Research Article

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Stratified heat transfer of magneto-tangent hyperbolic bio-nanofluid flow with gyrotactic microorganisms: Keller-Box solution technique

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Abstract: The purpose of the present investigation is to examine the heat, mass and microorganism concentration transfer rates in the magnetohydrodynamics (MHD) stratified boundary layer flow of tangent hyperbolic nanofluid past a linearly, uniform stretching surface comprising gyrotactic microorganisms as well as nanoparticles. The governing PDEs with relevant end point conditions are molded into a non-dimensional ordinary differential equation (ODE) form by means of the similarity transformation. The numerical solution of dimensionless problem is acquired within the frame of robust Keller-Box technique. The velocity, temperature, mass and motile microorganism density are investigated graphically within the context of different significant parameters. Numerical results have been inspected via plots and table (namely as the local Nusselt number, the local wall mass flux and the local microorganisms wall flux). This article proves that the energy, concentration and motile microorganism density reduce with increase in thermal, solutal and motile density stratification parameters.

The asserted outcomes are beneficial to enhance the cooling and heating processes, energy generation, thermal machines, solar energy systems, industrial processes etc.

Keywords: gyrotactic microorganisms, nanofluids, stratification, MHD, stretchable sheet, Keller-Box method

Nomenclature

C	nanoparticle volume fraction
C_w	wall dimensional concentration
C_∞	ambient dimensional concentration
D_B	coefficient of Brownian diffusion
D_m	microorganisms diffusivity
D_T	coefficient of thermophoretic diffusion
f	dimensionless stream function
We	Weissenberg number
N	concentration of microorganisms
k	k thermal conductivity
Lb	bioconvection Lewis number
Le	Lewis number
N_w	surface concentration of microorganisms
Nb	Brownian motion parameter
N_∞	ambient concentration of microorganisms
Nn_x	local density number of microorganisms
Nt	thermophoresis parameter
Nu_x	local Nusselt number
Pe	bioconvection Péclet number
Pr	Prandtl number
q_w	heat flux
q_m	wall mass flux
q_n	wall motile microorganisms flux
T_o	reference temperature
C_o	reference concentration of nanoparticles
Γ	time constant
St	thermal stratification parameter
Sm	motile density stratification parameter

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τ_w	shear stress
b	chemotaxis constant
Sh_x	local Sherwood number
T	temperature
T_∞	ambient temperature
u, v	velocity components
Wc	maximum speed of swimming cell
x, y	coordinate axis
n	power law index
C_f	skin friction coefficient
α	effective thermal diffusivity
χ	microorganisms fraction function
η	similarity variable
ϕ	rescaled nanoparticle volume fraction
Ω	bioconvection constant
θ	non-dimensional temperature
ν	kinematic viscosity
ρ_f	density of the base fluid
ρ_p	density of nanoparticles
$(\rho c)_f$	heat capacity of base liquid
$(\rho c)_p$	heat capacity of solid nanoparticles
τ	ratio of heat capacities of solid particles and base fluid
ψ	dimensional stream function
N_0	reference concentration of microorganisms
σ	electric conductivity
Sc	mass stratification parameter
a	stretching rate

1 Introduction

Bioconvection describes a macroscopic movement (convection) within the fluid caused by the density gradient produced by mutual floating of motile microorganisms [1,2]. Such a phenomenon takes place due to the fact that motile microorganisms are slightly denser as compared to fluid in suspensions so they usually run in the upward direction, leading to an unstable as well as dense upper surface. This results in the development of hydrodynamic unrest under specific circumstances. They are self-propelled and enhance the densities of the base fluids as a result of swimming in a specific direction in the fluids as a reaction to these stimuli like light, chemical gradients and gravity. Gyrotactic [3,4] is locomotion directed by the combination of balance of torques caused by shear flow as well as gravity functioning on a bottom-heavy cell. The purpose of study of swimming patterns of microorganisms in water as well as in other fluids denser as compared to water is their presence

in environmental and manufactured products such as environmental fuels, ethanol, fertilizers and fuel cells. Kuznetsov [5] presented the development of nanofluid bioconvection in suspensions having gyrotactic microorganisms as well as nanoparticles and increase fluid convection. Das *et al.* [6] have investigated the MHD bioconvection of gyrotactic microorganisms over a porous vertical sheet. It is noticed that the concentration of motile microorganisms increases with increase in the value of bioconvection Lewis number. Chen *et al.* [7] examined the time-dependent nanofluid flow caused by a stretching cylinder that consists of gyrotactic microorganisms and nanoparticles. They found that the concentration of microorganisms is seen to boost with greater bioconvection Peclet number together with lower Lewis number. Mosayebidorcheh *et al.* [8] have analyzed bioconvection fluid flow and heat transfer rate inside a horizontal channel utilizing least square technique. It is noted that thermophoresis parameter has a very small effect on temperature profile. Chakraborty *et al.* [9] have considered the impact of magnetohydrodynamics (MHD) within bioconvection of a nanofluid flow having gyrotactic microorganisms under convective end point conditions. Analysis shows that due to the increase in magnetic parameter the rate of motile microorganism flux decreases.

