Research Article

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Fractal calculation method of friction parameters: Surface morphology and load of galvanized sheet

<https://doi.org/10.1515/phys-2021-0042>

received March 26, 2021; accepted May 14, 2021

Abstract: In the forming process of galvanized sheet, the friction between the die and the blank often causes the zinc coating of galvanized sheet to peel off, scratch, and crack. The aim of this study is to evaluate and calculate the fractal characteristics of the surface morphology of galvanized sheet and the effect of pressure on the interfacial friction behavior. Two steel plates, GA and GI, produced by Shanghai Baosteel Company, were used as materials to conduct tribological experiments, measure the surface profile and three-dimensional shape of the galvanized sheet, and calculate the fractal dimension and fractal roughness parameters. According to the analysis results of friction surface damage of galvanized sheet, the damage failure parameters of galvanized sheet are calculated. On this basis, according to the adhesive friction theory, the total surface friction value of galvanized sheet is obtained, and the fractal calculation model of galvanized sheet friction is established. The simulation results show that the galvanized sheet has fractal characteristics. The average values of fractal dimension and scale factor of SP781BQ alloy hot-dip galvanized sheet are 1.52 and $0.23\ \mu\text{m}$, respectively. The average fractal dimension and scale coefficient of HC420/780DPD + Z hot-dip galvanized sheet are 1.60 and $0.11\ \mu\text{m}$, respectively. The friction coefficient calculated by the proposed method is consistent with the theoretical value, and the error is less than 10%, which proves the accuracy and feasibility of the friction fractal calculation method.

Keywords: galvanized sheet, surface morphology, pressure load, friction parameters, fractal calculation

1 Introduction

The surface of any friction pair in construction machinery is rough and uneven on the micro scale. The basic reason of friction and wear is that the two rough surfaces contact each other under the action of load, resulting in heat, deformation, adhesion, groove, and other effects. Therefore, the study of the contact behavior of micro rough surface has been one of the important topics in tribology [1–3].

In the physical field, in the literature [4], two different methods (GA and GI) were selected to evaluate the influence of surface morphology, pressure, and lubrication conditions on the friction behavior of galvanized steel plate interface. The surface morphology of the sample is measured by profilometer, fractal parameters are calculated by fractal theory, and the relationship between the actual contact area and the load is obtained. The contact model of galvanized plate and die is established; According to the contact model, the friction model of the relationship between the surface morphology and the load of the galvanized plate is established. Literature [5] introduced the deep learning representation and modeling method, characterized and modeled the galvanized sheet surface and fractal surface, and studied the modeling process of friction surface morphology. The results showed that the surface tribological parameters could not represent the surface treatment method, and the surface tribological parameters could not be reconstructed by the specified processing methods or parameters. Moreover, fractal surface modeling cannot achieve both fractal parameters and surface tribological parameters. Therefore, the subsequent simulation process cannot meet the requirements of scale and parameters. The depth learning representation and modeling method are introduced to characterize and model the surface of galvanized sheet and fractal surface. In the field of engineering, the main research contents of literature [6] include optimizing fractal surface modeling method and improving SSK distribution of fractal surface morphology. The multi-scale modeling and simulation of the surface topography of turning milling are carried out using the multi-directional continuous wavelet, and the

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compression sensing simulation is carried out to reduce the sampling rate. The super-resolution reconstruction process of rough surface topography scanning is complex. A method of fractal dimension calculation of 3D rough surface based on wavelet transform is proposed in document [7]. The method decomposes the 3D surface topography data by wavelet transform. The three-dimensional fractal dimension of rough surface is obtained by fitting the mean value of the square of wavelet coefficients under different decomposition scales. To verify the correctness and accuracy of the method, the 3D fractal surface is constructed under the condition of known parameters, and the results of fractal dimension calculation under different methods are compared and analyzed. The results provide the parameter basis for the establishment of fractal contact model of rough surface. However, in practical application, the friction coefficient calculated by the above traditional method is different from the theoretical value, and the fractal dimension and proportion coefficient of galvanized sheet are unreasonable.

