Research Article

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Analysis of the structure and properties of triangular composite light-screen targets

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Abstract: With the development of light-screen targets for ballistic projectiles, not only must accuracy in the measurement of projectiles be assured but also the installation of the system and error compensation in the initial calibrations must be considered. We have designed a triangular composition light-screen target that is easy to install, derived expressions for the speed and direction of ballistic trajectories based on intersecting-CCD vertical targets, and conducted analyses of the points of impact and projectile speed. Results show that errors in the point of impact are affected by the coordinates of the target plane. Positional errors gradually increase with the distance from the origin. The horizontal angle error with the trajectory line does not exceed 0.3° , whereas the vertical angle error does not exceed 0.2° . Errors in the speed of the projectile remain relatively stable when the vertical coordinates are greater. Errors in speed and direction, as well as error fluctuations, are smaller on the $x = 0$ plane, making it the ideal region for error compensation in initial calibrations.

Keywords: composite light-screen target, point of impact, ballistic trajectory, speed error

1 Introduction

With the rapid development of the arms industry, the precise positioning and testing of weapons and equipment [1] have become an important part of military development, as well as essential links in the process of weapons

development [2], production, and acceptance [3–5]. As such, it has drawn attention from many researchers. Optoelectronic testing technologies are an effective means for testing equipment [6–8]. Due to their advantages of non-contact, high accuracy, and good electromagnetic compatibility, these technologies are often used in ballistic range testing systems [9,10]. Light-screen targets measure the velocity of a bullet using the principles of optoelectronic testing. Due to their simple structures and ease of installation, light-screen targets are not only used in weapons testing systems but are also highly favored in the military training.

In recent years, relevant research have focused mainly on multi-light-screen targets [8,11] in which various technical indicators and data acquisition methods are used to enhance the accuracy in measuring the point of impact and speed. Actual tests in on-site locations are driving the rapid development of light-screen target technologies, for instance, expansion of the testing area from small target planes to large target planes [12,13], reduction in the size of measurable projectiles [14], and generation of optimization technologies for trigger interference signals [15]. Many novel light-screen targets have been developed, including light-screen targets with integrated transmission and receiving functions [16] and intelligent light-screen targets [17]. This diversity in light-screen targets can help to further optimize testing structures and precision.

Intersecting-charge-coupled devices (CCD) vertical target screens [2] are two-dimensional (2D) measurement planes formed from two linear CCD arrays. When a projectile penetrates the measurement plane, it would produce corresponding image points on the CCD arrays. As the primary optical axes of the CCD arrays form fixed angles with the horizontal axis, they allow us to calculate the location of the impact point. Using intersecting-CCD vertical target screens as the base structure, we have designed an easy-to-install and cost-efficient novel composite light-screen target with smaller error magnitudes. This method is a part of early research on the simultaneous measurement of multiple projectiles. We have conducted an error analysis on its structure, providing a theoretical foundation for the development of light-screen target technologies.

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2 Methodology

An intersecting-CCD vertical target screen is placed in the $z = 0$ plane. Two-point light sources, T_1 and S_1 , are placed symmetrically in front of the screen. Two linear CCD arrays are placed on the target plane, axisymmetric about the $x = 0$ plane and forming fixed angles of intersection with the $y = 0$ plane. These arrays receive signals from the light sources at T_1 and S_1 , respectively, to form two spatial planes, the S planes (the red plane) and T planes (the yellow plane), as shown in Figure 1.

When a projectile passes through this composite light screen in any arbitrary direction, for instance, the trajectory in Figure 1 marked in blue, it forms two points of intersection, K_2 and K_1 , on the two planes in front. This forms a trajectory on the intersecting-CCD target screen that passes through M_1 . The intersecting points K_2 and K_1 will respectively project within the T and S planes onto the linear CCD array, with the projected points being M_3 and M_2 . The coordinates of M_1 and the 2D coordinates can be expressed as

$$x = \frac{d \cos\left(\alpha_0 - \arctan\frac{h_1}{f}\right) \sin\left(\beta_0 + \arctan\frac{h_2}{f}\right)}{\sin\left(\alpha_0 + \beta_0 - \arctan\frac{h_1}{f} + \arctan\frac{h_2}{f}\right)} - \frac{f \sin\left(\arctan\frac{h_1}{f}\right)}{\sin\left(\alpha_0 - \arctan\frac{h_1}{f}\right)}, \quad (1)$$