More recently, nanofluids have acquired significant amounts of consideration on account of their ability in boosting the heat transfer. Nanosized particles, specifically, copper, silver, aluminum, gold and their oxides are widely-used as colloidal agents with many common base fluids such as methanol, water, glycerin, ethylene glycol and engine oil to ensure that thermophysical characteristics of heat transportation of the base fluid could be enhanced. Scientific research indicates that nanofluids generally require just incorporation up to a 5% nanoparticle volume fraction to make sure efficient heat transport enhancements [10]. Nanofluids give several diversified benefits in practical application such as microelectronics, optics, catalysis, fuel cell, nuclear reactors, bio drugs, renewable energy, transportation and many more [11]. Xuan *et al.* [12] imitated random motion as well as the aggregation procedure of the nanoparticles by using the idea of Brownian motion along with the model of diffusion-limited aggregation. Buongiorno [13] introduced convective transport in nanofluids considering thermophoresis and Brownian motion parameters. Furthermore, he revealed that these parameters are the most essential slip mechanisms. Kuznetsov and Nield [14] scrutinized the boundary layer nanofluid flow beyond a vertical surface. They noted that the heat transfer rate is a reducing

function of Brownian motion as well as thermophoresis numbers. Gireesha *et al.* [15] investigated the impact of Hall current on the unsteady fluid flow of nanofluid containing the dust as a nano particles. It is found that the effect of Hall current as well as unsteadiness is opposite for drag coefficient. More detailed information can be seen in Refs. [16–20].

In recent times, non-Newtonian fluids have received extensive interest as a consequence of their technological as well as industrial applications including cosmetic items, melts of polymers, glass fiber production, biological solutions, paints, asphalts, food processing and so on [21,22]. There exists not a single non-Newtonian liquid model that speculates entirely all features of non-Newtonian nanofluids. Hyperbolic tangent fluid belongs to the family of the non-Newtonian fluid, which has the ability to exhibit shear thinning (pseudo-plastic) phenomena. The unique benefit of hyperbolic tangent fluid covers the characteristics of physical robustness, simplicity and computational comfort. The hyperbolic tangent fluid has various benefits compared to various non-Newtonian models, including physical robustness, simplicity and computational comfort. Some of these fluids are composed of paints, ketchup, whipped cream, nail polish and blood etc. The different studies were presented on tangent hyperbolic fluid model utilizing various flow geometries. Veiga [23] illustrated the shear-dependent viscosity effect on flow of pseudo-plastic fluids under the non-slip Dirichlet boundary condition. Lin *et al.* [24] contemplated the Marangoni convective transferring heat in radiating nanofluid with non-uniform thermal conductivity. Kothandapani and Prakash [25] represented the MHD and the heat source impacts on peristaltic hyperbolic tangent nanofluid flow inside a tapering asymmetric channel. Gaffar *et al.* [26] investigated the MHD impacts on heat transport of hyperbolic tangent liquid in a cylinder considering partial slip. The impact of stratification on hyperbolic tangent fluid flow along a porous stretching cylinder is delineated by Nagendramma *et al.* [27]. Shafiq *et al.* [28] analyzed the bioconvective flow of tangent hyperbolic nanofluid flow under influences of the Newtonian heating as well as MHD.

The boundary layer fluid flow caused by continuous surfaces has excited the curiosity among a number of researchers for the last couple of decades because of its potential use in different engineering, for instance extrusion of polymer, in chemical engineering, metallurgy and bio-medical engineering. Material review demonstrates that it has important application for making medical equipment. Particularly, this type of flow occurs in spinning of drawing of plastic films, fibers, aerodynamics,

expansion of balloons, continuous casting, glass glowing and much more. Bachok *et al.* [29] studied time-dependent boundary layer fluid flow when the heat is transferred past a porous stretching/shrinking surface. Gupta and Gupta [30] analyzed the characteristics of flow, heat and mass transfer in the stretching sheet system, where the sheet is sucked and injected. The boundary layer heat transport flow along a continuous stretching surface has been discussed by Bhattacharyya *et al.* [31], which is bounded by slip and subjected to the stagnation point as well as heat transfer. The analysis demonstrated that the rise in slip enhances the choice of velocity ratio number for the presence of dual solution. Lok *et al.* [32] have explored the MHD flow when the sheet is shrieked. It is observed that the dual solutions occur for small values of magnetic parameter. Subsequently, various features of the problem of stretching sheet are examined in different findings [33–43].

Phenomenon of stratification is a crucial feature in heat as well as mass transfer analysis. The creation or deposition of the layers is termed the stratification. This phenomenon happens because of the difference in temperature or concentration, or differences in both, or existence of different liquids or various densities. This kind of phenomenon occurs in the natural flows, especially in the canals, rivers, seas and also water reservoirs etc. Furthermore, the impacts of solute as well as thermal stratifications are very important for solar energy because the better stratification means better power productivity. Bearon and Gunbaum [44] studied the influence of bioconvection in a stratified environment. They concluded that the unique part of this article is to look at the effect of salinity stratification on bioconvection. Kameswaran *et al.* [45] examined the thermal stratification factor affected by the heat transfer nanofluid flow due to a vertical wavy channel. It is determined that the thickness of thermal boundary layer is enhanced by the thermal stratification parameter. Madhu and Reddy [46] illustrated the effect of thermal stratification on MHD heat transfer flow past a continuous surface with an exponential velocity and it is stated that the temperature gradient enhances substantially with a boost in stratification. Recent additions considering nanofluids with heat and mass transfer in various physical situations are given by Refs. [47–53].