To solve the defects of traditional methods, this paper studies the genetic algorithm and genetic algorithm of galvanized sheet. Based on the fractal contact theory, the surface morphology of the galvanized plate is measured by optical profilometer, the damage and failure parameters of the galvanized plate are calculated, and the fractal calculation model of friction of the galvanized plate is established. The results are compared with the experimental data. The innovation is that according to the damage analysis results of the friction surface of the galvanized plate, the damage failure parameters of the galvanized plate are calculated. According to the adhesion friction theory, the total surface friction value of the galvanized plate is obtained, and the fractal calculation model of the friction of the galvanized plate is established.

2 Surface morphology and fractal characteristics of galvanized sheet

Two kinds of steel plates, GA and GI, produced by Shanghai Baosteel Co., Ltd., were used as experimental materials to study the tribological properties of the two kinds of steel plates, SP781BQ and HC420/780DPD + Z, respectively [8–10]. The two kinds of galvanized steel sheets were cut into rectangular samples with the size of 30 mm × 100 mm. The surface of all samples shall be cleaned with absolute ethanol, kept clean, and then dried with cold air. The surface morphology of galvanized sheet

was measured by Veeco Wyko nt1100 non-contact optical profilometer [11]. Three micro regions on the surface of the sample were randomly selected as the measurement surface to obtain the surface profile curve, and determine the surface roughness value and three-dimensional surface morphology. The sliding friction experiment was carried out. The experiment only studies the friction of the galvanized sheet under different mold roughness, different pressure, and no lubrication conditions. The surface roughness of the die is 0.4, 0.8, and 1.2 μm, respectively. Under the positive pressure of 1,000 N, the contact surface between the upper and lower die and the sample is about 2 mm × 30 mm rectangular area [12–14]. The friction sliding speed is set at 150 mm min⁻¹. To reduce the experimental error, the average value of all experimental data was taken after three measurements.

Three samples of GA and GI galvanized sheet were prepared. Randomly select two micro-zones on the sample surface as the measurement area (Figure 1), and use Myko NT1000 profiler to measure the surface profile and roughness of the sample (Figure 2); a better area was selected on the galvanized steel sample, and its three-dimensional morphology was measured by laser confocal microscope.

Figures 3 and 4 show the measured surface profiles of GA and GI plates (position 1 on the left and position 2 on the right).

It can be seen from the comparison of Figures 3 and 4 that the surface of the GA plate of the alloyed hot-dip galvanized steel sheet is relatively rough, and the degree of unevenness is relatively large. The surface of

Figure 1: Schematic diagram of sample size and measurement position.

Figure 2: Microscopic surface of zinc plate.

Figure 3: The surface profile of GA board SP781BQ in two measurement areas.

Figure 4: Surface profile of GI plate HC420/780DPD + Z in two measurement areas.

the pure zinc galvanized steel GI plate is relatively flat, and the degree of unevenness is small. The average surface roughness measured is 0.889 and $0.525 \mu\text{m}$, respectively. This is because the coating of GA plate is composed of hard brittle phase Zn Fe alloy, while the coating of GI plate is composed of soft Zn. The roughness of hard GA plate is larger than that of soft GI plate.

The fractal dimension and fractal roughness parameters calculated based on Figures 3 and 4 are shown in Table 1.

It can be seen from Table 1 that the fractal dimensions of the two kinds of galvanized sheets meet the requirements of $1 < d < 2$, so the two kinds of sheets have fractal characteristics [15]. The average fractal dimension of GA plate is smaller than that of GI plate, but the average value of scale coefficient is larger than that of GI plate. This is because the smooth surface has a deeper

self-similar fine structure, and thus has a higher fractal dimension.

