi_1}) + \rho \frac{\partial^2 u}{\partial t^2}.\end{aligned}\quad (24)$$

For linearity, $|\theta/T_0| \ll 1$ where $\theta = T - T_0$, then the governing equations will be in the forms

$$\begin{aligned}\tau_m c_{11} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \tau_m c_{22} \frac{u}{r^2} &= \frac{\beta_{11}}{1 + K_1 \theta} \frac{\partial \psi_1}{\partial r} \\ + (\beta_{11} - \beta_{22}) \frac{\psi_1}{r} + \rho \frac{\partial^2 u}{\partial t^2},\end{aligned}\quad (25)$$

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\xi\xi} \\ \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \tau_m c_{11} & \tau_m c_{12} & -\beta_{11} \\ \tau_m c_{12} & \tau_m c_{22} & -\beta_{22} \\ \tau_m c_{13} & \tau_m c_{23} & -\beta_{33} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial r} \\ \frac{u}{r} \\ \psi_1 \end{bmatrix}. \quad (26)$$

For convenience, the following dimensionless variables are considered.

$$\begin{aligned}\{u', r', R'\} &= \frac{c_0}{k} \{u, r, R\}, \quad \{t', \tau_0'\} = \frac{c_0^2}{k} \{t, \tau_0\}, \quad \theta' = \frac{\theta}{T_0}, \\ \sigma'_{ij} &= \frac{\sigma_{ij}}{c_{11}}, \quad K'_1 = T_0 K_1, \quad c_0'^2 = \frac{c_{11}}{\rho}.\end{aligned}\quad (27)$$

Using the quantities (27) in the governing Equations (23), (25), and (26) and suppressing dashes, we obtain

$$\tau_m \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \tau_m c_2 \frac{u}{r^2} = \varepsilon_1 \frac{\partial \psi_1}{\partial r} + \varepsilon_0 \frac{\psi_1}{r} + \frac{\partial^2 u}{\partial t^2}, \quad (28)$$

$$\begin{aligned}\left(1 + \tau_0 \frac{\partial}{\partial t} \right) \left[\frac{\partial^2 \psi_1}{\partial t^2} + \frac{\partial^2}{\partial t^2} \left(\varepsilon_4 \frac{\partial u}{\partial r} + \varepsilon_5 \frac{u}{r} \right) \right] \\ = \frac{\partial}{\partial t} \nabla^2 \psi_1 + \omega \nabla^2 \psi_1,\end{aligned}\quad (29)$$

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\xi\xi} \\ \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \tau_m & \tau_m c_1 & -\varepsilon_1 \\ \tau_m c_1 & \tau_m c_2 & -\varepsilon_2 \\ \tau_m c_3 & \tau_m c_4 & -\varepsilon_3 \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial r} \\ \frac{u}{r} \\ \psi_1 \end{bmatrix}, \quad (30)$$

where

$$\begin{aligned}c_1 &= \frac{c_{12}}{c_{11}}, \quad c_2 = \frac{c_{22}}{c_{11}}, \quad c_3 = \frac{c_{13}}{c_{11}}, \quad c_4 = \frac{c_{23}}{c_{11}}, \\ \varepsilon_1 &= \frac{T_0 \beta_{11}}{c_{11}}, \quad \varepsilon_2 = \frac{T_0 \beta_{22}}{c_{11}}, \quad \varepsilon_3 = \frac{T_0 \beta_{33}}{c_{11}}, \quad \varepsilon_4 = \frac{\beta_{11}}{\rho c_E}, \\ \varepsilon_5 &= \frac{\beta_{22}}{\rho c_E}, \quad \varepsilon_0 = \frac{T_0 (\beta_{11} - \beta_{22})}{c_{11}}, \quad \omega = \frac{K_0^*}{c_0'^2 K_0}.\end{aligned}\quad (31)$$