$$y = \frac{d \sin\left(\alpha_0 - \arctan\frac{h_1}{f}\right) \sin\left(\beta_0 + \arctan\frac{h_2}{f}\right)}{\sin\left(\alpha_0 + \beta_0 - \arctan\frac{h_1}{f} + \arctan\frac{h_2}{f}\right)}, \quad (2)$$

where

$$d = d_0 + \left[\frac{\sin\left(\arctan\frac{h_1}{f}\right)}{\sin\left(\alpha_0 - \arctan\frac{h_1}{f}\right)} + \frac{\sin\left(\arctan\frac{h_2}{f}\right)}{\sin\left(\beta_0 + \arctan\frac{h_2}{f}\right)} \right], \quad (3)$$

h_1 and h_2 are the image heights corresponding to the image heights for the two CCD arrays, α_0 and β_0 are the fixed angles of intersection between the main optical axis of the CCDs and the x -axis, d_0 is the central distance of the CCDs, and f is the shared focal length.

The point of impact will be $(x_m, y_m, 0)$, the values of which can be measured by the CCD target screen. The coordinates of light source S_1 are defined as $(x_1, 0, z_1)$. Two arbitrary reference points, $S_2(x_{1-1}, y_{1-1}, 0)$ and $S_3(x_{1-2}, y_{1-2}, 0)$, on the linear CCD array in the S plane are chosen. $M_2(x_3, y_3, 0)$ is the point projected by the light source S_1 onto the linear CCD array through the point K_1 . Similarly, the coordinates of light source T_1 are defined as $(x_2, 0, z_2)$. Two arbitrary reference points, $T_2(x_{2-1}, y_{2-1}, 0)$ and $T_3(x_{2-2}, y_{2-2}, 0)$, on the linear CCD array in the T plane are chosen. $M_3(x_4, y_4, 0)$ is the point projected by the light source T_1 onto the linear CCD array through the point K_2 .

The unit normal vector n_1 for the plane formed by S_1 , M_1 , and M_2 is

$$n_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_3 - x_1 & y_3 & -z_1 \\ x_3 - x_m & y_3 - y_m & 0 \end{vmatrix} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}. \quad (4)$$

The equation of this plane can then be expressed as

Figure 1: Structural schematic of the composite light-screen target.

$$A(x - x_1) + B(y) + C(z - z_1) = 0. \quad (5) \quad \beta = \arccos$$

The unit normal vector n_2 for the plane formed by T_1 , M_1 , and M_3 is

$$n_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_4 - x_2 & y_4 & -z_2 \\ x_4 - x_m & y_4 - y_m & 0 \end{vmatrix} = A'\mathbf{i} + B'\mathbf{j} + C'\mathbf{k}. \quad (6)$$

The equation of this plane can be expressed as

$$A'(x - x_2) + B'(y) + C'(z - z_2) = 0. \quad (7)$$

The intersection of the planes described by Eqs. (5) and (7) will be the ballistic trajectory. The direction vector of this trajectory line will be

$$s = n_1 \times n_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A & B & C \\ A' & B' & C' \end{vmatrix} = A''\mathbf{i} + B''\mathbf{j} + C''\mathbf{k} \quad (8)$$

The parametric equations for the trajectory line can be expressed as

$$\begin{cases} x = x_m + A''t \\ y = y_m + B''t \\ z = C''t. \end{cases} \quad (9)$$

Solving for the parameters of these two normal vectors yields

$$\begin{cases} A = -y_3z_m + z_1(y_3 - y_m) \\ B = -z_1(x_3 - x_m) \\ C = (x_3 - x_1)(y_3 - y_m) - y_3(x_3 - x_m), \end{cases} \quad (10)$$

$$\begin{cases} A' = -y_4z_m + z_2(y_4 - y_m) \\ B' = -z_2(x_4 - x_m) - x_2z_m \\ C' = (x_4 - x_2)(y_4 - y_m) - y_4(x_4 - x_m). \end{cases} \quad (11)$$

Let the linear vector of the trajectory be m, n, p , in which case

$$\begin{cases} m = BC' - B'C \\ n = A'C - AC' \\ p = AB' - A'B. \end{cases} \quad (12)$$

Thus, the horizontal angle of intersection between the trajectory line and the z -axis will be

$$\alpha = \arccos \left(\frac{BC' - B'C}{\sqrt{(BC' - B'C)^2 + (A'C - AC')^2 + (AB' - A'B)^2}} \right). \quad (13)$$