3 Relationship between surface morphology and load of galvanized sheet

3.1 Damage analysis of load friction surface of galvanized sheet under different load directions

The friction interface element of the galvanized sheet is only subjected to normal and two tangential load stresses, and its constitutive correlation [16] expression is recorded as:

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}. \quad (1)$$

Among them, t_1 , t_2 , and t_3 represent normal stress and two tangential stress in turn. k_1 , k_2 , and k_3 represent normal and two tangential stiffness in turn. δ_1 , δ_2 , and δ_3 represent the normal and the corresponding translation of the two tangential directions.

In a single form, in the bilinear constitutive model of the interface element on the surface of galvanized sheet, the analytical expression of the corresponding interface displacement δ_1^0 , δ_2^0 , and δ_3^0 relative to the initial point of interface damage is as follows:

$$\begin{cases} \delta_1^0 = \frac{N}{k_1} \\ \delta_2^0 = \frac{S}{k_2} \\ \delta_3^0 = \frac{T}{k_3}, \end{cases} \quad (2)$$

Table 1: Fractal parameters of galvanized sheet

Brand	Fractal dimension d	Scale coefficient G (μm)	Average fractal dimension	Average scale coefficient (μm)
GA (ST781BQ)	1.522	0.227	1.52	0.23
	1.528	0.229		
	1.497	0.232		
GA (ST781BQ)	1.597	0.114	1.6	0.11
	1.604	0.111		
	1.601	0.112		

where N is the normal strength of the interface and S and T are the shear strength of the interface.

The interface relative displacement of the global failure point control [17] is as follows:

$$\begin{cases} \delta_1^f = \frac{2G_{IC}}{N} \\ \delta_2^f = \frac{2G_{IIC}}{S} \\ \delta_3^f = \frac{2G_{IIIC}}{T} \end{cases} \quad (3)$$

In general, the damage initiation and broadening degree of interface elements are generated under mixed loading. It is necessary to consider the effect of coupling interface damage between the stress components of three different cracking states. If the shear strength of the two interfaces is equal, the energy release rate of crack strain and the interface stiffness are the same.

The analytical expression of the control displacement in the mixed mode is recorded as follows:

$$\delta_m = \sqrt{\langle \delta_1 \rangle^2 + \delta_2^2 + \delta_3^2} = \sqrt{\langle \delta_1 \rangle^2 + \delta_{shear}^2}, \quad (4)$$

where δ_{shear} is the tangential control displacement of the interface element and $\langle x \rangle$ is the operator. It is described as follows:

$$\langle x \rangle = \begin{cases} 0, & x \leq 0 \\ x, & x > 0. \end{cases} \quad (5)$$

Assuming the parameter $\beta = \delta_{shear}/\delta_1$, the damage initial principle is based on the quadratic stress theory, and the original damage displacement value under mixed mode can be obtained:

$$\delta_m^0 = \begin{cases} \delta_1^0 \delta_{shear}^0 \sqrt{\frac{1 + \beta^2}{(\delta_{shear}^0)^2 + (\beta \delta_1^0)^2}}, & \delta_1 > 0 \\ \delta_{shear}^0, & \delta_1 \leq 0. \end{cases} \quad (6)$$

The final failure uses the principle of secondary strain energy release rate to obtain the global damage displacement in the mixed state:

$$\delta_m^f = \begin{cases} \frac{2(1 + \beta^2)}{K \delta_m^0} \left[\left(\frac{1}{G_{IC}} \right)^2 + \left(\frac{\beta^2}{G_{IIC}} \right)^2 \right], & \delta_1 > 0 \\ \sqrt{(\delta_2^f)^2 + (\delta_3^f)^2}, & \delta_1 \leq 0. \end{cases} \quad (7)$$

For a representative interface, its area A can be regarded as the sum of undamaged area n and damaged area m , which is specifically expressed as [18]:

$$A = A^u + A^d = (1 - d)A + dA. \quad (8)$$

The stress in the representative interface is:

$$\tau = (1 - d)\tau^u + d\tau^d. \quad (9)$$

Among them, τ^d and τ^u are the stresses generated by the area of damaged interface and undamaged interface in turn.