4 Conditions of the problem

The initial and regularity conditions should be considered to solve the present problem. We suppose that the medium initially is at rest so that initial conditions of the problem have the form:

$$\begin{aligned}u(r, t)|_{t=0} &= 0 = \frac{\partial u(r, t)}{\partial t} \Big|_{t=0}, \\ \theta(r, t)|_{t=0} &= 0 = \frac{\partial \theta(r, t)}{\partial t} \Big|_{t=0}, \\ \psi_1(r, t)|_{t=0} &= 0 = \frac{\partial \psi_1(r, t)}{\partial t} \Big|_{t=0},\end{aligned}\quad (32)$$

$$u(r, t) = \theta(r, t) = \psi_1(r, t) = 0 \quad \text{at } r \rightarrow \infty. \quad (33)$$

To solve Equations (28) and (29), we will consider that the medium described above is quiescent and the following boundary conditions are considered.

The unbounded body surface is constrained, that is, the displacement is fixed in the surface

$$u(r, t) = 0 \quad \text{at } r = a. \quad (34)$$

The cavity surface $r = a$ is exponentially heated by the pulse decaying pulse heat [55] as

$$\theta(r, t) = f(t) = \frac{t^2}{16t_p^2} e^{-t/t_p} \quad \text{at } r = a, \quad t > 0. \quad (35)$$

where t_p is a characteristic time of the pulse heat flux. Using Equation (18), then we have

$$\psi_1(r, t) = f(t) + \frac{K_1}{2} (f(t))^2 \quad \text{at } r = a. \quad (36)$$

5 Solution of the problem

Taking into account the initial conditions (32) and assuming that $\beta_{11} = \beta_{22}$ ($\varepsilon_4 = \varepsilon_5 = \varepsilon$, $\varepsilon_0 = 0$) and $c_{11} = c_{22}$, we get Equations (28)–(30) under the Laplace transform process as:

$$\bar{\tau}_m \left(\frac{d^2 \bar{u}}{dr^2} + \frac{1}{r} \frac{d\bar{u}}{dr} \right) - \bar{\tau}_m \frac{\bar{u}}{r^2} - s^2 \bar{u} = \varepsilon_1 \frac{d\bar{\psi}_1}{dr}, \quad (37)$$

$$s^2 \bar{\psi}_1 + \varepsilon s^2 \left(\frac{d\bar{u}}{dr} + \frac{\bar{u}}{r} \right) = \frac{(s + \omega)}{(1 + \tau_0 s)} \nabla^2 \bar{\psi}_1, \quad (38)$$

$$\begin{bmatrix} \bar{\sigma}_{rr} \\ \bar{\sigma}_{\xi\xi} \\ \bar{\sigma}_{zz} \end{bmatrix} = \begin{bmatrix} \bar{\tau}_m & \bar{\tau}_m c_1 & -\varepsilon_1 \\ \bar{\tau}_m c_1 & \bar{\tau}_m c_2 & -\varepsilon_1 \\ \bar{\tau}_m c_3 & \bar{\tau}_m c_4 & -\varepsilon_3 \end{bmatrix} \begin{bmatrix} \frac{d\bar{u}}{dr} \\ \frac{\bar{u}}{r} \\ \bar{\psi}_1 \end{bmatrix}, \quad (39)$$

where $\bar{\tau}_m = 1 + t_0 s$.

Here, an over bar denotes the Laplace transform of the corresponding function, s is the Laplace variable. Equations (37) and (38) can be written in the forms

$$\left(DD_1 - \frac{s^2}{\bar{\tau}_m} \right) \bar{u} = \frac{\varepsilon_1}{\bar{\tau}_m} \frac{d\bar{\psi}_1}{dr}, \quad (40)$$

$$\varepsilon q D_1 \bar{u} = (D_1 D - q) \bar{\psi}_1, \quad (41)$$

where

$$D = \frac{d}{dr}, \quad D_1 = \frac{d\bar{u}}{dr} + \frac{\bar{u}}{r}, \quad q = \frac{s^2(1 + \tau_0 s)}{(s + \omega)}. \quad (42)$$