The vertical angle of intersection between the trajectory line and the z -axis will be

$$\times \left(\frac{A'C - AC'}{\sqrt{(BC' - B'C)^2 + (A'C - AC')^2 + (AB' - A'B)^2}} \right). \quad (14)$$

The velocity of the projectile can be derived from Eqs. (13) and (14). Unit normal vectors for the planes S and T can be expressed, respectively, in terms of points S_1, S_2, S_3 , and T_1, T_2, T_3 .

$$n_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_{1-1} - x_1 & y_{1-1} & -z_1 \\ x_{1-2} - x_1 & y_{1-2} & -z_1 \end{vmatrix} = A'''\mathbf{i} + B'''\mathbf{j} + C'''\mathbf{k}, \quad (15)$$

$$n_4 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_{2-1} - x_2 & y_{2-1} & -z_2 \\ x_{2-2} - x_2 & y_{2-2} & -z_2 \end{vmatrix} = A''''\mathbf{i} + B''''\mathbf{j} + C''''\mathbf{k}. \quad (16)$$

The equations of these plans are, respectively:

$$A'''(x - x_1) + B'''(y) + C'''(z - z_1) = 0, \quad (17)$$

$$A''''(x - x_2) + B''''(y) + C''''(z - z_2) = 0. \quad (18)$$

Solving Eqs. (15) and (16) yields

$$\begin{cases} A''' = -y_{1-1}z_1 + y_{1-2}z_1 \\ B''' = -z_1(x_{1-2} - x_1) + z_1(x_{1-1} - x_1) \\ C''' = y_{1-2}(x_{1-1} - x_1) - y_{1-1}(x_{1-2} - x_1), \end{cases} \quad (19)$$

$$\begin{cases} A'''' = -y_{2-1}z_2 + y_{2-2}z_2 \\ B'''' = -z_2(x_{2-2} - x_2) + z_2(x_{2-1} - x_2) \\ C'''' = y_{2-2}(x_{2-1} - x_2) - y_{2-1}(x_{2-2} - x_2). \end{cases} \quad (20)$$

Substituting Eq. (9) into Eq. (17) yields

$$A'''(x_m + A''t_2 - x_1) + B'''(y_m + B''t_2) + C'''(C''t_2 - z_1) = 0. \quad (21)$$

We can determine that the time taken for the projectile to travel from K_2 to M_1 is

$$t_2 = \frac{A'''(x_1 - x_m) - B'''y_m + C'''z_1}{A'''A'' + B'''B'' + C'''C''}. \quad (22)$$

Similarly, substituting eq. (9) into eq. (18) yields

$$A''''(x_m + A''t_1 - x_2) + B''''(y_m + B''t_1) + C''''(C''t_1 - z_2) = 0. \quad (23)$$

We can determine that the time taken for the projectile to travel from K_1 to M_1 is

$$t_1 = \frac{A''''(x_2 - x_m) - B''''y_m + C''''z_1}{A''''A'' + B''''B'' + C''''C''}. \quad (24)$$

Figure 2: Influence of impact point coordinates on horizontal coordinate errors of the point of impact.

We can easily use Eqs. (9), (22), and (24) to express the spatial coordinates of K_2 and K_1 as $K_2(x_{K2}, y_{K2}, z_{K2})$ and $K_1(x_{K1}, y_{K1}, z_{K1})$, respectively. The magnitude of the velocity of the projectile can then be expressed as

$$v = \frac{1}{2} \left(\frac{\sqrt{(x_{K2} - x_m)^2 + (y_{K2} - y_m)^2 + z_{K2}^2}}{t_2} + \frac{\sqrt{(x_{K1} - x_m)^2 + (y_{K1} - y_m)^2 + z_{K1}^2}}{t_1} \right). \quad (25)$$

Here the average value of the two distances, is taken. Alternatively, one can first compare the point distances between K_1M_1 and K_2M_1 and solve for velocity using the larger values. Both methods of solution aim to enhance the accuracy of the result.

3 Experimental procedures

Light screens primarily measure a projectile's point of impact and the speed at which it crosses the plane of the screen. As the point of impact in this system relies directly on the intersecting-CCD vertical target, the system parameters will have a direct impact on the measurement of positions. When the CCDs in the vertical targets are placed 2,000 mm apart, respective front optical lens focal lengths are 14 mm, CCD dimension is 6.4 mm, and baseline angles are 45° , and the horizontal coordinate errors and vertical coordinate errors for the point of impact are affected by the coordinates of the impact point, as shown in Figures 2 and 3.