In the case of neglecting the constitutive model of friction interface element, it is considered that the interface element will not generate normal interface damage under normal pressure. Therefore, when the interface is damaged, only normal compressive stress can exist in the interface area where debonding damage has occurred. However, when the interface unit is debonded, the debonded surface becomes rougher. At this time, when the interface is subjected to normal compression load, the corresponding slip of the interface with respect to the other phases will generate strong friction force in the debonding interface [19–21].

Considering the external friction of galvanized sheet at debonding interface, the representative interface internal stress in equation (9) can be transformed into:

$$\tau = \tau_s + \tau_f, \quad s = 1, 2, 3. \quad (10)$$

Among them, τ_f is the damage generated in the debonding interface, then:

$$\tau_{f1} = 0, \quad (11)$$

$$\tau_{fr} = \begin{cases} 0 & \delta_1 \geq 0 \\ 0 & \delta_m^{\max} \leq \delta_m^0, \delta_1 < 0 \\ -\mu K d \delta_1 \delta_r / |\delta_r| & \delta_m^0 < \delta_m^{\max} < \delta_m^f, \delta_1 < 0 \\ -\mu K \delta_1 \delta_r / |\delta_r| & \delta_m^{\max} \geq \delta_m^f, \delta_1 < 0, \end{cases} \quad (12)$$

where μ is the interfacial friction index.

3.2 Fractal calculation of friction damage failure parameters

According to the fractal calculation results of friction damage on the surface of galvanized sheet, the improved Hashin's friction damage failure principle is used as the judgment standard of fiber bundle failure, because the material strength evaluation value obtained using Hashin friction damage failure principle is far less than the experimental value. By adding the weight element of shear stress component [22,23], the bad effect of shear stress component on the failure of fiber element is reduced. The judgment basis of the improved three-dimensional Hashin friction damage failure principle is as follows.

The expression of fiber cracking failure in galvanized sheet fiber bundle is as follows:

$$\left[\frac{\sigma_1}{X_T(n)} \right]^2 + \alpha \left[\frac{\sigma_{12}}{S_{12}(n)} \right]^2 + \alpha \left[\frac{\sigma_{13}}{S_{13}(n)} \right]^2 \geq 1. \quad (13)$$

The analytical formula of resin fracture failure in galvanized sheet fiber bundle is as follows:

$$\left[\frac{\sigma_2 + \sigma_3}{Y_T(n)} \right]^2 + \frac{\sigma_{23}^2 - \sigma_2 \sigma_3}{S_{23}^2(n)} + \left[\frac{\sigma_{12}}{S_{12}(n)} \right]^2 + \left[\frac{\sigma_{13}}{S_{13}(n)} \right]^2 \geq 1. \quad (14)$$

Among them, σ_i and σ_{ij} are the normal stress and shear stress in each main direction of the fiber bundle under the n -th cycle load; $X_T(n)$ and $Y_T(n)$ in turn indicate the residual shear strength of the fiber bundle in each main direction under the M -th cyclic load; α is the failure weight element.

The mechanical properties of different volumes of galvanized sheet were tested. The residual stiffness and strength models of longitudinal and surface shear friction damage of unidirectional galvanized sheet with different fiber volume components were obtained by fitting.

The galvanized sheet unidirectional area longitudinal tensile friction damage test can obtain the residual stiffness and strength models related to the number of friction damage n , which are described as:

$$\frac{E(n)}{E_0} = 1 - 0.02 \times \left[0.82 \times \left(\frac{n}{N} \right)^{0.27} + 0.18 \times \left(\frac{n}{N} \right)^{21.8} \right], \quad (15)$$

$$\frac{X_T(n)}{\sigma_u} = 1 - \left(1 - \frac{\sigma_u}{X_T} \right) \left[0.82 \times \left(\frac{n}{N} \right)^{12.62} + 0.18 \times \left(\frac{n}{N} \right)^{0.31} \right]. \quad (16)$$