Introducing the thermoelastic potential function ϕ , defined by

$$u = \frac{d\phi}{dr}, \quad (43)$$

into Equations (40) and (41), we get

$$\left(D_1 D - \frac{s^2}{\bar{\tau}_m} \right) \bar{\phi} = \frac{\varepsilon_1}{\bar{\tau}_m} \bar{\psi}_1, \quad (44)$$

$$\varepsilon q D_1 D \bar{\phi} = (D_1 D - q) \bar{\psi}_1. \quad (45)$$

Eliminating $\bar{\psi}_1$ from Equations (44) and (45), one gets

$$\left((D_1 D)^2 - \left(q + \frac{s^2}{\bar{\tau}_m} + \frac{\varepsilon q \varepsilon_1}{\bar{\tau}_m} \right) (D_1 D) + \frac{q s^2}{\bar{\tau}_m} \right) \bar{\phi} = 0. \quad (46)$$

Which can be written in the form

$$(\nabla^2 - m_1^2)(\nabla^2 - m_2^2) \bar{\phi} = 0, \quad (47)$$

where m_1^2 and m_2^2 are the roots of the equation

$$m^2 - \left(q + \frac{s^2}{\bar{\tau}_m} + \frac{\varepsilon q \varepsilon_1}{\bar{\tau}_m} \right) m + \frac{q s^2}{\bar{\tau}_m} = 0, \quad (48)$$

where

$$A = q + \frac{s^2}{\bar{\tau}_m} + \frac{\varepsilon q \varepsilon_1}{\bar{\tau}_m}, \quad B = \frac{q s^2}{\bar{\tau}_m}, \quad (49)$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}.$$

The solutions of Equation (47) under the regularity conditions can be written in the form

$$\bar{\phi} = \sum_{i=1}^2 A_i K_0(m_i r), \quad (50)$$

where $K_0(m_i r)$ are the modified Bessel functions of the first kinds of order zero. A_i , $i = 1, 2$ are two parameters depending on the parameter s of the Laplace transform. Using Equations (43) and (50) we get

$$\bar{\psi}_1 = \frac{\bar{\tau}_m}{\varepsilon_1} \sum_{i=1}^2 \left(m_i^2 - \frac{s^2}{\bar{\tau}_m} \right) A_i K_0(m_i r). \quad (51)$$

Substituting from Equation (50) into the Laplace transform of Equation (43), we obtain

$$\bar{u} = - \sum_{i=1}^2 m_i A_i K_1(m_i r). \quad (52)$$

The well-known relations of the Bessel function

$$x K_n'(x) = -x K_{n+1}(x) + n K_n(x), \quad (53)$$

are used to derive the stresses with the aid of the displacement $\bar{u}$ and the function $\bar{\psi}_1$. They will be given by

$$\bar{\sigma}_{rr} = \bar{\tau}_m \sum_{i=1}^2 \left[\frac{s^2}{\bar{\tau}_m} A_i K_0(m_i r) + \frac{m_i(1 - c_1)}{r} A_i K_1(m_i r) \right], \quad (54)$$

$$\begin{aligned} \bar{\sigma}_{\xi\xi} = \bar{\tau}_m \sum_{i=1}^2 & \left[\left(m_i^2(c_1 - 1) + \frac{s^2}{\bar{\tau}_m} \right) A_i K_0(m_i r) \right. \\ & \left. - \frac{m_i(c_1 + 1)}{r} A_i K_1(m_i r) \right], \end{aligned} \quad (55)$$

$$\bar{\sigma}_{zz} = \bar{\tau}_m \sum_{i=1}^2 \left[\left(m_i^2 \left(c_3 - \frac{\varepsilon_3}{\varepsilon_1} \right) + \frac{\varepsilon_3 s^2}{\varepsilon_1 \bar{\tau}_m} \right) A_i K_0(m_i r) - \frac{m_i(c_3 + c_4)}{r} A_i K_1(m_i r) \right]. \quad (56)$$