The findings of the triple MHD stratified boundary layer non-Newtonian tangent hyperbolic nanofluid flow past a stretching surface comprising nanometer-sized particles and gyrotactic microorganisms were never analyzed in the literature based on the aforementioned survey of past literature and authors' knowledge. The flow model is characterized by intricated and highly

nonlinear ODEs for which numerical solution is accurately accomplished with the assistance of classical Keller-Box technique. Assessment of heat and mass transport amelioration by employing the Brownian motion and thermophoresis consequences. Graphical behaviors of velocity, temperature mass and density of microorganisms profiles through appropriate parameters are presented and described. The rates of heat transfer, mass transfer and motile density are analyzed and described by referring to the final graphs and tables.

2 Mathematical formulation

Current investigation assesses the flow and heat transport analysis in the steady 2D MHD flow of an incompressible tangent hyperbolic nanofluid. The fluid system is bounded by a stretching sheet, having velocity $u_w(x) = ax$ along the x -axis direction saturated by motile microorganisms. A magnetic field and a stretching sheet are perpendicular to each other, where the induced magnetic field is ignored by considering really small magnetic Reynolds number (Figure 1). For the mathematical formulation in this article, the Cartesian coordinate is chosen. The heat, concentration and motile density of microorganisms transport rates are analyzed due to the effect of Brownian motion, thermophoresis and stratification effects. The flow phenomena are carried out by the subsequent equations:

Figure 1: Schematic view of current flow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \nu(1-n) \frac{\partial^2 u}{\partial y^2} + \sqrt{2} \Gamma \nu n \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho} B_0^2 u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left(D_B \left(\frac{\partial C}{\partial y} \right) \left(\frac{\partial T}{\partial y} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right), \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right), \quad (4)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} + \frac{bW_c}{(C_w - C_\infty)} \frac{\partial}{\partial y} \left(N \frac{\partial C}{\partial y} \right) = D_m \frac{\partial^2 N}{\partial y^2}. \quad (5)$$

The associative end point conditions [54] are shown as follows:

$$\left. \begin{aligned} u &= u_w(x) = ax, \quad v = 0, \quad T = T_w = T_0 + e_1 x, \\ C &= C_w = C_0 + m_1 x, \\ N &= N_w = N_0 + n_1 x \quad \text{at } y = 0, \\ u &\longrightarrow 0, \quad T \longrightarrow T_\infty = T_0 + e_2 x, \\ C &\longrightarrow C_\infty = C_0 + m_2 x, \\ N &\longrightarrow N_\infty = N_0 + n_2 x \quad \text{as } y \longrightarrow \infty, \end{aligned} \right\} \quad (6)$$

where $(e_1, e_2, m_1, m_2, n_1, n_2)$ are the dimensional constants.

Introducing the following similarity transformations which converts Eqs. (1)–(5) into the set of ODEs

$$\left. \begin{aligned} \eta &= \sqrt{\frac{a}{\nu}} y, \quad \psi = -\sqrt{a\nu} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\ \phi(\eta) &= \frac{C - C_\infty}{C_w - C_\infty}, \quad \chi(\eta) = \frac{N - N_\infty}{N_w - N_\infty}. \end{aligned} \right\} \quad (7)$$

Dimensional stream function ψ is given by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (8)$$

Following Ref. [55], we have

$$u = ax f'(\eta), \quad v = -\sqrt{a\nu} f(\eta). \quad (9)$$

By making use of Eqs. (7) and (9), Eq. (1) satisfied automatically, whereas Eqs. (2)–(5) take the form

$$((1-n) + nWef'')f''' - (f')^2 + ff'' - M^2 f' = 0, \quad (10)$$

$$\theta'' - \text{Pr} f' \theta - \text{Pr} \text{St} f' + \text{Pr} f \theta' + \text{Pr} \text{Nb} \phi' \theta' + \text{Pr} \text{Nt} (\theta')^2 = 0, \quad (11)$$

$$\phi'' - \text{Pr} \text{Le} f' \phi - \text{Pr} \text{Le} \text{Sc} f' + \text{Pr} \text{Le} f \phi' + \frac{\text{Nt}}{\text{Nb}} \theta'' = 0, \quad (12)$$

$$\chi'' - \text{Lb } f' \chi - \text{Lb } \text{Sm} f' + \text{Lb } f \chi' - \text{Pe}(\chi' \phi' + \phi''(\Omega + \chi)) = 0, \quad (13)$$

with the transformed dimensionless boundary conditions:

$$\left. \begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 - \text{St}, \\ \phi(0) = 1 - \text{Sc}, \quad \chi(0) = 1 - \text{Sm} \text{ at } \eta = 0, \\ f'(\eta) \longrightarrow 0, \quad \theta(\eta) \longrightarrow 0, \quad \phi(\eta) \longrightarrow 0, \\ \chi(\eta) \longrightarrow 0 \text{ as } \eta \longrightarrow \infty. \end{aligned} \right\} \quad (14)$$

Parameters involved in final ordinary differential Eqs. (10)–(13) are given in Table 1.