As can be seen, the position error is smallest at the origin and increases with the value of the coordinates.

Figure 3: Influence of impact point coordinates on vertical coordinate errors of the point of impact.

Figure 4: Horizontal angle error between the trajectory line and the z-axis.

The horizontal error increases significantly as the x -coordinate changes, but changes in the y -coordinate can always keep lateral error within 1 mm. Similarly, the vertical error increases significantly as the y -coordinate changes, but changes in the x -coordinate can also keep vertical error within 1 mm. In practical applications, effective detection target surfaces generally have dimensions of $\pm 500 \text{ mm} \times \pm 500 \text{ mm}$. This can keep position errors within a relatively small range and ensure measurement accuracy.

When the placement distance error of the CCDs in the vertical target screen is 1 mm, the focal length error of the front optical lens is 0.14 mm, and the screen angle error is 0.1° , and the horizontal angle error between the trajectory detection direction and the z -axis is as shown in Figure 4.

Horizontal errors within the target plane can be controlled within 0.3° , as shown in Figure 4. Such errors are smaller in regions closer to the $x = 0$ plane, but increase as the horizontal coordinates of the point of impact take on larger values. However, errors are not symmetrically distributed from left to right on the $x = 0$ plane. This is because the outward direction of the projectile is random. Projected points M_3 and M_2 on the T and S planes may not be axisymmetric. By contrast, the vertical coordinates of the point of impact have a smaller influence on the horizontal angle errors. Error fluctuations are usually within 0.05° .

Given the same system parameter values, the vertical angle error between the trajectory line and the z -axis is as shown in Figure 5.

Figure 5: Vertical angle error between the trajectory line and the z-axis.

Figure 6: Influence of the point of impact on the speed error of the projectile.

As shown in Figure 5, vertical angle errors in the target screen generally do not exceed 0.2° , smaller than horizontal angle errors. When y -coordinates are smaller, angle errors can be kept within 0.1° , while x -coordinates have a lesser effect on fluctuations in vertical angle errors. As y -coordinates increase, angle errors tend to increase, while x -coordinates will have a larger impact on vertical angle errors. However, peak error values are in the vicinity of 0.2° and can ensure error precision.

When the same values are set for system parameters, errors in the speed of the projectile similarly vary with changes in the coordinates of the target surface, as shown in Figure 6.

As can be seen, changes in the x -coordinate have a greater impact on errors in the speed of the projectile when the y -coordinate is smaller. The impact of the x -coordinate on errors in the speed of the projectile is relatively stable. Even though the horizontal and vertical coordinates will simultaneously have an impact on errors in projectile speed, combined with the measured projectile direction, we can easily find that the speed and the direction errors in the vicinity of the $x = 0$ plane and are relatively small. Effective measurement target planes are selected symmetrically with the $x = 0$ plane as the axis. This selection method not only makes installation easier and reduces the time needed for calibration, but can also effectively reduce measurement errors.

4 Conclusion

Based on intersecting-CCD vertical target screens, this study has designed a triangular composite light-screen

target and derived the horizontal and vertical angles of intersection between the trajectory line and the z -axis for such light screens. Through parametric equations, we derived an expression for the speed of the projectile. Through error analysis, we discovered that errors in the horizontal and vertical coordinates with respect to the point of impact are affected by the coordinates of impact, with error values increasing with the two orthogonal coordinates. However, x -coordinates have a more significant impact on the horizontal axis error of the point of impact than y -coordinates. Similarly, y -coordinates have a more significant impact on the vertical axis error of the point of impact than x -coordinates. In an effective receiving target screen, points of impact near the origin are more precise. Greater error compensation is required during initial system calibration for regions further from the origin.

Through a simulation analysis of the horizontal and vertical angle errors between the trajectory line and the z -axis, we find that horizontal angle errors can be kept within 0.3° , whereas vertical angle errors generally do not exceed 0.2° . In addition, horizontal coordinates of the point of impact have a more significant influence on the horizontal angle error. Vertical coordinates have a more significant impact on the vertical angle error. Errors in the speed of the projectile are also affected by the orthogonal coordinates. When y -coordinates are larger, fluctuations in speed error are relatively stable. On the whole, errors in speed and direction are smaller in the vicinity of the $x = 0$ plane. This provides some convenience for the initial system calibration.

In future work, a further analysis on this system and building of experimental platforms in the field to carry

out specific analysis by combining simulation data with experimental results, will be conducted. The system with a sound calibration data compensation to make measurements more precise, will be provided.

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