The boundary conditions (35) and (36), after using the Laplace transform operator, become

$$\bar{\psi}_1(a, s) = \frac{t_p}{8(1 + s t_p)^3} + \frac{3K_1 t_p}{64(2 + s t_p)^3} = \bar{G}(s), \quad (57)$$

$$\bar{u}(a, s) = 0.$$

The substitution of Equations (51) and (54) into the above conditions gives two equations in the unknown parameters, A_i , $i = 1, 2$ as

$$\sum_{i=1}^2 \left(m_i^2 - \frac{s^2}{\bar{\tau}_m} \right) A_i K_0(m_i a) = \frac{\varepsilon_1}{\bar{\tau}_m} \bar{G}(s), \quad (58)$$

$$\sum_{i=1}^2 m_i A_i K_1(m_i a) = 0. \quad (59)$$

After solving the above equations, we have the constants A_i , $i = 1, 2$ in the form

$$A_1 = \frac{\bar{G}(s) m_2 \varepsilon_1 K_1(m_2 a)}{m_1(s^2 - m_2^2 \bar{\tau}_m) K_0(m_2 a) K_1(m_1 a) + m_2(-s^2 + m_1^2 \bar{\tau}_m) K_0(m_1 a) K_1(m_2 a)}, \quad (60)$$

$$A_2 = -\frac{\bar{G}(s) m_1 \varepsilon_1 K_1(m_1 a)}{m_1(s^2 - m_2^2 \bar{\tau}_m) K_0(m_2 a) K_1(m_1 a) + m_2(-s^2 + m_1^2 \bar{\tau}_m) K_0(m_1 a) K_1(m_2 a)}.$$

So the solution of the problem will be completed in the Laplace transform domain. In addition, the temperature $\bar{\theta}$ can be obtained by solving Equation (18) after applying the Laplace transform as

$$\bar{\theta} = \frac{\sqrt{2k_1 \bar{\psi}_1 + 1} - 1}{K_1}. \quad (61)$$

6 Numerical results and discussion

In this section, the distributions of the studied fields such as temperature, displacement, and stresses will be inside the medium in their inverted forms. To invert the Laplace transform in Equations (51)–(56), a numerical inversion method based on a Fourier series expansion [56] should be adopted. Any expression in Laplace domain can be inverted in this method to the time domain as

$$f(r, t) = \frac{e^{ct}}{t} \left(\frac{1}{2} \bar{f}(r, c) + \operatorname{Re} \sum_{n=1}^m (-1)^n \bar{f} \left(r, c + \frac{in\pi}{t} \right) \right). \quad (63)$$

Numerous numerical experiments have shown that the value of c should satisfy the relation $ct \cong 4.7$ [45]. So, one will use the same value of c for the numerical evaluation purpose.

Numerical evaluations are made by choosing an orthotropic material such as *cobalt*. The properties of the cobalt material are given in SI units [57,58] as

$$\{c_{11}, c_{12}, c_{22}, c_{13}, c_{23}\} = \{3.071, 1.650, 1.027, 1.150, 3.581\} \times 10^{11} \text{ kg m}^{-1} \text{ s}^{-2},$$

$$\{\beta_{11}, \beta_{22}, \beta_{33}\} = \{7.04, 6.90\} \times 10^6 \text{ kg m}^{-2} \text{ s}^{-2},$$

$$K_0 = 96 \text{ W m}^{-1} \text{ K}^{-1}, \quad \rho = 8,836 \text{ kg m}^{-3},$$

$$T_0 = 298 \text{ K}, \quad K_0^* = 2 \text{ W m}^{-1} \text{ K}^{-1} \text{ s}^{-1}, \quad \theta_0 = 1.$$

The numerical results of nondimensional temperature θ , radial displacement u , and radial and hoop stresses σ_{rr} and $\sigma_{\theta\theta}$ variations are performed along with the radial distance r . The results are illustrated graphically in Figures 1–16 and Tables 1–4. Numerical calculations are performed for four cases as follows:

6.1 Effect of the variable thermal material properties

The analysis of nondimensional temperature, displacement, and thermal stress using different values of the K_1 coefficient of difference is based in general MGT thermal viscoelasticity, while the relaxation time τ_0 and the

Figure 1: Temperature θ versus the variability of thermal conductivity.