The main physical parameters [56] occurred in this problem, namely the drag coefficient, the local heat and mass fluxes and the local motile microorganisms flux are defined as

$$\left. \begin{aligned} C_f &= \frac{2\tau_w}{\rho u_w^2}, \quad \text{Nu}_x = \frac{xq_w}{k(T_w - T_o)}, \\ \text{Sh}_x &= \frac{xq_m}{D_B(\phi_w - \phi_o)}, \quad \text{Nn}_x = \frac{xq_n}{D_m(N_w - N_o)}, \end{aligned} \right\} \quad (15)$$

in which

$$\left. \begin{aligned} \tau_w &= \mu(1-n) \left(\frac{\partial u}{\partial y} \right)_{y=0} + \mu \frac{n\Gamma}{\sqrt{2}} \left(\frac{\partial u}{\partial y} \right)_{y=0}^3, \\ q_w &= -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, \\ q_m &= -D_B \left(\frac{\partial C}{\partial y} \right)_{y=0}, \quad q_n = -D_m \left(\frac{\partial N}{\partial y} \right)_{y=0}. \end{aligned} \right\} \quad (16)$$

Table 1: Parameters involved in final ordinary differential equations

Parameter	Name	Expression
We	$We = \sqrt{\frac{2a^3}{\nu}} \Gamma x$	Weissenberg number
M	$M = \sqrt{\frac{\sigma B_0^2}{a \rho_f}}$	Magnetic parameter
Pr	$Pr = \frac{\nu}{\alpha}$	Prandtl number
Nb	$Nb = \frac{\tau D_B(\phi_w - \phi_o)}{\nu}$	Brownian motion parameter
Nt	$Nt = \frac{\tau D_B(T_w - T_o)}{T_o \nu}$	Thermophoresis parameter
Le	$Le = \frac{\alpha}{D_B}$	Lewis number
St	$St = \frac{e_2}{e_1}$	Thermal stratification parameter
Lb	$Lb = \frac{\nu}{D_m}$	Bioconvection Lewis number
Sc	$Sc = \frac{m_2}{m_1}$	Mass stratification parameter
Pe	$Pe = \frac{bWc}{D_m}$	Bioconvection Lewis number
Sm	$Sm = \frac{n_2}{n_1}$	Motile density stratification parameter
Ω	$\Omega = \frac{n_{\infty}}{n_w - n_{\infty}}$	Microorganism concentration difference parameter

Using the similarity transform presented above, Eq. (15) can be described as:

$$\left. \begin{aligned} \frac{1}{2} C_f \text{Re}_x^{1/2} &= (1-n)f''(0) - \frac{1}{2} n \text{We}(f''(0))^3, \\ \text{Nu}_x \text{Re}_x^{-1/2} &= -\theta'(0), \\ \text{Sh}_x \text{Re}_x^{-1/2} &= -\phi'(0), \quad \text{Nn}_x \text{Re}_x^{-1/2} = -\chi'(0), \end{aligned} \right\} \quad (17)$$

whereas the local Reynolds number is provided by $\text{Re}_x = \frac{u_w x}{\nu}$.

3 Numerical simulations

In this study, Keller-Box scheme [57,58] is employed to solve Eqs. (10)–(13) together with the related end point conditions (14) for various values of the significant parameters. The physical interpretation of Keller-Box method is shown in the flow chart diagram (shown in Figure 2).

We introduce dependent variables $\tilde{z}_1, \tilde{z}_2, \tilde{p}, \tilde{q}$ and $\tilde{g}$ such that

$$\frac{df}{d\eta} = \tilde{z}_1, \quad \frac{d\tilde{z}_1}{d\eta} = \tilde{z}_2, \quad \frac{d\theta}{d\eta} = \tilde{p}, \quad \frac{d\phi}{d\eta} = \tilde{q}, \quad \frac{d\chi}{d\eta} = \tilde{g}. \quad (18)$$

So that Eqs. (10)–(13) can be presented as

$$((1-n) + n \text{We } \tilde{z}_2) \frac{d\tilde{z}_2}{d\eta} - \tilde{z}_1^2 + f\tilde{z}_2 - M^2 \tilde{z}_1 = 0, \quad (19)$$

$$\frac{d\tilde{p}}{d\eta} - \text{Pr} \tilde{z}_1 \theta - \text{Pr } \text{St} \tilde{z}_1 + \text{Pr } f\tilde{p} + \text{Pr } \text{Nb} \tilde{p} \tilde{q} + \text{Pr } \text{Nt} \tilde{p}^2 = 0, \quad (20)$$

$$\frac{d\tilde{q}}{d\eta} - \text{Pr } \text{Le} \tilde{z}_1 \phi - \text{Pr } \text{Le } \text{Sc} \tilde{z}_1 + \text{Pr } \text{Le} f\tilde{q} + \frac{\text{Nt}}{\text{Nb}} \frac{d\tilde{p}}{d\eta} = 0, \quad (21)$$

Figure 2: Flow chart illustrating Keller-Box method.