The friction damage times n , stress level p , and fiber volume component V_f were obtained by fitting the unidirectional shear test of galvanized sheet. The residual stiffness model and strength model of shear friction damage in the relevant unidirectional plate surface are:

$$\frac{E(n, p, V_f)}{E_0} = 1 - \left[1 - \left(\frac{V_f}{1.59} \right)^{0.19} \right] \times \left[p^{1.51} V_f^{-0.36} \times \left(\frac{n}{N} \right)^{p^{2.95}} + (1 - p^{1.51} V_f^{-0.36}) \times \left(\frac{n}{N} \right)^{p^{-5.46}} \right], \quad (17)$$

$$\frac{S_{12}(n, p, V_f)}{S_{12}} = 1 - \left(1 - \frac{\sigma}{S_{12}} \right) \times \left[p^{-0.04} V_f^{1.3} \times \left(\frac{n}{N} \right)^{p^{1.2}} + (1 - p^{-0.04} V_f^{1.3}) \times \left(\frac{n}{N} \right)^{p^{-5.46}} \right]. \quad (18)$$

The surface of galvanized sheet is pushed by friction damage load. With the increase in the number of friction damage loading cycles n , the unit in the fiber bundle will gradually damage, and its material properties will gradually decline with the increase in n . Here, equations (16) and (17) can be used to degrade the material properties of the unit in a gradual mode.

With the increase in the number of friction damage, the properties of unit materials in the cell gradually decline. After a certain degree of friction damage, the element will generate damage failure, and the material properties of the element will degrade suddenly after failure. To accurately judge the friction damage of materials, it is necessary not only to evaluate the failure mode principle of single cell, but also to define the basis of final damage on the surface of galvanized sheet. Under the action of friction damage load, when the damage is widened to a fixed level, the surface of galvanized sheet will not have the bearing capacity, and the global structure will eventually be damaged.

When simulating progressive damage under static load, when the stress-strain curve has an inflection point, the average unit cell strain will reach the cracking strain. If the damaged unit has covered the surface of all fiber bundles, the single cell structure cannot be sustained. The static cracking strain is chosen as the basis to judge the failure of single cell structure. That is to say, under the action of friction damage load, when the average cell strain reaches the static cracking strain, the global failure of the surface structure of galvanized sheet is judged. In the friction damage calculation program, only the influence of the maximum friction damage stress load on the single friction damage performance is considered. The residual strength model and stiffness model also complete degradation under the highest friction damage stress load.

3.3 Fractal calculation model for friction of galvanized sheet

According to the adhesive friction theory, the total friction can be expressed as follows:

$$F = F_a + F_e = A_r \tau_b + F_e, \quad (19)$$

where F is the total friction force; F_a is the adhesion; F_e is the furrow force; and τ_b is the shear strength of soft metals. In general, the ploughing force is only a few percent of the friction force, so it can be ignored. The friction coefficient μ can be simplified as follows:

Figure 5: Friction coefficient calculated by friction model under different loads.

$$\mu = \frac{F}{P} = \frac{A_r \tau_b}{p}. \quad (20)$$

When the shear strength of the zinc layer of the alloyed hot-dip galvanized sheet GA is 113 MPa, the friction coefficient is $\mu = \frac{F}{P} = 113 \frac{A_r}{p}$. For hot-dip galvanized sheet GI, when its shear strength is 90 MPa, its friction coefficient is $\mu = \frac{F}{P} = 90 \frac{A_r}{p}$.

The relationship between the friction coefficient and the positive pressure of two different galvanized sheets is shown in Figure 5. It can be seen from the figure that the friction coefficient of the two galvanized sheets decreases with the increase in load. The friction coefficient of GA plate decreases greatly in the initial stage, but decreases slowly when the load reaches 3,000 N. In contrast, the friction coefficient of GI plate is relatively gentle in the whole process of decline.

4 Simulation experiment design and result analysis

4.1 The accuracy of friction damage judgment of galvanized sheet surface under the proposed method

Simulation experiments are designed to verify the accuracy of the fractal calculation method of friction damage. According to the static test of the specimen, the static strength used in this paper is 480 MPa. According to the load movement curve of the static test, the cracking strain can be calculated to be 1.68%.