Figure 2: Displacement u distribution versus the variability of thermal conductivity.

Figure 5: Temperature θ for different thermoelasticity models.

Figure 3: The stress σ_{rr} versus the variability of thermal conductivity.

Figure 6: Displacement u for different thermoelasticity models.

Figure 4: The stress $\sigma_{\xi\xi}$ versus the variability of thermal conductivity.

Figure 7: The stress σ_{rr} for different thermoelasticity models.

Figure 8: The stress $\sigma_{\xi\xi}$ for different thermoelasticity models.

Figure 11: The stress σ_{rr} distribution for different values of the viscosity parameter t_0 .

Figure 9: Temperature θ distribution for different values of the viscosity parameter t_0 .

Figure 12: The stress $\sigma_{\xi\xi}$ distribution for different values of the viscosity parameter t_0 .

Figure 10: Displacement u distribution for different values of the viscosity parameter t_0 .

Figure 13: Temperature θ distribution for different time of pulse heat parameter t_p .

Figure 14: Displacement u distribution for different time of pulse heat parameter t_p .

Figure 15: The stress σ_{rr} distribution for different time of pulse heat parameter t_p .

Figure 16: The stress $\sigma_{\xi\xi}$ distribution for different time of pulse heat parameter t_p .

viscosity t_0 parameters remain constant. Three different values of parameter K_1 are applied to the viscoelastic material. The values of $K_1 = -0.5, -1$ are taken for temperature-independent thermal conductivity and $K_1 = 0$ for fixed thermal conductivity.

Changes with spatial coordinates are shown in Figures 1–4. Under all thermoelasticity models, the essence of variations in the fields varies dramatically in time and has a prominent impact of time on all profiles. The results show that the element of variance affects all the fields examined. There is also a finite propagation speed phenomenon in any figure. This is different from cases in which combined and uncoupling models of thermoelasticity have an endless speed of spread and thus all functions in the infinite media have nonzero value.

The following relevant facts are also observed:

- The coefficient of thermal conductivity variation has significant impact on the rate of spread of the wave for all areas studied. Thermomechanical reactions are greatly influenced by the dependence of thermal conductivity on temperature.
- Temperature difference via distance r is shown in Figure 1. Figure 1 displays the initial change and decrease in the magnitudes of temperature θ variations. The front of the heat wave is moving with time at a final velocity. The figure also shows that with the decrease of parameter K_1 , the temperature value decreases. The figure shows that the temperature has a nonzero value at a given time in a limited space field only. And the turbulence disappears outside this region, and the heat turbulence is not felt in the area. The nonzero region, in turn, moves over time at many points.
- In Figure 2, it is shown that displacement variance u starts at zero in all surface cavity cases $r = 1$ that consistently decreases to the lowest value and is consistently compatible with the limiting condition.
- The displacement u starts from zero to the highest value at $r = 1.1$ close to the surface, and decreases to zero gradually. As Figure 2 shows, the displacement u decreases, since the parameter K_1 is reduced, when r is 1.7 it enters a steady state.
- Figure 3 shows that stress σ_{rr} begins with negative values on the cavity's surface and the stress activity then rises in the range 1 to 1.7, and eventually reaches the steady state when r goes up. The vector thermal conductivity parameter K_1 can also be seen in the figure to raise the stress magnitude σ_{rr} .
- The thermal stress $\sigma_{\xi\xi}$ starts with negative values in all cases and rises steadily to zero and to stable case. Figure 4 shows also that the $\sigma_{\xi\xi}$ rises as the values of the parameter K_1 are increased.