Figure 3: Difference approximation grid structure for the Keller-Box scheme.

$$\frac{d\check{g}}{d\eta} - \text{Lb}\check{z}_1\chi - \text{Lb}\text{Sm}\check{z}_1 + \text{Lb}f\check{g} - \text{Pe}\left(\check{g}\check{q} + \frac{d\check{q}}{d\eta}(\sigma + \chi)\right) = 0. \quad (22)$$

The associated end point conditions turn out to be

$$\left. \begin{aligned} f(0) = 0, \quad \check{z}_1(0) = 1, \quad \theta(0) = 1 - \text{St}, \\ \phi(0) = 1 - \text{Sc}, \quad \chi(0) = 1 - \text{Sm} \text{ at } \eta = 0, \\ \check{z}_1(\eta) \rightarrow 0, \quad \theta(\eta) \rightarrow 0, \quad \phi(\eta) \rightarrow 0, \\ \chi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty. \end{aligned} \right\} \quad (23)$$

A one-dimensional computational mesh is displayed in Figure 3 and the grid points are defined by $\eta_0 = 0$, $\eta_j = h + \eta_{j-1}$, $j = 0, 1, 2, 3, \dots, J$, where h is $\Delta\eta$ spacing. By means of the finite-difference approximation at mid-point $\eta_{j-1/2}$ we have

$$\frac{f_j - f_{j-1}}{h} = \frac{(\check{z}_1)_j + (\check{z}_1)_{j-1}}{2}, \quad (24)$$

$$\frac{(\check{u}_1)_j - (\check{u}_1)_{j-1}}{h} = \frac{(\check{u}_2)_j + (\check{u}_2)_{j-1}}{2}, \quad (25)$$

$$\frac{\theta_j - \theta_{j-1}}{h} = \frac{\check{p}_j + \check{p}_{j-1}}{2}, \quad (26)$$

$$\frac{\phi_j - \phi_{j-1}}{h} = \frac{\check{q}_j + \check{q}_{j-1}}{2}, \quad (27)$$

$$\frac{\chi_j - \chi_{j-1}}{h} = \frac{\check{g}_j + \check{g}_{j-1}}{2}, \quad (28)$$

$$\left. \begin{aligned} & \left(\frac{\check{p} - \check{p}_{j-1}}{h} \right) - \text{Pr} \left(\frac{(\check{u}_1)_j + (\check{u}_1)_{j-1}}{2} \right) \left(\frac{\check{\theta}_j + \check{\theta}_{j-1}}{2} \right) \\ & - \text{Pr} \text{St} \left(\frac{(\check{z}_1)_j + (\check{z}_1)_{j-1}}{2} \right) \\ & + \text{Pr} \left(\frac{f_j + f_{j-1}}{2} \right) \left(\frac{\check{p}_j + \check{p}_{j-1}}{2} \right) \\ & + \text{Pr} \text{Nb} \left(\frac{(\check{p})_j + (\check{p})_{j-1}}{2} \right) \left(\frac{q_j + q_{j-1}}{2} \right) \\ & + \text{Pr} \text{Nt} \left(\frac{(\check{p})_j + (\check{p})_{j-1}}{2} \right)^2 = 0, \end{aligned} \right\} \quad (30)$$

$$\left. \begin{aligned} & \left(\frac{\check{q} - \check{q}_{j-1}}{h} \right) - \text{Pr} \text{Le} \left(\frac{(\check{u}_1)_j + (\check{u}_1)_{j-1}}{2} \right) \left(\frac{\check{\phi}_j + \check{\phi}_{j-1}}{2} \right) \\ & - \text{Pr} \text{Le} \text{Sc} \\ & \left(\frac{(\check{u}_1)_j + (\check{u}_1)_{j-1}}{2} \right) + \text{Pr} \text{Le} \left(\frac{f_j + f_{j-1}}{2} \right) \left(\frac{\check{q}_j + \check{q}_{j-1}}{2} \right) \\ & + \frac{\text{Nt}}{\text{Nb}} \left(\frac{\check{p} - \check{p}_{j-1}}{h} \right) = 0, \end{aligned} \right\} \quad (31)$$

$$\left. \begin{aligned} & \left(\frac{\check{g} - \check{g}_{j-1}}{h} \right) - \text{Lb} \left(\frac{(\check{u}_1)_j + (\check{u}_1)_{j-1}}{2} \right) \left(\frac{\check{\chi}_j + \check{\chi}_{j-1}}{2} \right) \\ & - \text{Lb} \text{Sm} \left(\frac{(\check{z}_1)_j + (\check{z}_1)_{j-1}}{2} \right) \\ & + \text{Lb} \left(\frac{f_j + f_{j-1}}{2} \right) \left(\frac{\check{g}_j + \check{g}_{j-1}}{2} \right) \\ & - \text{Pe} \left(\frac{\check{g}_j + \check{g}_{j-1}}{2} \right) \left(\frac{\check{q}_j + \check{q}_{j-1}}{2} \right) \\ & - \text{Pe} \left(\sigma + \frac{\chi_j + \chi_{j-1}}{2} \right) \left(\frac{\check{q} - \check{q}_{j-1}}{h} \right) = 0. \end{aligned} \right\} \quad (32)$$