Figure 6 shows the S - N (friction stress) curve of friction damage calculation.

It can be seen from Figure 6 that the friction damage and stress level curves of the calculation method in this paper have good linearity, and the S - N curve close to the surface of the galvanized sheet under various stress tracking results is linear. In the simulation experiment, it can be seen that the shear load has more influence on the friction damage. In this paper, the improved three-dimensional Hashin principle is used to reduce the influence of shear model on the evaluation of friction damage, so as to enhance the accuracy of friction damage calculation.

Figure 7 shows the transformation of the ratio of friction damage elements of galvanized sheet with normalized damage under different stress levels.

It can be seen from Figure 7 that under the friction damage stress level, the element expansion rate of galvanized sheet is faster, and the element damage ratio increases exponentially. The reason is that when a certain element has damage failure, after the stiffness matrix is reduced by the principle of sudden drop of stiffness, because of the phenomenon of stress accumulation. As a

Figure 6: Schematic diagram of S - N curve for friction damage calculation.

Figure 7: Schematic diagram of stress level fiber damage development speed.

result, the adjacent elements will grow rapidly under the next cycle load, and then the expansion rate will increase significantly. Under different stress levels, it is reliable to judge the structural failure using the failure damage ratio of single cell structure, and it also shows that the crack strain used in this method as the evaluation standard of structural failure is more reasonable. At the same time, it can be seen that the higher the friction damage stress level, the faster the fiber bundle from the initial damage to the final failure of the structure, and the faster the failure rate.

4.2 The accuracy test of the friction fractal calculation result of the proposed method

To carry out the sliding friction experiment of galvanized sheet, the friction coefficient of galvanized sheet under different loads was measured. SP781BQ galvanized sheet and HC420/780DPD + Z galvanized sheet were used in this experiment. The diversity of galvanized sheet materials can enhance the reliability of the experimental results. The friction coefficient is calculated using the friction model established above. The comparison between the theoretical value and the experimental value is shown in Tables 2 and 3.

It can be seen from Tables 2 and 3 that the theoretical value of friction coefficient of the two kinds of galvanized sheet is close to the experimental value, and the error

Table 2: Comparison of theoretical and experimental values of friction coefficient of SP781BQ galvanized sheet

Category	Load (N)		
	1,000	1,500	2,000
Theoretical value	0.1853	0.1625	0.1451
Experimental value	0.2058	0.1751	0.1554
Error (%)	7.5	9.1	7.5

Table 3: Comparison of theoretical and experimental values of friction coefficient of HC420/780DPD + Z galvanized sheet

Category	Load (N)		
	1,000	1,500	2,000
Theoretical value	0.1350	0.1388	0.1276
Experimental value	0.1501	0.1459	0.1365
Error (%)	10	7.4	5.7

between them is less than 10%. The theoretical value of the friction coefficient is slightly less than the experimental value because the influence of ploughing force is ignored.

5 Conclusion

- (1) The galvanized sheet has fractal characteristics. The average fractal dimension and scaling factor of SP781BQ hot-dip galvanized sheet were 1.52 and 0.23 μm respectively, and the average fractal dimension and scaling coefficient of HC420/780DPD Z hot-dip galvanized sheet were 1.60 and 0.11 μm respectively.
- (2) In this paper, a friction model is established and the stress-friction curve and the load-damage unit curve obtained are consistent with the general law of damage expansion. Therefore, the frictional damage fractal calculation method proposed in this paper can accurately and effectively judge the friction damage of galvanized sheet surface, and has high applicability and robustness.
- (3) The test results are verified by comparing the sliding friction of the plate. The results show that the theoretical values of friction coefficients are in good agreement with the experimental values. The average error is less than 10%, which proves the accuracy and feasibility of the frictional fractal model.

Acknowledgment: The study was supported by “Youth Teacher Growth Program of Hunan University (2018).”

Conflict of interest: Authors state no conflict of interest.

Data availability statement: All data generated or analysed during this study are included in this article.

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