Table 1: The temperature θ contrary to distance r

r	CTE	LS	GN-II	GN-III	MGTE
1	0.112287	0.109485	0.111373	0.112925	0.110421
1.1	0.0752025	0.0484334	0.0696591	0.0818029	0.0580705
1.2	0.0477732	0.0102731	0.0306165	0.0562197	0.0139437
1.3	0.0299267	0.00677321	0.0107474	0.0380369	0.0105894
1.4	0.0184762	0.00431137	0.00885532	0.0254152	0.0084062
1.5	0.0113664	0.0021397	0.00716433	0.0169794	0.00527508
1.6	0.00701441	0.000639865	0.00530149	0.0114454	0.0018516
1.7	0.00437618	0.000223965	0.00339989	0.00784973	0.000618303
1.8	0.00277639	0.000151559	0.00170345	0.00550368	0.000552207
1.9	0.00179679	0.000085594	0.000596068	0.003946	0.000420289
2	0.00118576	3.40409×10^{-5}	0.000389723	0.00288244	0.000217534

Table 2: The displacement u contrary to distance r

r	CTE	LS	GN-II	GN-III	MGTE
1	0	0	0	0	0
1.1	0.0240802	0.0276999	0.0259519	0.0215585	0.0270082
1.2	0.0206	0.0170611	0.0174863	0.0192808	0.0166129
1.3	0.0147977	0.0122142	0.0133833	0.0148498	0.0144965
1.4	0.00988051	0.00589623	0.00959885	0.0107208	0.0087272
1.5	0.0065697	0.0024589	0.00637108	0.00775281	0.00420253
1.6	0.00435189	0.000975351	0.00381671	0.00560828	0.00170302
1.7	0.00289483	0.000512914	0.00207843	0.00407864	0.00108501
1.8	0.00192958	0.000255428	0.00107912	0.00297307	0.000712038
1.9	0.00128812	0.000109748	0.000643028	0.00216876	0.000384228
2	0.000859968	4.22114×10^{-5}	0.000454568	0.00158052	0.000157299

Table 3: The stress variation σ_{rr} counter to distance r

r	CTE	LS	GN-II	GN-III	MGTE
1	-0.350963	-0.285408	-0.329551	-0.365883	-0.307272
1.1	-0.294993	-0.218316	-0.287248	-0.320049	-0.254473
1.2	-0.191927	-0.0417496	-0.125454	-0.223217	-0.0533499
1.3	-0.123564	-0.0348947	-0.0459588	-0.153961	-0.047408
1.4	-0.0767182	-0.0222753	-0.0384682	-0.103202	-0.0388767
1.5	-0.0475105	-0.0110903	-0.0313232	-0.0692144	-0.0248776
1.6	-0.0294519	-0.00339436	-0.0231963	-0.0467759	-0.00876084
1.7	-0.0184546	-0.00124336	-0.0148677	-0.0321572	-0.00283757
1.8	-0.0117504	-0.000820589	-0.00745115	-0.0225884	-0.00256927
1.9	-0.0076238	-0.000452974	-0.00261438	-0.016214	-0.00198005
2	-0.00503774	-0.000179587	-0.00170999	-0.0118489	-0.00103551

- We can see that in one part of the cylinder, compression and stress in another part are both types of stress. Tensile stresses are imposed at the medium adjacent to the cylinder surface, which rise with time. There is no question that the highest value of the fields examined close to

the cavity surface and their magnitude are decreased as the radial distances are increased. The figures indicate that the thermal conductivity factor of change significantly affects all fields, making it increasingly important to recognize the thermal conductivity variable.

Table 4: The stress variation $\sigma_{\xi\xi}$ contrary to distance r

r	CTE	LS	GN-II	GN-III	MGTE
1	-0.391694	-0.351417	-0.378539	-0.400861	-0.364851
1.1	-0.292789	-0.202875	-0.278379	-0.318371	-0.239783
1.2	-0.188201	-0.0398277	-0.121611	-0.22038	-0.0527266
1.3	-0.119654	-0.0302275	-0.0432811	-0.15066	-0.0437141
1.4	-0.0741107	-0.0194283	-0.0361359	-0.100854	-0.0356002
1.5	-0.0457618	-0.0096999	-0.0294587	-0.0675245	-0.022695
1.6	-0.0283118	-0.00293236	-0.0218748	-0.045583	-0.00797461
1.7	-0.0177065	-0.00104854	-0.0140506	-0.0313061	-0.00259161
1.8	-0.0112574	-0.0007037	-0.00704202	-0.0219759	-0.00234833
1.9	-0.00729729	-0.00039371	-0.00245696	-0.0157705	-0.00180897
2	-0.0048208	-0.00015644	-0.00160628	-0.0115266	-0.000944247

- These figures show that the quantities of the field depend not only on variables in time and space, but also on changes in thermal conductivity parameters. All these figures illustrate the phenomenon of small propagation rates.