Next, the substitution as shown in Eq. (33) is introduced. This substitution is used to linearize the nonlinear Eqs. (23)–(32) by Newton's method, and is shown as follows:

$$\left. \begin{aligned} & \left((1 - n) + n \text{We} \left(\frac{(\check{z}_2)_j + (\check{z}_2)_{j-1}}{2} \right) \right) \left(\frac{(\check{z}_2)_j - (\check{z}_2)_{j-1}}{h} \right) - \left(\frac{(\check{z}_1)_j + (\check{z}_1)_{j-1}}{2} \right)^2 \\ & + \left(\frac{f_j + f_{j-1}}{2} \right) \left(\frac{(\check{z}_2)_j + (\check{z}_2)_{j-1}}{2} \right) - M^2 \left(\frac{(\check{z}_1)_j + (\check{z}_1)_{j-1}}{2} \right) = 0, \end{aligned} \right\} \quad (29)$$

$$\left. \begin{aligned} (\tilde{z}_1)_j^{n+1} &= (\tilde{z}_1)_j^n + \delta(\tilde{z}_1)_j^n, \quad (\tilde{z}_2)_j^{n+1} = (\tilde{z}_2)_j^n + \delta(\tilde{z}_2)_j^n, \\ (\tilde{g})_j^{n+1} &= (\tilde{g})_j^n + \delta(\tilde{g})_j^n, \\ f_j^{n+1} &= f_j^n + \delta f_j^n, \quad \theta_j^{n+1} = \theta_j^n + \delta \theta_j^n, \\ \phi_j^{n+1} &= \phi_j^n + \delta \phi_j^n, \quad \chi_j^{n+1} = \chi_j^n + \delta \chi_j^n, \\ p_j^{n+1} &= p_j^n + \delta p_j^n, \quad q_j^{n+1} = q_j^n + \delta q_j^n. \end{aligned} \right\} \quad (33)$$

The linearized set of equations can be written in the form of block tridiagonal structure

$$A\delta = K, \quad (34)$$

where

$$A = \begin{bmatrix} [M_1] & [P_1] & & & \\ [N_2] & [M_2] & [P_2] & & \\ & \ddots & \ddots & \ddots & \\ & & [N_{j-1}] & [M_{j-1}] & [P_{j-1}] \\ & & & [N_j] & [M_j] \end{bmatrix}$$

$$\delta = \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{j-1}] \\ [\delta_j] \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} [K_1] \\ [K_2] \\ \vdots \\ [K_{j-1}] \\ [K_j] \end{bmatrix}.$$

Matrix A is factorized in the following form:

$$A = LU, \quad (35)$$

where

$$L = \begin{bmatrix} [a_1] & & & & \\ [b_2] & [a_2] & & & \\ & \ddots & \ddots & \ddots & \\ & & [a_{j-1}] & & \\ & & [b_j] & [a_j] \end{bmatrix},$$

$$U = \begin{bmatrix} [I] & [c_1] & & & \\ & [I] & [c_2] & & \\ & & \ddots & \ddots & \\ & & & [I] & [c_{j-1}] \\ & & & & [I] \end{bmatrix}.$$

In Eq. (34) each $[a_i]$, $[b_i]$, $[c_i]$ is a 9×9 matrix and $[I]$ is a 9×9 identity matrix. By applying LU factorization process on Eq. (34), we obtain the solution δ . We choose the grid size $h = 0.01$ while we execute the numerical simulation until $\eta_{\max} = 14$. The convergence criterion is fixed at 10^{-5} as the difference between the present and prior iterations for the desirable accuracy. The outcomes

Table 2: Comparison value of $-f''(0)$ for various values of M

M	Ali <i>et al.</i> [59]	Ibrahim [60]	Present study
0.0	1×10^0	1×10^0	1×10^0
1	$14,142 \times 10^{-4}$	$14,142 \times 10^{-5}$	$14,142 \times 10^{-6}$
5	$24,495 \times 10^{-4}$	$244,948 \times 10^{-5}$	$2,449,501 \times 10^{-6}$

of Keller-Box method have been compared with that of `bvp4c` solution. The results are verified by comparing the present outcomes of $-f''(0)$ to previous results of Ali *et al.* [59] and Ibrahim [60] for $We = n = 0$ as shown in Table 2. As a conclusion, it is found that the outcomes show a very good agreement with the previous researchers.