6.2 A comparison of different thermoelasticity models

In the second case, the different physical distributions against distance r are investigated and the different theories of thermal elasticity are compared if the coefficient of thermal conductivity K_l is established. The differences in physical domains are shown in Figures 5–8, respectively. We have the following models of thermoelasticity as special cases: the coupled thermoelasticity (CTE) ($\tau_0 = K^* = 0$), the Lord and Shulman model (LS) ($K^* = 0$). In addition, when $K^* = 0$, we recover type I of Green and Naghdi theory (GN-I), in the meantime type II of Green and Naghdi theory (GN-II) can be achieved when $K = 0$. The generalized MGTE is attained when $\tau_0, K^* > 0$.

This section includes some findings for comparisons between and for practical purposes between different models of thermoelasticity. For future research comparisons, tables can be used. The physical fields for CTE, LS, GN-II, GN-III, and MGTE models vary against the distance r when the time $t = 0.12$ is shown in Tables 1–4. The figures and tables show that:

- The thermal parameters τ_0 and K^* have a significant impact on the distribution of the field number. The combined (CTE, LS, GNII, and MGTE) and generalized theory results close to the surface of the cylinder, where

boundaries are dominant. The results are very similar. In the cylinder, the solution is drastically different. It is because thermal waves propagate at an unlimited rate of propagation in the generalized models as against a finite speed in coupled model.

- Table 1 and also Figure 5 display the inconsistencies between GN-III forecasts and MGTE theories. The GN-III magnitude also exceeds MGTE, while the graphs LS and MGTE reflect comparable results in both models.
- In the case of a GN-II model, the temperature change and other distributions are substantially different from other generalized convergence models (LS and MGTE).
- The results of generalization GN-III model thermoelasticity indicate that they differ considerably from low energy-spending GN-II thermoelasticity models.
- The presence of the relaxation parameter on the LS and MGTE models may also indicate a slowing temperature decay.
- The results of the generalized model of thermoelasticity GN-III also show convergence from the conventional elasticity model (CTE) which does not fade away in heat quickly inside the body, in contrast to other generalized models of thermoelasticity. This is completely consistent with what Quintanilla [23] was told about.
- The figures and tables show that the distributions of changes in temperature and physical quantities studied in the theories of thermal elasticity MGTE and LS follow similar patterns in the case of viscous solids. In general, both theories are very similar in behavior and convergence, but differ only slightly in amount.
- As the distance increases, the results are very similar, which is in line with the generalized thermoelasticity theories.

6.3 Influence of the viscoelastic parameters

Viscoelasticity is the combination of viscous and elastic properties in a material, depending on the amount of time, temperature, stress, and pressure. Viscoelasticity is elastic materials' time-dependent behavior. This means that the response to a stimulus is deferred and the substance has a lack of energy. Normally, viscoelastic behavior occurs inside the same substance at various timescales.

The current case will be based on how the nondimensional shapes of the temperature, displacement, and stress vary from mechanical relaxation $t \rightarrow 0$ due to the term $1 + t_0 \frac{\partial}{\partial t}$ in fundamental equations of all viscosity. The values $K_1 = -0.5$ and $\tau_0 = 0.1$ are used in this numerical analysis. The distributions of the viscosity t_0 are tested for validation at three different dimensional periods without mechanical relaxation.

In this case, only MGTE theory is used. Comparisons of dimensionless physical fields are rendered in both different cases: (i) in the case of $t_0 = 0.1$ and $t_0 = 0.2$ the thermos-viscoelastic model (MGVTE) and (ii) in the case of the thermoelastic Solid Model (TMS) for $t_0 = 0$. For the two separate thermoelastic and thermo-viscoelastic models, field comparisons investigated in respect of the distance r are shown in Figures 9–12 for $t = 0.12$. We also notice the following relevant notes from the figures:

- Figures 9, 11, and 12 demonstrate the effects of the parameter viscosity on temperature and thermal stresses. The effect on temperature and thermal stress has been known to be highly influenced by the viscosity parameter.
- Figure 9 suggests that the viscosity parameter tends to increase the temperature distribution. In contrast to the MGTE theory, the temperature distribution in MGVTE theory is strong.
- The absolute values of stresses σ_{rr} and $\sigma_{\xi\xi}$ in Figures 11 and 12 increase as the value of the parameter viscosity increases. The effect of viscosity also tends to vanish from the surface of the cavity over time and inside the body.
- Figure 10 provides contrasts of the displacement u with the viscosity modulus t_0 . It was found that the displacement u decreases with increased mechanical time of relaxation t_0 . The displacement distribution of the MGTE is also found to be higher than the MGVTE distribution. The radial displacement factor has the same pattern in both theories [59].
- These results are helpful for materials science studies, material designers, low temperature physicists, and those interested in hyperbolic thermo-viscoelasticity (MGVTE).

6.4 Influence of the time of the pulse parameter

The effect of the time of the pulse heat parameter t_p on the dimensionless temperature θ , displacement u as well as thermal stresses σ_{rr} and $\sigma_{\xi\xi}$ are illustrated in Figures 13–16. Once again, the values of θ and u are decreasing as t_p increases (Figures 13 and 14), while the thermal stresses σ_{rr} and $\sigma_{\xi\xi}$ are decreasing as t_p decreases (Figures 15 and 16). The behavior of the three cases of the time of the pulse heat t_p is generally quite similar and t_p has a major influence on all the physical fields. It is clear that the displacement u no more rises in the radial direction and has its highest value at the position regardless of the value of t_p .

7 Conclusion

In this article, a new model of generalized thermoelasticity has been derived by introducing relaxation time into Green and Naghdi's theory of the third type. The introduced model is based on MGTE model proposed by Quintanilla [23]. To study and investigate the introduced model, an infinite viscous body with a cylindrical cavity has been studied. The physical properties of the medium are viscous and variable and depend on the change of temperature. The surface of the cylindrical cavity is restricted and exposed to thermal flux in the form of laser pulses that diminish exponentially with time. In the domain of the Laplace transform, the problem is solved and the numerical values of the physical field of the different studied fields are obtained using a suitable approximate numerical technique. Variations in displacement, thermal stress distribution, and temperature are represented graphically for different variability of thermal conductivity parameter, viscosity, and laser pulse. In addition to the above, the results of the current model have been compared with the previous thermoelastic models. The following findings can be summarized in the following observations from a study of the hypothetical and the numerical estimation of this work:

- The change of the coefficient of thermal conductivity greatly affects the velocity of wave propagation for all studied fields. The magnitudes of the change in temperature decrease as the variability coefficient increases.
- In all distributions of physical fields the influence of viscosity is strongly noticeable.
- As a special case from the implemented model, the conventional thermoelastic theory and other generalized thermoelasticity theories can be extracted.

- In the case of the generalized GMT thermoelastic model, heat propagates as a wave with a finite speed in the medium rather than an infinite speed.
- The results indicate that due to the presence of relaxation factor in the LS and MGTE models, the propagation of heat waves is slowed down. We also found that there is a convergence in the behavior of the different distributions in the case of the two models.
- In addition, significant differences have been observed in field variables with laser pulse change.
- The findings presented here should finally prove useful for scientists, engineers, and those working on the development of solid mechanics. These theoretical findings will provide interesting information to experimental scientists and researchers who work on this topic.
- The model presented in this work may be applied in the future when the medium is functionally graded and heterogeneous.

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