4 Results and discussion

This section deals with graphical features of velocity profile f' , temperature profile θ , concentration profile ϕ , motile microorganism density profile χ , heat transfer rate “ θ ,” mass transfer rate “ ϕ ” and density of motile microorganism rate for essential parameters. For numerical simulation, we considered the values of various emerging parameters such as $Pr = 10$, $Lb = 1$, $Nb = 0.2$, $Nt = 0.1$, $Le = 0.5$, $We = 0.1$, $n = 0.3$, $\sigma = 3$, $Pe = 1$, $Sc = 0.1$, $St = 0.1$, $Sm = 0.1$, $M = 0.5$, $\Omega = 3$ unless others specified. It is observed that an increase in M reduces $f'(\eta)$, as shown in Figure 4. Physically magnetic parameter produces a resistive Lorentz force, which has the impact of slowing flow. As a result velocity profile decays. Figure 5 displays the impact of n on flow field $f'(\eta)$. It is detected that a surge in M reduces the velocity profile. Meanwhile, amplitude of the shear thinning

Figure 4: Influence of M on f' .

Figure 5: Influence of n on f' .

phenomenon reduces by rising n which accordingly decays the profile, followed by the decrement of thickness of hydrodynamic boundary layer. The effect of Weissenberg number We on $f'(\eta)$ is presented in Figure 6. It is noted that an escalation in We causes a decrement in velocity profile. It is mainly because that the predominant values of We cause augmentation in the relaxation time. Subsequently, the cohesive forces among the fluid particles become weaker and the fluid motion is opposed.

In this figure, the impact of Prandtl number Pr on thermal profile $\theta(\eta)$ is depicted. It can be seen from Figure 7 that an enhancement in Pr decays thermal profile, as the thermal conductivity of the mixture decreases by augmenting Pr . Hence, the rate of heat transfer slows down, which drops the temperature of nanofluid. Influence of Nb on $\theta(\eta)$ is shown in Figure 8. An increment in values of Brownian motion parameter Nb induces an enlargement in concentration temperature $\theta(\eta)$. Basically

Figure 7: Influence of Pr on θ .

additional heat is generated by the liquid particles, which move randomly inside the model of larger Brownian motion parameter Nb . For that reason temperature $\theta(\eta)$ enhances. Figure 9 clearly shows that the temperature profile $\theta(\eta)$ is amplified when bigger thermophoresis parameter Nt . This is because the heated particles are being transferred from warm to cold region, under the effect of thermophoresis phenomenon.

Figure 10 shows the impacts of Nb on $\phi(\eta)$. It is noted that fluid concentration reduces with higher Nb . Actually, higher Nb causes the fluid particles collide with great momentum among them. As a result, less mass is shifted and the concentration is reduced. Figure 11 depicts a concentration profile due to the effect of thermophoresis parameter. This figure shows that the concentration increases for larger Nt . That is because the thermophoresis force increases to a higher rate for thermophoresis parameter, which results in greater diffusive impact.

Figure 6: Influence of We on f' . Figure 8: Influence of Nb on θ .

Figure 9: Influence of Nt on θ . Figure 10: Influence of Nb on ϕ . Figure 11: Influence of Nt on ϕ .

Thus, concentration distribution increases. Figure 12, depicts the impact of Lewis number Le on concentration distribution $\phi(\eta)$. From Figure 12 it is clear that fluid concentration decreases with increase in Lewis number Le . Physically, it is true since increase in Lewis number Le means decrease in molecular diffusivity, which leads to decrease of concentration boundary layer. Therefore, the concentration is smaller for bigger values of Le . Figure 13 elucidates that ϕ and thickness of concentration boundary layer increase with the growth of magnetic parameter M . The reason is that the temperature gradient inherent in the viscosity of the fluid.

Figure 14 displays the impact of Peclet number Pe on the motile density profile $\chi(\eta)$. This profile diminishes with an increment in Pe . In fact, a rise in Peclet number Pe induces a decrement in the diffusivity of micro-organisms. Figure 15 depicts the influence of σ on $\chi(\eta)$.

Figure 12: Influence of Le on ϕ . Figure 13: Influence of M on ϕ .

Figure 14: Influence of Pe on χ . Figure 16: Influence of Lb on χ . Figure 15: Influence of σ on χ . Figure 17: Influence of M on χ .

This profile reduced with the effect of σ . Figure 16 highlighted the influence of bioconvection Lewis number Lb on motile density $\chi(\eta)$. The decay of motile density can be seen during an increment in Lb . The diffusivity of micro-organisms reduces for bigger Lb and consequently motile density of fluid declines. From Figure 17, we noticed that motile density profile $\chi(\eta)$ enhances for larger magnetic parameter M .

Figure 18 highlights the effect of thermal stratification parameter St on the temperature field $\theta(\eta)$. It is determined that temperature field decays steadily with the boost in St . In physical terms, the existence of stratification effect minimizes the effective temperature difference within the surface and the ambient fluid, which causes a weaker temperature field. Figure 19 is delineated to examine the variation in temperature profile for various

Figure 18: Influence of St on θ .

solulal stratification parameter Sc . It is noted from the figure concentration profile ϕ reduces with the increment in the value of solulal stratification parameter Sc . This is simply because of the decline of convective potential within ambient fluid and surface. Figure 20 signifies the impact of the motile density stratification parameter Sm on the motile density profile $\chi(\eta)$. An increment in Sm causes a deceleration in concentration difference of microorganisms within the surface and far from the surface. Hence, reduction in the density profile is observed.

Figure 21 demonstrates the variation of the local Nusselt number with Nb and Pr . It is noted that the local Nusselt number reduces with an increment in Nb parameter, however, opposite behavior with the rise in Pr . Because increasing the Brownian motion as well as thermophoretic effect means more collisions which create

Figure 19: Influence of Sc on ϕ .

Figure 21: Variation in $-\theta'(0)$ with Pr and Nb .

Figure 22: Variation in $-\phi'(0)$ with Nt and Le .

Figure 20: Influence of Sm on χ .

Figure 23: Variation in $-\chi'(0)$ with Sm and Lb .

Table 3: Variation of $Nu_x^{-1/2}$, $Shu_x^{-1/2}$ and $Nn_x^{-1/2}$ for different parameters when $Lb = 0.5$ and $\Omega = 3$

M	Le	Nt	Nb	We	n	Pr	Pe	St	Sc	Sm	$-\theta'(0)$	$-\phi'(0)$	$-\chi'(0)$
0.5	0.5	0.1	0.1	0.4	0.3	10	2.5	3	0.1	0.1	2.24978	1.16562	10.41580
1											2.21904	0.98787	8.85685
1.5											2.17331	0.75694	6.74985
	2										1.76923	4.78884	45.36783
	4										1.58134	7.41846	70.94515
	6										1.48820	9.34790	89.73649
		0.1									2.24978	1.16562	10.41580
		0.15									2.08605	0.74902	6.50579
		0.2									1.94483	0.41811	3.44864
			0.1								2.24978	1.16562	10.41580
			0.2								1.72430	1.98638	18.25522
			0.3								1.34063	2.21600	20.45385
				0.1							2.25679	1.19033	10.63528
				0.2							2.25471	1.18262	10.56697
				0.3							2.25239	1.17441	10.49404
					0.1						2.26767	1.26319	11.24977
					0.2						2.26036	1.22097	10.89209
					0.3						2.24978	1.16562	10.41580
						3					1.55334	0.06357	0.10356
						5					1.91753	0.34782	2.71949
						10					2.24978	1.16562	10.41580
							1				2.24978	1.16562	4.28797
							2				2.24978	1.16562	8.32506
							3				2.24978	1.16562	12.54019
								0.2			2.22956	1.14396	10.20473
								0.3			2.20807	1.12279	9.99834
								0.4			2.18524	1.10215	9.79709
									0.2		2.27423	1.07552	9.62548
									0.3		2.29896	0.98515	8.84000
									0.4		2.32395	0.89450	8.06031
										0.2	2.24978	1.16562	10.12935
										0.3	2.24978	1.16562	9.84290
										0.4	2.24978	1.16562	9.55645

additional heat, the temperature escalates and heat transfer rate will be reduced. From Figure 22, an increase in the local Sherwood number is noted with an increase in Le and Nt . This might be related to the concept that when the concentration at the sheet is greater than in the fluid, the mass is transferred from the higher concentration to the lower region. Figure 23 examines the influence of Sm and Lb on the local density number. With an increment in Sm , the value of density number reduces. Further density number escalates with an increment of bioconvection Lewis number Lb . It is because of the higher concentration of the motile microorganisms close to the boundary region.

Table 3 portrays the numerical results of the local Nusselt number, the local Sherwood number and the local density number for various values of M , Le , Nt , Nb , We , n , Pr , Pe , St , Sc and Sm . Tabular values reveal

that local Nusselt number diminishes by enhancing M , Le , Nt , Nb , We , n and St although it enhances for bigger values of Pr and Sc . The magnitude of mass transfer rate is augmented for bigger values of Le , Nb and Pr , whereas it reduces for M , Nt , We , n , St and Sc . Besides, the local density number boosts for the larger values of Le , Nb , Pr and Pe , but it declines for bigger values of M , Nt , We , n , St , Sc and Sm .

5 Conclusion

In the present research, a computational MHD boundary layer flow and heat transport of tangent hyperbolic nanofluid with gyrotactic microorganisms, past a stretching surface is discussed in the presence of stratification.

The summed up major outcomes of present research are highlighted as follows:

- The local density rate of the motile microorganisms enhances with increasing bio-convection Lewis number.
- The density of motile microorganisms decelerates by increasing bio-convection Lewis number, Peclet number and microorganism concentration difference parameter.
- The flow of fluid velocity profile of nanofluid appears as a reducing function of magnetic parameter, power law index and Weissenberg number.
- The temperature field displays declining attitude toward the Prandtl number although it shows notable increment for Brownian motion and thermophoresis parameters.
- Concentration profile reduces against Lewis number and Brownian motion parameter, whereas both thermophoresis number and magnetic parameter boost it.
- Temperature, mass and motile concentration decrease for the bigger values of St , Sc and Sm .
- An increase in Brownian motion parameter, thermophoresis number results in an escalation in the mass transfer rate, however, a reduction in the rate of heat transfer.

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